Radiative and Dynamical Processes: Mini-problem 4 (due Wednesday)

An optically thick protoplanetary disk has a column density $\Sigma = \int \rho(z) \mathrm{d}z$ and an effective temperature at the photosphere $T_{\rm eff}$. We will approximate the energetics of the disk by assuming that energy is liberated at the mid-plane, $z = 0$, and transported to the photosphere by radiative diffusion. Making this approximation, write down the (constant) vertical flux through the disk, $F(z)$.

Now consider the radiative diffusion equation,

$$F(z) = -\frac{16\sigma T^3}{3\kappa_R \rho} \frac{\mathrm{d}T}{\mathrm{d}z}.$$

Assuming the opacity, κ_R, is a constant, write down the optical depth τ to the mid-plane, assuming that $\tau \gg 1$. By integrating the radiative diffusion equation, show that the central temperature is related to the surface temperature by an expression of the form $T_c/T_{\rm eff} \sim \tau^{1/4}$.

(Try this especially if you saw this problem before in Origins!) A more realistic form for the opacity in a protoplanetary disk is $\kappa_R = \kappa_0 T^2$, with κ_0 a constant. How does this change the answer?