

### Problem set 3 (due Thursday March 12th)

1. In class, we derived an expression for the radial drift speed as a function of the dimensionless stopping time (here, we ignore the gas velocity),

$$v_r = -\frac{\eta v_K}{\tau_{\text{fric}} + \tau_{\text{fric}}^{-1}},$$

where  $\eta$  describes the deviation from Keplerian velocity of the gas. We also showed (or, at least, asserted) that for  $\tau > \alpha$  the thickness of the equilibrium particle layer is,

$$\frac{h_d}{h} \simeq \sqrt{\frac{\alpha}{\tau}}.$$

Suppose that the relative velocities between particles are determined by differential radial drift, and that a “typical” collision is between a particle with some stopping time  $\tau$ , and a smaller particle with  $\tau/2$ . If we were to assume that all collisions lead to coagulation, is it generally true that there are *enough collisions* for particles to grow faster than they drift radially? In other words, is the “meter-sized barrier” to planetesimal formation purely a matter of drift time scale, or is there implicitly an assumption about collision outcomes being inefficient?

2. Consider a disk whose temperature profile, given purely by stellar irradiation, can be written as,

$$T(r) = 150 \left( \frac{r}{2.7 \text{ AU}} \right)^{-1/2}.$$

Suppose that the opacity at low temperatures is given by (Bell & Lin 1994),

$$\kappa = 2 \times 10^{-4} T^2 \text{ cm}^2 \text{ g}^{-1}.$$

If the surface density of gas at the snow line is  $500 \text{ g cm}^{-2}$ , estimate the minimum accretion rate above which heating by accretion would alter the central temperature by a non-negligible amount (say 10%).

3. Consider particles with stopping time  $\tau = 0.1$  orbiting at 1 AU within a disk with  $\Sigma = 10^3 (r/1 \text{ AU})^{-1} \text{ g cm}^{-2}$  and  $h/r = 0.03$ . Assume that the particle drag is in the Epstein regime, so that the drag force is linear in the velocity difference between the particle motion and that of the local gas. Determine, either analytically or numerically, how the particle *eccentricity* evolves if it is initially non-zero.