ASTR 5820: Problem Set #1

(due Thursday 29th January)

1. The IAU definition of a Solar System "planet" includes the requirement that a planet be massive enough that gravity forces the body to adopt a "near-spherical" shape. Consider a rocky body that has a mean density ρ and a yield strength P_{yield} (the yield strength can be considered to be the maximum pressure that the rock can resist due to its internal strength before it deforms). By dimensional consideration of the hydrostatic equilibrium equation (or otherwise) determine an expression for the minimum radius of a body that would be expected to be roughly spherical.

   For a density of 3.5 g cm^{-3} and a yield strength of 2×10^9 dynes cm^{-2} what is the minimum radius? Is this consistent with observations of Solar System objects?

2. A giant planet such as Jupiter can derive energy from solar irradiation and from other sources, such as gravitational contraction. Estimate the luminosity that would result from contraction of the planet at a rate dR_J/dt. How fast would Jupiter need to contract for this luminosity source to match that due to Solar irradiation?

3. *Chondrules* are mm-sized inclusions, found within meteorites, that appear to have solidified from a molten state following rapid heating (perhaps from a shock wave). One of the interesting properties of chondrules is that a small fraction (about 5%) are "compound", suggesting that the molten drops collided and stuck together before they solidified. As we will see in this question, this places interesting limits on the density of the chondrule precursors within the disk.

   (a) Assume that chondrules are spheres of radius s and density ρ, and that they have number density (number per cm^3) n and random velocities v. Write an expression for the characteristic collision time t.
   (b) Various arguments based on the chemical properties and mineralogy of chondrules suggest that they cooled on a time scale of the order of 10^4 s. Assuming that s = 1 mm, ρ = 3 g cm^{-3} and that v = 10 cm s^{-1}, use the fact that 5% of chondrules are compound to estimate the volume density (in g cm^{-3}) of chondrules within the disk at the time when they were molten.
   (c) At 1 AU the *gas* surface density is of the order of 10^3 g cm^{-2}. The vertical thickness of the protoplanetary disk at this radius is $h \sim 0.05$ r, and a rough estimate is that the solid to gas ratio is 1%. Use these numbers to estimate the expected volume density of solids at 1 AU.
   (d) By comparing your results from (b) and (c), determine by what factor chondrules need to be concentrated (compared to the mean solid density) to account for the observation of compound chondrules.