ASTR 3830: Problem Set 1

Due in class Thursday February 1st

(1) A spiral galaxy has a rotation curve which rises linearly from zero at the center to 200 km/s at 5 kpc from the center. The rotation curve then remains flat at a constant value of 200 km/s out to 15 kpc, beyond which it can't be measured.

(a) Make a plot of M(r), the total mass enclosed within radius r, as a function of radius. Assume that the mass distribution in the galaxy is spherically symmetric. Note: 1 kpc = 3.086×10^{21} cm. Use Solar masses for the y-axis on the plot.

(b) Assume that the form of the rotation curve is dominated by dark matter. Plot the density of the dark matter in g cm^{-3} versus radius.

(c) If this model applied to our own Galaxy, what would be the estimated mass in dark matter interior to the Earth's orbit in the Solar System (i.e. within 1 AU of the Sun)?

(2) For stellar masses above a Solar mass, the Initial Mass Function of stars is a power law:

$$\frac{dN}{dM} = kM^{-\alpha}$$

with k and α constants. Recall that the number of stars N with mass M between M_1 and M_2 is given by integrating this expression between appropriate limits:

$$N = \int_{M_1}^{M_2} \frac{dN}{dM} dM$$

(a) Suppose that stars with masses $M_{SN} < M < M_{GRB}$ end their lives as Type II supernovae, while more massive stars with $M > M_{GRB}$ instead explode as gamma-ray bursts (up to arbitrarily high masses). Show that the ratio of the number of supernovae to the number of gamma-ray bursts, N_{SN}/N_{GRB}, is given by (you may assume that a is greater than 1):

$$\frac{N_{SN}}{N_{GRB}} = \left[\left(\frac{M_{SN}}{M_{GRB}} \right)^{1-\alpha} - 1 \right]$$

(b) Sensible values for the parameters are M_{SN} = 8 Solar masses, and $\alpha = 2.25$. If the number of supernovae exceeds the number of gamma-ray bursts by a factor of 10, determine the minimum mass for a star that ends its life as a gamma-ray burst.

3) A galaxy has a flat rotation curve, $v(r) = v_c$, with v_c a constant, out to some radius R. Interior to R the dominant contribution to the potential is dark matter, with a spherically symmetric distribution. Outside R, the density is zero. Show that the escape velocity from the galaxy for r < R is given by,

$$v_e^2 = 2v_c^2\left(1 + \ln\frac{R}{r}\right)$$