

ASTR 3830: Problem Set 4

Due in class Friday April 25th

1) Consider a star of radius R_* a distance r away from a supermassive black hole with mass M_{BH} . The star has mean density ρ_* .

(a) Calculate the *difference* in the gravitational force from the black hole at distance r and distance $(r+R_*)$ – this is called the *tidal force*. By equating this to the force that gas on the stellar surface feels due to the *star's own gravity*, show that the tidal force will be able to overcome the star's own self-gravity and rip the star apart if:

$$\rho_* < \frac{CM_{\text{BH}}}{r^3}$$

where C is a numerical factor which you should determine but whose exact value is not too important. You may assume, very reasonably, that $r \gg R_*$.

(b) Hence calculate the minimum distance that a Solar type star could approach the supermassive black hole at the Galactic Center before being tidally destroyed. The mass of the Galactic Center black hole is about 4.5 million Solar masses.

(c) Recalling that a black hole has a Schwarzschild radius given by $2GM_{\text{BH}}/c^2$, find the value of the maximum black hole mass for which tidal disruption of a Solar type star can occur (above this mass the tidal disruption radius is inside the Schwarzschild radius, and the black hole would be able to swallow stars whole).

2) When two galaxies containing supermassive black holes merge, the holes are expected to spiral inward until they eventually form a binary system at the center of the new galaxy. For a binary made up of two black holes of mass m_1 and m_2 , orbiting in a circular orbit with separation a , the rate of inspiral due to gravitational radiation is:

$$\frac{da}{dt} = -\frac{64G^3 m_1 m_2 (m_1 + m_2)}{5c^5 a^3}$$

For a binary with initial separation a_0 , find an expression for the time remaining until merger t_{merge} . Hence, determine the maximum initial separation, in units of their Schwarzschild radius, that a binary made up of million Solar mass black holes can have if it is to merge within 1 Gyr.

3. The Navarro-Frenk-White (NFW) profile is a theoretical model for the distribution of dark matter within a galaxy cluster. The profile can be written as:

$$\rho = \frac{C}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

Here, ρ is the density of dark matter at radius r from the cluster center. C and r_s are both constants.

(a) Plot a graph of $\log \rho$ as a function of $\log (r/r_s)$.

(b) The mass $M(r)$ interior to radius r is given by the usual integral $M(r) = \int_0^r 4\pi r^2 \rho dr$.

Carry out this integral to find an expression for $M(r)$ [Hint: one way to do this is to first simplify things by making the substitution $x \equiv r/r_s$, and then substitute $t = 1 + x$ to put the integral into tractable form...]