Extraterrestrial Life: Problem Set #1 Solutions

1) Explain briefly how the terrestrial planets (such as the Earth) differ from the giant planets (such as Jupiter). Describe how these differences are thought to arise as a consequence of the theory of the formation of the Solar System.

The terrestrial planets are low mass planets (the most massive is the Earth) whose composition is predominantly rock. The atmospheres of the terrestrial planets make up a very small fraction of the mass. The giant planets have larger masses (though there is a wide range between Jupiter at the massive end and Uranus / Neptune at the low mass end) and have substantial (many Earth mass) gaseous envelopes. In the Solar System the giant planets all lie in the outer Solar System, exterior to the orbits of the terrestrial planets.

It is thought that these differences arise from the timing of the planet formation process. The cores of the gas giants formed relatively quickly, and became massive enough to capture their massive envelopes before the gas in the protoplanetary disk was lost. The terrestrial planets formed more slowly, and so never gained massive primordial atmospheres. Going further, the more rapid formation of large bodies in the outer Solar System may be due to an increased density of solid materials beyond the snowline – the radius in the protoplanetary disk beyond which it is cold enough for ices to condense as well as rocky materials.

2) It is possible (although unproven by observations so far) that planetary systems may exist around high mass stars as well as around low mass stars. Why are planetary systems around high mass stars unpromising locations to search for life?

This one is simple: stellar lifetime is by far the most important factor. Massive stars have short main sequence lifetimes, so even if they harbor planets it is less likely that there will be enough time for life to arise and evolve before the star ends its life.

3) Suppose that a single supernova explosion results in the ejection of 10 Solar masses of heavy elements (all those elements other than hydrogen and helium) back into space. If the $10^{11}$ stars in the Galaxy have an average heavy element content that is 1% of their mass, how many supernovae must have exploded during the lifetime of the Galaxy to create those heavy elements? [assume that the average stellar mass is 1 Solar mass for the purposes of this question]

The total mass of heavy elements in the Galaxy’s stars is:

$$10^{11} \text{ stars} \times 0.01 \text{ Solar masses / star} = 10^9 \text{ Solar masses}$$

If each supernova yields 10 Solar masses of heavy elements:
Number of supernovae = $10^9$ Solar masses / 10 Solar masses per star = $10^8$ stars

This is about right – the observed rate of supernova in the Galaxy is about 1 per 100 years and the Galaxy is 10 billion ($10^{10}$) years old… so there’s been time for around 100 million supernovae.

4) The distance to the nearest star system, Alpha Centauri, is 4.4 light years. How fast would a spacecraft need to go (in km per second) to reach Alpha Centauri in 1000 years?

Distance = velocity x time

Solving for the velocity needed to travel 4.4 light years in 1000 years we get,

$$v = \frac{d}{t} = \frac{4.4 \text{ ly} \times 9.5 \times 10^{12} \text{ km/ly}}{1000 \text{ yr} \times 3.16 \times 10^7 \text{ s/yr}} = 1300 \text{ km/s}$$

...making use of the number of seconds in a year (3600 s/hr x 24 hr/day x 356.25 days/yr = $3.17 \times 10^7$ s/yr). It’s good to be careful about units when dealing with problems like this – numbers in astronomy are often very large and it’s easy to make mistakes!

5) Suppose that the average density of stars in a region of the Galaxy is 1 star per cubic light year (i.e. a cube, one light year on a side, contains on average one star). Write down an expression for the number of stars within a sphere of radius $r$. If only 1 in a million stars are surrounded by planets that harbor life, how far away would you expect the nearest life-bearing planet to be?

The volume of a sphere of radius $r$ is:

$$V = \frac{4}{3} \pi r^3$$

Suppose that the density of stars (the average number of stars per unit volume) is $n$. The within the sphere the number of stars will be:

$$N = Vn$$

Substituting for $V$ and rearranging to solve for $r$, we find that the radius of a sphere that contains $N$ stars is given by,
\[ N = \frac{4}{3} \pi r^3 n \]
\[ r^3 = \frac{3N}{4\pi n} \]
\[ r = \left( \frac{3N}{4\pi n} \right)^{\frac{1}{3}} \]

If 1 in a million stars harbors life, we have a good shot at finding life within a sphere big enough to contain a million other stars \((N = 10^6 \text{ stars})\). The density is 1 star per cubic light year. Using these numbers,

\[ r = \left( \frac{3 \times 10^6 \text{ stars}}{4\pi \times 1 \text{ star/ly}^3} \right)^{\frac{1}{3}} = 62 \text{ ly} \]

Note that I fudged the numbers to make the question easier – at least near the Sun the density of stars is much less than 1 per cubic light year! But the basic principle is right.