ASTR 3730: Astrophysics 1 – Problem Set #4

Due in class Thursday November 3rd

1) (a) The gravitational binding energy of a star of mass M and radius R is, approximately, $\Omega \sim GM^2 / R$. In the textbook, it is shown that the virial theorem can be written in the form:

$$\bar{P} = -\frac{1}{3}\frac{\Omega}{V}$$

where $\bar{P}$ is the typical pressure. Use this to show that,

$$P \sim \left(\frac{4\pi}{3^4}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

where ρ is a characteristic density.

(b) Show that if the radiation pressure equals the gas pressure, the total pressure is,

$$P = 2\left(\frac{3}{a}\right)^{1/3}\left(\frac{k\rho}{\bar{m}}\right)^{4/3},$$

where $\bar{m}$ is the mean particle mass.

(c) By equating these expressions for the pressure, determine the mass scale at which radiation pressure dominates in a star.

2) A simple model of a star of radius R assumes (not very realistically!) that the density ρ is constant. We further assume that the star is made up of pure hydrogen, obeying the ideal gas law.

(a) Adopting a boundary condition P(R) = 0, solve the equations of stellar structure to get the pressure profile P(r).

(b) Find the temperature profile T(r)

(c) If the nuclear energy generation rate scales with temperature as $\varepsilon \sim T^4$, determine the radius at which ε drops to 10% of its central value.