ASTR 3830: Problem Set 2

Due in class Wednesday February 19th

1. For stellar masses above a Solar mass, the Initial Mass Function of stars can be approximated as a power law:

$$\frac{dN}{dM} = kM^{-\alpha}$$

with k and α constants. Recall that the number of stars N with mass M between M_1 and M_2 is given by integrating this expression between appropriate limits:

$$N = \int_{M_1}^{M_2} \frac{dN}{dM} dM$$

(a) Suppose that stars with masses $M_{SN} < M < M_{GRB}$ end their lives as Type II supernovae, while more massive stars with $M > M_{GRB}$ instead explode as gamma-ray bursts (up to arbitrarily high masses). Show that the ratio of the number of supernovae to the number of gamma-ray bursts, N_{SN}/N_{GRB}, is given by (you may assume that α is greater than 1):

$$\frac{N_{SN}}{N_{GRB}} = \left[\left(\frac{M_{SN}}{M_{GRB}}\right)^{1-\alpha} - 1\right]$$

(b) Sensible values for the parameters are M_{SN} = 8 Solar masses, and $\alpha = 2.25$. If the number of supernovae exceeds the number of gamma-ray bursts by a factor of 10, determine the minimum mass for a star that ends its life as a gamma-ray burst.

2. The area inside the Einstein ring is given by $A = \pi d_L^2 \theta_E^2$, where d_L is the distance to the lens and θ_E is the angular size of the Einstein ring. Assume that the distance to the **sources**, d_S, is fixed, and write $d_L = x d_S$:

Sketch how A varies with x for $0 < x < 1$ (don't forget that θ_E depends on both d_L and d_{LS}).

By differentiating and setting the resultant expression to zero, find the value of x that maximizes the physical area inside the Einstein ring.

(this proves one of the most important results for the study of gravitational lensing, which we'll return to later in the semester)

3. Assume that early in the history of the Milky Way, a short burst of star formation formed a population of stars in the halo with a Salpeter mass function,

$$\xi(M) = \xi_0 M^{-2.35},$$

with a lower mass cutoff at 1 Solar mass. By now, all these stars have died. If every star with a mass less than 8 Solar masses leaves a 0.6 Solar mass white dwarf, compute the **fraction** of the total mass that was originally in stars that is now locked up in white dwarfs.

(hint: to do this, first find the total mass in stars by integrating the IMF. This will depend on ξ_0. Then find the mass that ends up in white dwarfs, remembering that since each low mass star yields the same mass white dwarf, it's the number of such stars rather than their integrated mass that must be computed.)