ASTR 5120: Problem Set 1

1. In practice, the collisionless Boltzmann equation and Fokker-Planck equations are useful when the system under consideration is symmetric enough to cut down the number of degrees of freedom to something more manageable. Consider a spherically symmetric cluster (perhaps of galaxies) that is in a steady state and in which $\overline{v_r}$ and $\overline{v_\theta}$ vanish. By expressing the collisionless Boltzmann equation in spherical co-ordinates and taking moments (as we did to in the general case) show that the system obeys,
$$\frac{\mathrm{d}(\rho\overline{v_r^2})}{\mathrm{d}r} + \frac{\rho}{r}\left[2\overline{v_r^2} - \left(\overline{v_\theta^2} + \overline{v_\phi^2}\right)\right] = -\rho\frac{\mathrm{d}\Phi}{\mathrm{d}r}.$$

2. Suppose that some event (a lightning strike maybe!) impulsively ionizes a small volume of scale Δr in an otherwise weakly ionized and unmagnetized fluid. Without detailed calculations, write down expressions for the time scale on which the ions and electrons would be expected to diffuse outward. Will the time scale for the ions normally equal that for the electrons? If not, what about charge neutrality?

3. (for discussion Wednesday – no written answer needed) A fraction of Kuiper Belt Objects are known to be binaries. It is not thought that these objects could have formed as binaries, and direct physical collisions seem to be too rare to account for the observed number. How could they have formed via collisionless gravitational processes? If we model the disk as N point masses orbiting the Sun with some mean rotational velocity and specified velocity dispersion, how might we go about calculating the rate at which binaries form?

   [think about this, then read Goldreich, Lithwick & Sari (2002)]