ASTR 5120: Problem Set 2

1. A young star of radius R_* and effective temperature T_{eff} is surrounded by a flat and razor-thin disk of gas and dust that extends from infinity to the equator. The disk absorbs all intercepted radiation from the star and re-emits it locally as thermal radiation at temperature $T_{\rm disk}(r)$. By making use of (a) the definition of flux in terms of an integral of intensity over solid angle and (b) the fact that intensity is constant along rays, derive the temperature profile in the disk (which will be of the form $T_{\rm disk}/T_{eff} = f(r, R_*)$.

   [You can assume that the star is a sphere of uniform brightness, and will need to know the standard result that the flux from a surface of brightness B is $F = \pi B$.]

2. The interiors of large planets are (or were) molten, whereas the same is not true of very small bodies. First of all, explain physically why this should be so?

   Let us try and construct a simple model to estimate how large a body needs to get before the interior melts. Suppose that the source of heat is radioactive decay, which results in a release of energy at a rate $\dot{u}$ per unit mass [units erg s^{-1} g^{-1}]. The heat is transported diffusively out of the interior according to the equation,

   $$\mathbf{q} = -\kappa \nabla T$$

   where $\mathbf{q}$ is the heat flux, and κ the thermal conductivity. For a body of mass m, radius r and specific heat capacity c, estimate (or derive exactly, if you prefer!) the equilibrium central temperature T.

   Optional extra: suppose that the source of heat is the decay of the short-lived isotope ^{26}Al, which has a fractional abundance relative to hydrogen of 10^{-10}. The half-life is 0.7 Myr with each decay liberating about 3 MeV of energy. Bearing in mind that asteroids are not usually made of hydrogen estimate $\dot{u}$. Find out reasonable numbers for κ and c and see if the estimate you derived previously works.