

Exploring a quantum degenerate gas of fermionic atoms

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We predict novel phenomena in the behavior of an ultracold, trapped gas of fermionic atoms. We find that quantum statistics radically changes the collisional properties, spatial profile, and off-resonant light scattering properties of the atomic fermion system, and we suggest how these effects can be observed.

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The experimental ability to cool a dilute gas of atoms to temperatures well below one microKelvin has introduced a new area of research, highlighted by the observation and study of Bose-Einstein condensates (BEC) [1–3]. Interparticle interactions in these quantum degenerate gases are weak and therefore amenable to theoretical treatment; in addition, the interaction strength can be altered experimentally [4,5]. A quantum degenerate trapped gas of fermionic atoms, i.e., atoms with an odd total number of constituent fermions, is relatively unexplored theoretically as well as experimentally. An intriguing possibility is that at sufficiently low temperature the atoms will develop pairwise correlations, analogous to Cooper pairing of electrons, and undergo a phase transition to a superfluid-like state. Theoretical studies have concluded that this new fermionic condensate will only occur at very low temperatures and will be difficult to observe experimentally [6]. We explore a trapped gas of fermionic atoms that is quantum degenerate but uncondensed, a regime that is interesting in its own right.

Experimental techniques similar to those used in trapping and cooling bosonic atoms such as ^{87}Rb , ^{23}Na , or ^7Li can be employed to trap and cool fermionic atoms, with possible experimental candidates including ^{40}K , ^6Li , ^2H , and metastable ^3He . We focus on ^{40}K , which has a large number of trappable spin states, as an example for the effects discussed herein. We consider 10^6 atoms in a cylindrically symmetric harmonic trap with a radial trap frequency $\omega_r = 2\pi \times 400\text{Hz}$ and an axial frequency $\lambda\omega_r$ where $\lambda = 0.1$. The characteristic temperature for quantum degeneracy, the Fermi temperature, is given by $T_F = \frac{\hbar\omega_r}{k_B}(6\lambda N)^{1/3} = 1.6\mu\text{K}$ [7]. (Note that the criteria for quantum degeneracy for fermions is approximately the same as for bosons, since the BEC phase transition occurs at $T_c = \frac{\hbar\omega_r}{k_B}(\lambda N/1.202)^{1/3}$). While there is no

phase transition for fermions at T_F , the behavior of the system changes below T_F where quantum statistics becomes important.

We first examine collisional properties of the trapped, fermion gas. In the usual BEC experiment, the atomic sample is cooled optically then loaded into a magnetic trap. To avoid losses due to spin-exchange collisions, typically only atoms in a single spin state are loaded into the trap. Cooling then proceeds by forced evaporation, i.e., removal of the most energetic atoms and rethermalization of the gas via elastic collisions. For bosonic atoms at these low temperatures the elastic collisions are predominately binary and s -wave in character. For fermionic atoms the dominant contribution must be p -wave since Fermi-Dirac (FD) statistics prohibits s -wave collisions between identical particles. However, the p -wave collision cross-section is suppressed at low temperature due to the centrifugal barrier. The threshold collision energy $E_{th}(l)$ for a given partial wave l to contribute to the scattering can be approximated by the height of the centrifugal barrier,

$$E_{th}(l) = \frac{\hbar^2 l(l+1)}{2mb^2} - \frac{C_6}{b^6}, \quad b^2 = \left(\frac{6C_6 m}{\hbar^2 l(l+1)}\right)^{1/2}. \quad (1)$$

Using the C_6 coefficient for K-K collisions [8] gives a p -wave threshold energy $E_{th}(l=1) = 100\mu\text{K}$. Thus, below $100\mu\text{K}$, the elastic collision rate in a gas of identical ^{40}K atoms will rapidly vanish, unless there exists an accidental p -wave resonance.

Sympathetic cooling [9] can be used to circumvent this obstacle to evaporative cooling. If bosonic atoms as well as fermionic atoms are loaded into a magnetic trap, the bosonic atoms can be cooled evaporatively and the fermionic atoms will be cooled “sympathetically” via (s -wave) elastic collisions with the bosons. A second alternative is to load two species of fermionic atoms into the trap, allowing s -wave elastic collisions between non-identical fermions to rethermalize the gas during evaporative cooling. For the remainder of this paper, we assume one can successfully cool a gas of fermionic atoms to the quantum degenerate regime and that the background gas used for sympathetic cooling can be removed, without significantly disturbing the fermionic gas.

The quantum statistical suppression of s -wave interactions between identical fermions makes an ultracold, trapped gas of fermionic atoms an excellent realization

of an ideal gas. This could be seen experimentally with a measurement of the elastic rethermalization rate [10] of a single component compared to a two-component (bosonic and fermionic atoms, or two species of fermionic atoms) gas. Alternatively, a study of shape excitations of the trapped atom cloud [11] will reveal this effect. For a gas of identical fermions in a harmonic trap, shape oscillations will occur at exact multiples of the characteristic trap frequency and will damp at a rate determined by the elastic collision rate in the cloud. At temperatures below the threshold for p -wave collisions ($\approx 100\mu\text{K}$), which is still well above the temperature of quantum degeneracy, the ratio of the p -wave to s -wave elastic cross-section σ_p/σ_s scales as $(ka)^4$ where k is the atomic wave vector and a is a characteristic range of the potential. For example, a collision energy of $100\mu\text{K}$ and a range $a = 50$ Bohr radii gives $\sigma_p/\sigma_s \approx 10^{-3}$. The excitation lifetimes at low T , therefore, will be limited by the lifetime of atoms in the trap (typically 100's of seconds). For comparison, in a two-component fermion cloud these excitations will have shorter lifetimes (a reasonable estimate is ≈ 100 ms, the lifetime seen for shape oscillations in non-condensed, bosonic atom clouds [11]) and at sufficiently high densities should exhibit a small frequency shift [12] due to the s -wave interactions. Interestingly enough, anomalously low damping rates could be observed in a dilute gas of fermions before they are observed in atomic BEC (due to different physics).

Quantum statistics also changes the collisional properties of multi-component fermionic atom clouds. Atoms in two (or more) spin states could either be loaded initially into a magnetic trap or created by an rf or microwave pulse applied to a cold single-component cloud. At low temperatures ($T < 100\mu\text{K}$) quantum statistics modifies the inelastic collisional loss rates by prohibiting s -wave spin-exchange collisions involving an initial or final state consisting of identical fermions. For example, ^{40}K atoms in the two spin states $|F = 9/2, m_F = 9/2\rangle$ and $|9/2, 7/2\rangle$ are stable against spin exchange at low temperatures (note the inverted ground-state structure shown in Fig. 1). Furthermore, at quantum degenerate temperatures otherwise allowed spin-exchange collisions can be suppressed through final state occupation. For example, an ensemble of ^{40}K atoms in the three spin states $|9/2, 9/2\rangle$, $|9/2, 7/2\rangle$, and $|9/2, 5/2\rangle$ is subject to the following spin-exchange collision: $|9/2, 5/2\rangle + |9/2, 7/2\rangle \rightarrow |9/2, 3/2\rangle + |9/2, 9/2\rangle$. This process will limit the lifetime of the $|9/2, 5/2\rangle$ atoms in the trap. However, for $T/T_F \ll 1$ the Pauli exclusion principle will suppress this m_F changing collision since the final state, $|9/2, 9/2\rangle$ with an appropriate energy and momentum conserving external state, is likely to be occupied. This suppression of spin-exchange could be observed as an increased lifetime for the $|9/2, 5/2\rangle$ atoms in the three-component cloud at quantum degenerate temperatures.

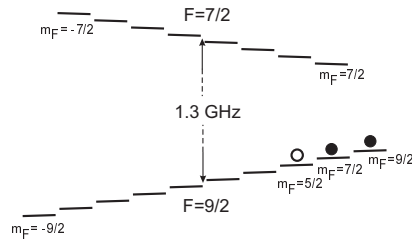


FIG. 1. Schematic of the ground-state hyperfine levels of ^{40}K (shown with exaggerated Zeeman splittings).

We now consider the interaction of atoms with light, which is a powerful tool for probing trapped atomic gases and has been studied theoretically for ultracold bosons, and to a lesser extent, fermions [13–17]. One effect of quantum statistics on light scattering, which has been essential for studying BEC with optical imaging techniques, is simply that the scattered light reflects the spatial profile of the atom cloud, which in a harmonic trap is directly related to the quantum statistical occupation of trap energy levels. For example, light scattering images of trapped, bosonic atoms reveal deviations from the classical gaussian spatial profile even at temperatures slightly above the BEC phase transition [3,18]. The analog of this effect for fermions provides a straight-forward measure of the quantum statistics, which is useful to quantify for comparison to other probes of quantum degeneracy.

A trapped fermion cloud will have a larger spatial extent and a lower peak density relative to Maxwell-Boltzman (MB) particles (see Fig. 2 inset). This effect will be small unless T is well below T_F [7,20,21], but may be revealed in careful studies of the cloud profile as a function of temperature. For example one could observe the momentum distribution by imaging the atoms after a period of free expansion from the trap [1]. The expected two-dimensional expanded cloud image can be calculated from the semiclassical momentum distribution for fermions in a harmonic trap [21], integrated through in one dimension. For sufficiently long expansion times the initial spatial distribution can be ignored. Interactions can also be ignored for a single-component fermion cloud for the reasons discussed previously. A simple measure of the extent to which FD statistics makes the expanded cloud image non-gaussian can be extracted by comparing a gaussian surface fit to the entire image and a fit to only the outer wings of the cloud image. Fitting the full cloud image reveals the enhanced width due to FD statistics (larger average energy) while a fit to the outer edges of the cloud gives a width that more closely

reflects the cloud temperature. In Fig. 2 we plot the ratio of the gaussian widths from these two fits, as a function of reduced temperature T/T_F .

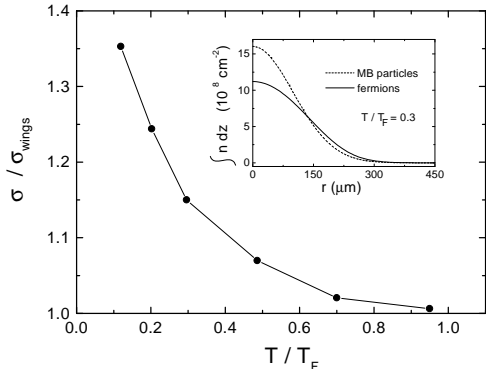


FIG. 2. Ratio of the widths derived from gaussian surface fits to the full expanded fermion cloud image and to only the outer edges of the cloud σ/σ_{wings} vs. T/T_F . The wings were chosen to include approximately one fifth of the total number of atoms. The inset shows traces through the cloud image for FD and MB particles, calculated for ^{40}K , $T/T_F = 0.3$, and a $10 \mu\text{s}$ expansion.

A second effect of atom quantum statistics on light scattering comes from an enhanced or decreased probability for scattering atoms into occupied trap levels. Assuming a two-level atomic system, an atom that scatters a photon receives recoil momentum that projects it onto new harmonic trap levels, which may or may not be already occupied. (Experimentally a two-level system may be approximated by driving the atomic cycling transition with circularly polarized light whose quantization axis coincides with the direction of the trap magnetic field.) For bosons an enhanced probability of scattering into occupied final states is predicted to give an extremely broad line width [15]. For fermions, the Pauli exclusion principle does not allow scattering into occupied final states resulting in a blocking effect [16,17,19] which implies a narrowing of the linewidth.

We explore this effect by calculating the scattering rate of weak, off-resonant, monochromatic light from an optically thin cloud of fermionic atoms. This subject has also been treated by Javanainen and Ruostekoski [17] who look at the spectrum of light scattered from a spatially homogeneous gas. We focus here instead on the angular distribution of scattered light, and treat a harmonically confined gas. We start with the spectral density function (Eq. 47 in [17]) describing the scattered light intensity at \mathbf{r} with frequency $\omega_L + \omega$, where ω_L is the frequency of the probe laser,

$$\mathbf{S}(\mathbf{r}, \omega) = \frac{1}{2\pi} I(r) \frac{R^2}{\delta^2} \mathbf{M}(\hat{\mathbf{e}}, \hat{\mathbf{n}}) \int dt \int d^3r_1$$

$$\times \int d^3r_2 \exp^{i\omega t + i\Delta\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2)} \times \langle \psi^\dagger(\mathbf{r}_1, 0) \psi(\mathbf{r}_1, 0) \psi^\dagger(\mathbf{r}_2, t) \psi(\mathbf{r}_2, t) \rangle, \quad (2)$$

where $\hbar\Delta\mathbf{k}$ is the recoil momentum of the atom, $\frac{1}{2\pi} \frac{R^2}{\delta^2} I(r) \mathbf{M}(\hat{\mathbf{e}}, \hat{\mathbf{n}})$ is the scattered intensity from a single free atom, and ψ denotes the atomic ground-state field operator. In Eqn. 2, the excited-state field operators have been replaced by an adiabatic solution in terms of the ground-state field operators [17]. We then use the finite temperature version of Wick's theorem [22] to evaluate the expectation value of the four field operators, taken with respect to the atoms' many-body wave function. We use the unperturbed Green's function for fermionic atoms confined in a harmonic trap, and ignore coherent scattering which is relevant only for very small scattering angles and does not depend on the atom quantum statistics. Taking the ratio of the scattered intensity for FD compared to MB particles gives

$$\frac{\mathbf{S}_{FD}}{\mathbf{S}_{MB}}(\theta, \phi, \omega) = \frac{1}{N} \sum_{\mathbf{i}} \sum_{\mathbf{n}} [1 - f(E_{\mathbf{n}}, T)] f(E_{\mathbf{i}}, T) \times \left| \int d^3p \phi_{\mathbf{n}}^*(\mathbf{p} + \hbar\Delta\mathbf{k}) \phi_{\mathbf{i}}(\mathbf{p}) \right|^2 \delta(E_{\mathbf{n}} - E_{\mathbf{i}} - \hbar\omega). \quad (3)$$

Here $\mathbf{i} = (i_x, i_y, i_z)$ and $\mathbf{n} = (n_x, n_y, n_z)$ denote the initial and final harmonic trap levels of the atom, $f(E, t) = \frac{1}{z \exp \frac{E}{k_B T} + 1}$ is the FD function, and z is the fugacity. The single particle, momentum-space wave function for a harmonically confined atom is denoted by $\phi_{\mathbf{n}}(\mathbf{p})$, and the integral above has an analytic form [23]. Eqn. 3 can also be understood from Fermi's Golden Rule, where one integrates over all possible final states \mathbf{n} and averages over initial states \mathbf{i} . Experimentally one would detect the scattered photons, with the detector position defining the scattering angle (θ, ϕ) relative to the incident probe beam (along \hat{x}) and conservation of momentum determining the atom recoil. For $\lambda_L = 767 \text{ nm}$ light at the ^{40}K cycling transition, the recoil energy $E_{recoil} = \frac{\hbar^2}{2m\lambda_L^2}$ is $21\hbar\omega_r$. The ratio E_{recoil}/E_F , an important parameter in characterizing the light scattering properties, is then 0.25 for our calculations.

Figure 3 shows the angular dependence of the scattered light intensity, integrated over all scattered photon frequencies, for fermions relative to that for MB particles. (For the temperatures we consider the scattering is essentially isotropic in ϕ , reflecting the isotropy of the momentum distribution at thermal equilibrium.) The blocking effect of FD statistics appears as a suppression of the photon scattering rate, predominantly at small angles where the small recoil momentum means that the final states are mostly occupied. This effect could be studied by monitoring the scattering rate at small angle as a function of temperature. Alternatively, one could measure the ratio of the scattered light intensity at a for-

ward angle to that at a backward angle, revealing the enhanced backward scattering at low reduced temperatures [24]. In addition to modifying the angular dependence of the scattered light, FD statistics also suppresses the total scattering rate (see Fig. 3). At $T/T_F = 0.24$, for instance, the lifetime of the atomic excited state grows to 1.7 times the natural atomic transition lifetime. This effect is reminiscent of enhanced atomic excited state lifetimes seen in cavity experiments [25], although in our case the lifetime grows because of a suppression of atom, rather than photon, final states. The spectrum of scattered light, while more challenging to measure, can be calculated straight-forwardly from Eqn. 3. The blocking effect of FD statistics shifts the spectrum to lower frequency as shown in the inset to Fig. 3.

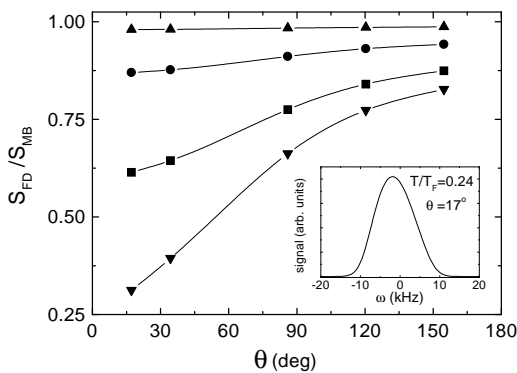


FIG. 3. Angular dependence of scattered light intensity for trapped, fermionic atoms, relative to that for classical particles, shown for $E_{recoil} = 21\hbar\omega_r$, $E_F = 84\hbar\omega_r$, and $T/T_F=1.0$ (up triangles), 0.47 (circles), 0.24 (squares), and 0.10 (down triangles). The error on the calculated points, due to truncating the summation in Eqn. 3, is less than 0.5 %. The inset shows the spectrum of the scattered light for $T/T_F = 0.24$ and $\theta = 17^\circ$.

The large impact of FD quantum statistics on light scattering can be used to probe quantum degeneracy. A large blocking effect requires both small T/T_F (see Fig. 3) and E_{recoil} less than or comparable to E_F . This second condition implies that the experiment is best performed with large N and a tightly confining trap. Another way to enhance the observed effect is to use a focused probe beam with a waist smaller than the cloud size, thus interrogating atoms in lower harmonic oscillator levels preferentially.

In conclusion, we have examined the emergence of quantum statistical effects on the behavior of a trapped gas of fermionic atoms. Collisional properties as well as light scattering exhibit striking changes, such as the suppression of radiative and non-radiative decay processes, and provide convenient probes of quantum degeneracy.

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