



Supporting Online Material for  
**Quantum State Engineering and Precision Metrology Using State-  
Insensitive Light Traps**

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**Supporting Online Material –**  
**Quantum State Engineering and Precision Metrology**  
**Using State-Insensitive Light Traps**

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## **1. Preserving the coherence of light – matter interactions**

Improvement of spectroscopic resolution has been a driving force behind many scientific and technological breakthroughs, including the invention of laser and the realization of ultracold atoms. The recent development of optical frequency combs has greatly facilitated the distribution of optical phase coherence across a wide range of electromagnetic spectrum. Many excellent references on optical frequency combs have appeared, including (2-5). For the state-of-the-art performance in optical phase transfer and comparison, see (6, 7).

To preserve the coherence of light-matter interactions, control of the atomic center-of-mass wavefunction is equally important as for the internal states. Trapped ions enjoy the benefit of deep potentials for tight localization of the center-of-mass wavefunction, while the traps normally do not perturb the internal atomic states used for spectroscopy or quantum information processing (8). For neutral atoms, the realization of state-insensitive optical traps allows many individual atoms be trapped under a condition like an ion trap. Indeed, experiments reported in (9) demonstrate that the level of measurement uncertainties achieved with neutral atom systems can now rival trapped ions. The use of many atoms in neutral systems allows for strong enhancement of the collective signal-to-noise ratio, thereby creating a powerful paradigm to explore precision metrology and quantum measurement and control. Early developments on the magic wavelength optical trap were paralleled in the Caltech group (10) and the Tokyo group (11, 12). For detailed calculations of magic-wavelength for the Sr optical clock, please refer to (13) and (14).

Sr atoms are precooled to  $\mu\text{K}$  temperatures before they are loaded into an array of optical traps, a one-dimensional optical lattice, formed by an optical standing wave with its axis oriented in the vertical direction. The resulting potential difference between neighboring lattice sites removes the degeneracy of the otherwise translation-symmetric lattice. The formation of localized Wannier-Stark states strongly inhibit tunneling between lattice sites, eliminating a potential problem of accuracy for the optical lattice clock (15).

Although both clock states have electronic angular momentum  $J=0$ , the nuclear spin  $I=9/2$  permits ten nuclear spin sublevels, all of which are populated in the ground clock state after cooling. However, a single spin state can be easily achieved by optical pumping. The Stark shifts cannot be completely compensated for all of the magnetic sublevels simultaneously. Or equivalently, the magic wavelength varies slightly for different sublevels. Typically, for the 1D lattice, the laser polarization is linear and coincides with a transverse magnetic field (if it is used to lift the spin degeneracy) to jointly define the quantization axis. Under this configuration, the nuclear spin-dependent vector light shift or the linear Zeeman shift is canceled by averaging the frequencies of a pair of transitions from opposite-signed magnetic sublevels, e.g.,  $m_F = \pm 9/2$  (16-18). The tensor light shift is the same for  $m_F = \pm 9/2$  and its effect is thus absorbed into the scalar polarizability that defines the magic wavelength for the  $\pm 9/2$  spin states.

The typical lattice trap depth is 30-50 photon recoil energy, sufficient to confine atoms in the Lamb-Dicke regime, as the axial trap frequency (tens of kHz) far exceeds the photon recoil (5 kHz), resulting in recoil-free atomic absorptions (19). The typical atomic density

ranges from  $10^{11} \text{ cm}^{-3}$  to  $10^{12} \text{ cm}^{-3}$ . The laser probe is aligned precisely parallel to the lattice axis to avoid transverse excitations and the probe polarization is parallel to that of the lattice laser. The Doppler effect is manifested as modulation sidebands of the unshifted atomic transition (carrier transition) and it is removed completely via resolved-sideband spectroscopy in which the trap frequency is much greater than the narrow linewidth of the clock transition probed by a highly coherent laser. The use of the magic wavelength allows atoms confined in the perturbation-free lattice to preserve the coherence of the  $^1\text{S}_0$  and  $^3\text{P}_0$  superposition for 1 s (20).

## 2. Level Structure for Cavity QED in a FORT

Altogether, there is a nontrivial set of constraints that should be satisfied for a suitable trapping mechanism in cavity QED, including the possibility for efficient cooling of atomic motion. The important benefits from operation at  $\lambda_L$  are clarified from a more detailed examination of the energy level structure for one atom trapped in a cavity in a regime of strong coupling. There is correspondingly a complex interplay of the atom-cavity coupling  $g(\vec{r})$  and the ac-Stark shifts  $U_g(\vec{r})$ ,  $U_e(\vec{r})$  for ground and excited electronic levels ( $g, e$ ). For an atom trapped by a FORT with wavelength  $\lambda_F$ , denote the ac-Stark shifts for the ground and excited levels  $g, e$  by  $\delta_{g,e}(\vec{r}) = U_{g,e}(\vec{r})/\hbar$ . With reference to Fig. 4(a) in (1), assume that the lower manifold  $g$  consists of two levels  $a, b$  (e.g., hyperfine levels) with equal FORT shifts  $\delta_g(\vec{r})$  but with only level  $b$  coupled to the cavity mode via the excited state  $e$ . That is, the atom-cavity coupling  $g(\vec{r})$  refers to the  $b \leftrightarrow e$  transition as in Fig.

4(a), with the  $a \leftrightarrow e$  transition having negligible coupling, which is a good approximation for many experiments.

It is then straightforward to find the position-dependent eigenvalue structure for the atom-cavity system, which consists of a ladder of states with successive rungs  $\dots, n-1, n, n+1, \dots$ , where  $n = 0, 1, 2, \dots$  gives the number of quanta of excitation shared between atom and cavity field (21). The transition frequencies from the ground state with no excitation ( $n = 0$ ) to the first excited manifold with two states and 1 quantum of excitation ( $n = 1$ ) are given by

$$\Delta^\pm(\vec{r}) = \frac{1}{2}(\delta_e(\vec{r}) - \delta_b(\vec{r})) \pm \left[ \frac{1}{4}(\delta_e(\vec{r}) - \delta_b(\vec{r}))^2 + g(\vec{r})^2 \right]^{1/2}, \quad (1)$$

where  $\Delta^\pm(\vec{r})$  is measured relative to the “bare,” free-space atomic resonance absent the FORT (i.e., the actual optical frequencies are  $\omega^\pm(\vec{r}) = \omega_A + \Delta^\pm(\vec{r})$ ). Here, we take  $\omega_A = \omega_C$  and neglect dissipation ( $\gamma, \kappa$ ).

For a conventional FORT,  $\delta_F^b(\vec{r}) < 0$  thereby providing confinement for an atom in its ground state  $b$ . However, for the excited state,  $\delta_e(\vec{r}) \approx -\delta_b(\vec{r}) \equiv \delta_0(\vec{r})$  leading to (10, 22-24)

$$\Delta^\pm(\vec{r}) \approx \delta_0(\vec{r}) \pm \left[ \delta_0(\vec{r})^2 + g(\vec{r})^2 \right]^{1/2}. \quad (2)$$

In general the external trapping potential  $\delta_0(\vec{r})$  and the atom-cavity coupling  $g(\vec{r})$  have quite different form and magnitude, resulting in complex spatial structure for  $\Delta^\pm(\vec{r})$ .

An example of the large variation in  $\Delta^\pm(\vec{r})$  along the cavity axis is given in Figs. 4, 5 of Ref. (24), with excursions in  $\Delta^\pm(\vec{r})$  exceeding even the maximum coupling  $g_0$ . In this case, probe spectra to record the vacuum-Rabi splitting as in Fig. 4(d) of (1) would have a quite different form dominated by the spatial variation in  $\Delta^\pm(\vec{r})$  and not by the coupling-induced interaction  $\pm g(\vec{r})$ . Moreover, measurements that require well-defined values for a probe frequency relative to  $\Delta^\pm(\vec{r})$  (e.g., photon blockade as in Fig. 4(b)) would become much more problematic.

This said, we should stress that the variation in  $\Delta^\pm(\vec{r})$  in a conventional FORT is not without potential benefits. For example, with dissipation  $(\gamma, \kappa)$  incorporated into the analysis, new regimes not found for free-space optical forces arise, including mechanisms for heating and cooling of atomic motion within the setting of cavity QED (24-28). Here, excitation is provided by driving either the cavity (near  $\omega_c$ ) or atom (near  $\omega_A$ ).

By contrast, in a FORT operated with  $\lambda_F$  near a magic wavelength  $\lambda_L$ ,  $\delta_e(\vec{r}) \approx \delta_b(\vec{r}) < 0$ , with then (10, 23, 24)

$$\Delta^\pm(\vec{r}) \approx \pm g(\vec{r}), \quad (3)$$

so that the transition frequencies to the dressed states depend only on the location  $\vec{r}$  of the atom within the cavity mode  $\psi(\vec{r})$  (here, for the  $n=1$  manifold, but also for arbitrary  $n$ ). A probe beam therefore monitors directly the physics associated with the coherent coupling  $g(\vec{r})$  free from the complexity brought by the spatially dependent detuning  $\delta_e(\vec{r}) - \delta_b(\vec{r})$  evidenced in Eq. 2. Admittedly, the atom's equilibrium position  $\vec{r}_0$  is determined by the

structure of the FORT (via  $\delta_{a,b}(\vec{r})$ ), but it is possible to localize the atom such that

$$g(\vec{r}_0) \approx g_0 \quad (29).$$

An important practical advantage of operation at a magic wavelength is that powerful techniques for laser cooling and trapping of neutral atoms in free space can be directly applied to the setting of cavity QED (30). Until very recently (31), strong coupling had been achieved only in Fabry-Perot cavities, which necessarily have limited geometrical access to the mode volume (32) and hence restrictions in the ability to illuminate the atom with external control fields. Having the toolbox of free-space cooling techniques available by way of a FORT at the magic wavelength greatly expands the options for cooling within the constraints imposed by cavity QED.

### 3. Strong coupling in cavity quantum electrodynamics

Strong coupling in cavity QED requires  $g_0 \gg (\gamma, \kappa)$ , where  $2g_0$  is the one-photon Rabi frequency for the oscillatory exchange of one quantum of excitation between atom and cavity field,  $\gamma$  is the atomic decay rate to modes other than the cavity mode, and  $\kappa$  is the decay rate of the cavity mode itself (32). In this circumstance, the number of photons

required to saturate an intracavity atom is  $n_0 \sim \frac{\gamma^2}{g_0^2} \ll 1$ , while the number of atoms

required to have an appreciable effect on the intracavity field is  $N_0 \sim \frac{\kappa\gamma}{g_0^2} \ll 1$ .

For a dipole-allowed atomic transition,  $g_0$  is given by



$$g_0 = \sqrt{\frac{|\vec{\varepsilon} \cdot \vec{\mu}_{ij}|^2 \omega_C}{2\hbar\varepsilon_0 V_m}}, \quad (1)$$

where  $\vec{\mu}_{ij}$  is the transition-dipole moment between atomic states  $i, j$  with transition frequency  $\omega_A$ , and  $\omega_C$  is the resonant frequency of the cavity field with polarization vector  $\vec{\varepsilon}$  and mode volume  $V_m$ . If we denote the spatial dependence of the cavity mode by  $\psi(\vec{r})$ , then the interaction energy  $\hbar g(\vec{r})$  likewise becomes spatially dependent, with  $g(\vec{r}) = g_0 \psi(\vec{r})$  and  $V_m = \int d^3 r |\psi(\vec{r})|^2$ . A photon of energy  $\hbar\omega_C$  in a volume  $V_m$  has an associated electric field  $E_1 \sim (\hbar\omega_C/V_m)^{1/2}$ . Thus for strong coupling, very high- $Q$  cavities ( $Q \geq 10^8$ ) of small volume are required (32).

#### 4. State-insensitive traps for cold molecules

The state-insensitive optical traps can be applied directly to research on cold molecules, which are expected to play increasingly important roles in studies of novel quantum dynamics, precision measurement, and ultracold collisions and chemical reactions. Cold molecules can be created through photoassociation processes using a weak electronic transition. The narrow transition linewidth requires precise and long-duration atom-light interactions. This condition is fulfilled in a state-insensitive trap (33). For example, narrow-line photoassociation near the  $^1S_0 - ^3P_1$  dissociation limit in  $^{88}\text{Sr}$  is an ideal system to test theory - experiment correspondence without the complication of nuclear spins. The wavelength of a state-insensitive lattice trap for the  $^1S_0 - ^3P_1$  transition is  $\sim 914$  nm (19), permitting a recoil- and Doppler-free photoassociation process. The 15 kHz natural width of the molecular line can resolve every vibrational level located near the dissociation limit. The combination of a narrow linewidth least-bound state and its strong coupling to the

scattering state should allow efficient tuning of the ground state scattering length with the optical Feshbach resonance technique. The other important feature of this narrow-line photoassociation is relatively large Franck-Condon overlapping factors between vibrational levels of the excited and ground electronic potentials. This favorable overlap leads to efficient productions of ultracold ground-state molecules confined in a lattice field, which can then serve as a basic system for precision test of possible time-dependent drifts of fundamental physical constants. The scalar nature of the molecular vibrational levels in the electronic ground potential permits a straightforward search for a magic lattice wavelength where the polarizabilities of two particular vibrational levels match, thus facilitating accurate measurements of the vibrational energy intervals in the ground potential. This molecular clock system is particularly suitable for measurement on possible variations of the proton-electron mass ratio. The expected constraint reaches  $1 \times 10^{-15}/\text{year}$  (34), similar to that provided by atomic frequency metrology. However, tests based on molecular vibration frequencies provide more independence from theory models than atomic tests.

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