Problem Set 4 (due Tuesday April 17th)

(1) This question asks how fast black holes can grow by accretion, under the assumption that the accretion rate is limited by the Eddington limit.

Consider a black hole accreting gas through a thin accretion disk with a constant radiative efficiency $\eta = 0.1$. (i.e. a mass accretion rate $\dot{M}$ produces a luminosity $L = \eta \dot{M} c^2$.) Suppose that as the black hole grows, it *always* accretes gas at a rate such that $L = L_{\rm Edd}$, where $L_{\rm Edd}$ is the Eddington limiting luminosity for the current mass M. Under those conditions, show that the black hole grows in mass according to,

$$M = M(t=0)e^{t/\tau},$$

where $M(t=0)$ is some initial mass and τ is a constant, known as the *Salpeter time*. Calculate the value of τ in years.

A quasar is observed at high redshift, at a time when the universe was only 500 Myr old. If the black hole in the quasar is estimated to have a mass of $10^9\ M_\odot$, could it have grown via Eddington limited accretion from a 10 $M_\odot$ seed in the time available?

(2) This question works out the basics of *stellar tidal disruption* by supermassive black holes. We won't cover this in detail in class, but it can be worked out from quite basic Newtonian gravity principles.

A star of radius r_* and mass m_* approaches at distance $d \gg r_*$ a black hole of mass $M_{\rm BH}$. By considering the *difference* in the gravitational force from the black hole at distance d and distance $(d + r_*)$, find an expression for the *tidal force* that would tend to rip the star apart. (In the appropriate limit where $d \gg r_*$, you should be able to simplify this to a form that scales as r_*/d^3.)

By comparing the tidal force due to the black hole to the force of self-gravity that is trying to hold the star together, show that the tidal force will win and the star will be destroyed if,

$$d < \left(\frac{M_{\rm BH}}{m_*}\right)^{1/3} r_*.$$

(Up to numerical constants that aren't too important.) Calculate this *tidal radius* in units of the Schwarzschild radius for a Solar-type star approaching the supermassive black hole in the Galactic Center.