ASTR 3830: Midterm #1

Monday February 24th, 10am, G131

Attempt all questions. No notes or books, but calculators are OK.

(1) The stars in an elliptical galaxy are distributed in a single spheroidal component. What are the three main **stellar** components of a spiral galaxy such as the Milky Way?

Bulge
Disk
Halo

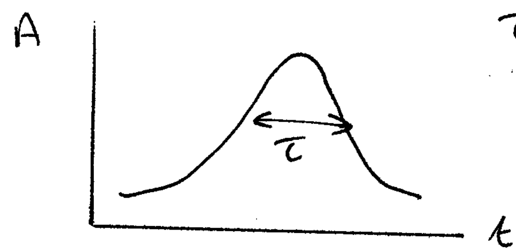
3

(2) What observable parameter of microlensing light curves provides information about the mass of the lens?

The duration

$\tau \propto M^{1/2}$

2



(3) When considering gravitational scattering events between stars in a cluster, what is the definition of the "strong encounter radius" (either a mathematical definition, or an explanation of the concept, is fine).

The distance inside which encounters lead to a large (~1 radian) deflection of the orbit

$$r_{strong} = \frac{2GM}{V^2}$$

2

(4) A very young cluster is made up of stars whose distribution with mass follows a normal ("Salpeter") mass function. Which type of stars are expected to dominate the (a) mass and (b) luminosity output of the cluster?

(a) Mass : low mass stars

(b) Luminosity : high mass stars

2

(5) Briefly explain the physical significance of the **relaxation time** of a stellar cluster.

Relaxation time is the timescale over which star-star gravitational perturbations lead to large angle deflections of the orbit.

2

(6) Explain why a spherical mass distribution with density profile ρ(r) proportional to $1 / r^2$ yields a flat rotation curve.

$$M(r) = \int 4\pi r^2 \rho \, dr = Kr \quad \text{for } \rho \propto 1/r^2,$$

with K = constant.

Rotation gives:

$$\frac{V^2}{r} = \frac{GM}{r^2}$$

3

so $M(r) \propto r$

$\Rightarrow V^2 =$ constant

(7) Consider a stellar population whose initial mass function is:

$$\frac{dN}{dM} = kM^{-3}$$

between 1 Solar mass and 100 Solar masses. If stars with masses above 8 Solar masses end their lives as supernovae, determine the *fraction* of all stars between 1 and 100 Solar masses that go supernova.

Number of stars is given by integrating the IMF between specified limits

$$N = \int_{M_1}^{M_2} kM^{-3}\, dM = -\frac{k}{2}\left[M^{-2}\right]_{M_1}^{M_2} = -\frac{k}{2}\left[M_2^{-2} - M_1^{-1}\right]$$

Fraction:

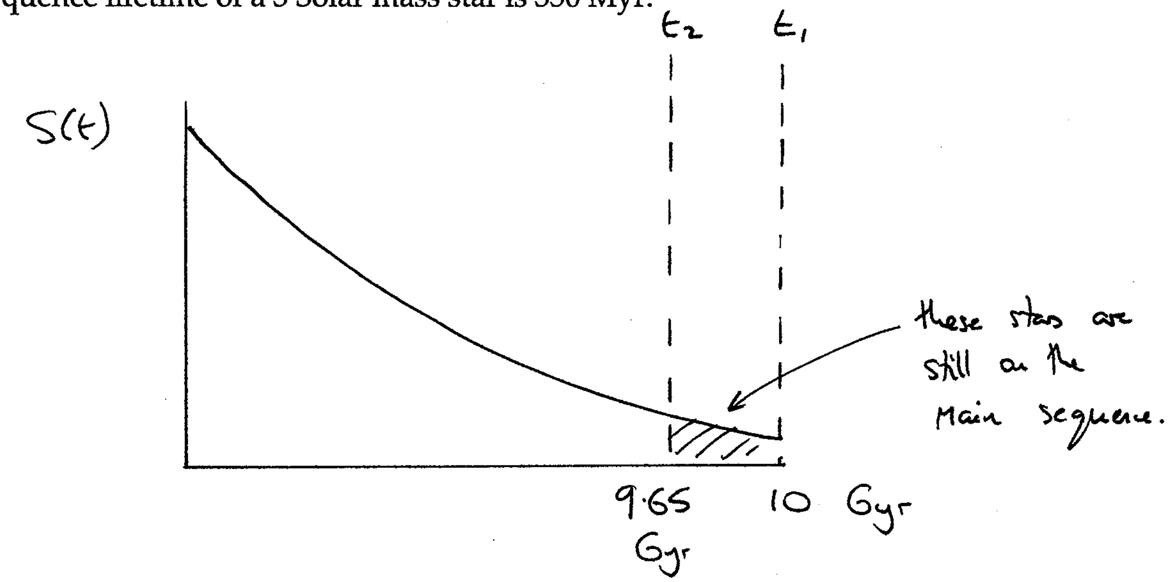
$$f = \frac{\int_8^{100} kM^{-3}\, dM}{\int_1^{100} kM^{-3}\, dM} = \frac{100^{-2} - 8^{-2}}{100^{-2} - 1^{-2}}$$

$$= \underline{\underline{0.0155}}$$

3

(8) Suppose that the rate of star formation in the Galaxy is decreasing with time according to the expression $S(t) = k\exp[-t/t_*]$, where k and t_* are constants and S(t) is the star formation rate in units of Solar masses per year.

If the age of the Galaxy is 10 Gyr, find the fraction of all the 3 Solar mass stars ever made that are still on the main sequence. Assume that $t_* = 3$ Gyr, and that the main sequence lifetime of a 3 Solar mass star is 350 Myr.



Integral of the star formation rate:

$$N = \int_{t_i}^{t_f} S(t)\,dt = \int_{t_i}^{t_f} k e^{-t/t_*}\,dt$$

$$= -kt_* \left[e^{-t/t_*} \right]_{t_i}^{t_f}$$

$$\text{Fraction}^{-1} = \frac{\int_0^{10} S(t)\,dt}{\int_{9.65}^{10} S(t)\,dt} = \frac{e^{-10/3} - 1}{e^{-10/3} - e^{-9.65/3}}$$

3

$$\underline{\underline{\text{Fraction} = 4.6 \times 10^{-3}}}$$

NAME:

ASTR 3830: Midterm #2

Monday April 7th, 10am, G131

This is an open book exam: notes, books and calculators are OK. No consulting the internet please! There are 2 questions, each with 2 parts.

1) A black hole of mass M that accretes spherically symmetrically (rather than from a disk) has an accretion rate that is given by,

$$\dot{M} = 4\pi R_B^2 \rho c_s$$

where ρ is the density of gas far away from the black hole, which has sound speed c_s. The radius R_B, called the Bondi radius, is given by $R_B = GM/c_s^2$.

(a) At time t=0, the black hole has a mass M_0. If the black hole accretes spherically, determine the mass of the black hole as a function of time M(t), assuming that the density and sound speed of the gas that it accretes remain constant.

$$\dot{M} = 4\pi R_B^2 \rho c_s \qquad R_B = \frac{GM}{c_s^2}$$

$$\dot{M} = \frac{4\pi G^2 M^2 \rho}{c_s^3} \qquad \dot{M} = kM^2 \quad \text{with } k = \frac{4\pi G^2 \rho}{c_s^3}$$

$$\int_{M_0}^{M} \frac{dM}{M^2} = k\int_0^t dt'$$

$$\left[-\frac{1}{M}\right]_{M_0}^{M} = kt$$

$$\frac{1}{M_0} - \frac{1}{M} = kt$$

$$\frac{1}{M_0} - kt = \frac{1}{M}$$

$$\frac{1 - kM_0 t}{M_0} = \frac{1}{M} \Rightarrow M(t) = \frac{M_0}{1 - kM_0 t}$$

$$\Rightarrow M(t) = M_0\left(1 - \frac{4\pi G^2 \rho}{c_s^3} M_0 t\right)^{-1}$$

(b) Suppose that the black hole is a Schwarzschild black hole (i.e. with no spin). If the accreting gas has an angular velocity Ω at the Bondi radius, determine the *minimum* Ω needed in order for a rotationally supported gas disk to form outside the last stable orbit. You may assume that angular momentum is conserved as gas falls inside the Bondi radius, and that the angular velocity of a gas disk around a black hole can be approximated by its Newtonian value.

Bondi radius $\quad R_B = \frac{GM}{c_s^2}$

Specific angular momentum $\quad l = vr = \Omega r^2$

$$l_B = \left(\frac{GM}{c_s^2}\right)^2 \Omega .$$

@ last stable orbit

$$l_{MS} = (G R_{MS} M)^{1/2} \qquad R_{MS} = \frac{6GM}{c^2}$$

$$\therefore l_{MS} = \left(6 \frac{G^2M^2}{c^2}\right)^{1/2}$$

Equating.

$$6^{1/2} \frac{GM}{c} = \frac{G^2M^2}{c_s^4} \Omega$$

$$\Rightarrow \quad \Omega = \frac{6^{1/2} c_s^4}{GMc}$$

Check dimensions $\frac{GM}{c^3}$ is $[t]$ ... so Ω has dimensions $1/[t]$

which is OK!

2) A star cluster around a supermassive black hole of mass M_{BH} has a spherically symmetric density profile,

$$\rho = \rho_0 r^{-2}$$

where ρ_0 is a constant.

(a) Determine the total mass in stars enclosed within radius r, M(r).

$$\rho = \rho_0 r^{-2}$$

$$M = \int_0^r 4\pi r^2 \rho \, dr$$

$$M = \int_0^r 4\pi \rho_0 \, dr$$

$$\underline{\underline{M = 4\pi \rho_0 r}}$$

(b) The black hole's "sphere of influence" within a galaxy of velocity dispersion σ is given by,

$$r_{BH} = \frac{GM_{BH}}{\sigma^2}.$$

Within this radius, the total mass of stars just equals the mass of the black hole (i.e. $M(r_{BH}) = M_{BH}$). If the stars all have masses M_*, determine the expected distance from the black hole of the *nearest* individual star.

Determine the normalization ρ_0.

$$M(r_{BH}) = M_{BH}$$

$$4\pi\rho_0 \frac{GM_{BH}}{\sigma^2} = M_{BH} \Rightarrow \rho_0 = \frac{\sigma^2}{4\pi G}.$$

$\therefore$ Mass enclosed: $M(r) = \frac{4\pi\sigma^2}{4\pi G} \cdot r = \frac{\sigma^2 r}{G}.$

Set this equal to M_*

$$\frac{\sigma^2 r}{G} = M_*$$

$$\underline{\underline{r = \frac{GM_*}{\sigma^2}}}$$

END – don't forget to write your name!