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Progress on the optical lattice clock

Les progrès accomplis dans le domaine des horloges optiques en réseau

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A B S T R A C T

Optical lattice clocks have made significant leaps forward in recent years, demonstrating the ability to measure time/frequency at unprecedented levels. Here we highlight this progress, with a particular focus on research efforts at NIST and JILA. We discuss advances in frequency instability and the characterization of key systematic effects, with a brief outlook to the future.

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R É S U M É

Les horloges optiques en réseau ont fait des progrès significatifs ces dernières années, en démontrant la possibilité de mesurer le temps et les fréquences à un niveau jamais atteint auparavant. Dans cet article, nous illustrons ces progrès en nous focalisant sur les efforts de recherches au NIST et au JILA. Nous discutons les avancées au niveau de l'instabilité de fréquence et de la caractérisation des effets systématiques clés, et nous donnons un bref aperçu des perspectives futures.

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1. Introduction

Atomic clocks based on optical transitions have long held the potential to measure time and frequency beyond the levels of state-of-the-art primary standards using a radio frequency transition in laser-cooled cesium atoms. Researchers have explored multiple architectures for realizing this type of advanced optical timekeeper. One type of system, the optical lattice clock, is based on large ensembles of ultracold neutral atoms confined in an optical lattice, and exhibits exceedingly high optical transition quality factors [1]. The lattice clock has now been developed for roughly a decade. The large atom number enables measurement with reduced noise from quantum projection of the atomic state. The tight atomic confinement in a specially designed laser potential enables atomic excitation free from Doppler and motional effects, which are pronounced for untrapped atoms. The far-detuned laser potential is operated at the magic wavelength, where light shifts on the electronic states being probed are canceled [2]. After the first proposal of an optical lattice clock [3], early demonstration

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experiments advanced rapidly. Today, scientists around the world have or are constructing dozens of these next-generation timekeepers based on laser-cooled strontium, ytterbium, mercury, and magnesium. Systems based on strontium and ytterbium have been accepted internationally as a secondary representation of the SI second. These clocks now push to uncharted domains of performance in the key figures of merit for optical frequency standards: frequency uncertainty and instability. In so doing, they enable new and more precise atomic clock measurements relevant for tests of fundamental physics and exploration of quantum physics and technology.

State-of-the-art primary frequency standards traditionally use laser-cooled cesium in an atomic fountain. Over the last decade, these systems have improved, reaching a fractional frequency uncertainty and instability at the 10^{-16} level. During the comprehensive evaluation of a strontium lattice clock at JILA in 2008 [4] and of an ytterbium lattice clock at NIST in 2009 [5], frequency uncertainty and instability also reached fractional levels of 10^{-16}. In order to move further, it became clear that several aspects of the optical lattice clock needed additional study and advancement. The measurement capability of these frequency standards, given by the stability, fell far short of the standard quantum limit. They were limited by an important technical noise, known as the Dick effect. Thus a major research effort has been launched at improving the stability of clock lasers—the optical local oscillators. Significant perturbative shifts of the electronic 'clock' states being probed resulted from the blackbody Stark effect and ultracold atomic interactions. Research efforts at JILA and NIST since that time have largely focused on these areas, and here we highlight that work. Improved clock performance and the study of these effects have helped evaluate a range of other physical effects that become important as we push the total clock uncertainty into the 10^{-18} regime [6,7]. Significant progress worldwide has made these timekeepers among the most promising next-generation atomic clocks.

2. Optical lattice clock stability

Like any other atomic clock today, the optical lattice clock exploits narrowband transitions between long-lived electronic states. These long-lived, highly-coherent states can be driven for long durations and can thus discriminate photon frequency very precisely: only a field tuned near-exactly in resonance with the transition frequency can transfer population. With electronic transitions at optical frequencies and with natural line widths at the mHz level, atoms in an optical lattice clock can be superb frequency discriminators. However this discrimination requires one to measure population transfer between quantum states, a process limited by the randomness of quantum measurement. The quantum projection noise of a two-level system sets the fundamental precision of an atomic clock measurement, and thus the fractional stability of any frequency standard using atoms. Fortunately, the optical lattice clock employs ensembles of thousands to millions of atoms, where the quantum projection noise can be averaged down by the square root of this large atom number. By so doing, optical lattice clocks based on strontium and ytterbium might achieve fractional frequency instability of $\leq 10^{-17}$ in a mere second of measurement. However, in practice, until recent years we typically observed instability of 10^{-15} at the timescale of one second [4,5].

A number of effects can compromise performance and prevent reaching the quantum limit. However, one effect is particularly troublesome and relevant to optical atomic clocks: the Dick noise [8]. Excitation of the clock transition is not continuous. Instead it is limited to a finite time, usually given by frequency instability of the laser field driving the excitation (typically ≤ 1 s). Furthermore, consecutive excitations of the clock transition are interrupted by operations such as cooling and trapping of atoms into the optical lattice, as well as atomic state preparation and readout. This periodic interrogation of the clock transition introduces unwanted sensitivity to frequency noise of the laser driving field: noise at harmonics of the measurement cycle frequency is aliased by the periodic atom–laser interaction. This aliased noise compromises the atomic response and thus the stability of the optical clock, and is known as the Dick effect. Even though laser frequency noise is typically reduced by pre-stabilization to high-finesse, ultra-stable optical cavities, the resulting noise levels were sufficient to limit normal lattice clock operation at the 10^{-15} level at one second of measurement.

The most obvious solution to the Dick effect is to reduce the laser frequency noise spectrum, consequently reducing the aliased noise. Such an approach requires enhanced pre-stabilization of the interrogation laser, which is limited by the length stability of the optical cavity to which the laser frequency is stabilized. This limit is typically given by Brownian thermo-mechanical fluctuations of the optical cavity mirrors [9]. In order to minimize the influence of these fluctuations on the fractional length stability of the optical cavity, the cavity length should be large and the mirror substrate material should have low mechanical loss (e.g., fused silica). Enhanced cavity-stabilized lasers were constructed for the NIST Yb lattice clock [10] and the JILA Sr optical lattice clock [11,12]. The reduced frequency noise spectrum enabled the Dick effect to be lowered by roughly one order of magnitude. As an added benefit, these lower-noise laser systems exhibit longer coherence times, allowing the spectroscopic interrogation time of the lattice-trapped atoms to be extended, resolving optical features at or below the Hz level [13,10]. Recent comparisons between two Yb lattice clocks [14] and between two Sr lattice clocks [6] highlight the benefits of the reduced noise. Frequency instability of 3.4×10^{-16} was realized at one second, averaging down as white frequency noise to $>10{,}000$ seconds, approaching an instability of 1×10^{-18} (see Fig. 1). A more recent experiment using 1 s clock interrogation time was able to demonstrate instability of 2.2×10^{-16} at one second [7].

Under such conditions, these clocks are still influenced by the Dick effect, with lingering room for improvement. This is especially true when long atomic interrogation times yield further reductions in the limit set by quantum projection

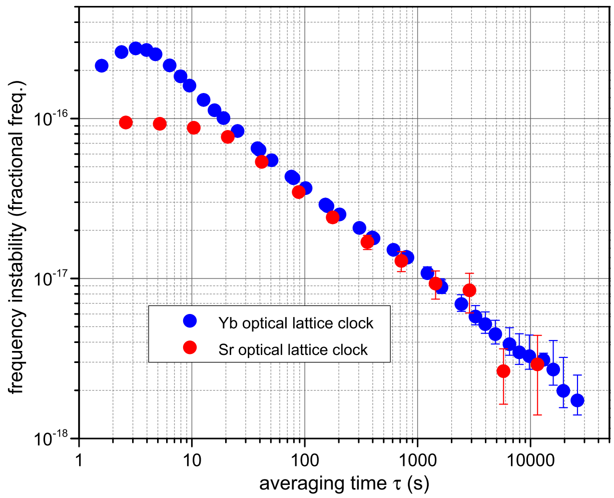

Fig. 1. (Color online.) Frequency instability of the Yb (blue) and Sr (red) optical lattice clocks. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

noise [7]. In order to advance further, one can explore even better cavity-stabilized laser systems. Thermal noise can be reduced with cryogenic cavities (e.g., [15]), newly-developed crystalline optical coatings [16], or cavities with large mode areas to transversely average the thermal noise [10,17]. Combining crystalline optical coatings with cryogenic operation can yield frequency stability at a few parts per 10^{17} level or below. Alternatively, a combination of two atomic systems can be used in a zero-dead-time interleaved fashion to strongly suppress the aliasing problem at the heart of the Dick effect [8,14]. Either way, the optical lattice clock seems destined to achieve frequency instability of $<10^{-16}$ in one second, facilitating a variety of ultra-precise time and frequency measurements on rapid timescales.

3. Understanding important systematic effects

Armed with superior frequency instability, the optical lattice clock can be adjusted to precisely study systematic effects that set the uncertainty at which the natural atomic frequency can be realized or measured. We highlight here two of the most important systematic effects, which have been studied extensively in recent years for both Sr and Yb clocks.

3.1. Blackbody Stark effect

The difference in the polarizabilities of the two atomic clock states gives rise to a host of systematic effects, including the AC Stark shifts due to the lattice trapping field and the clock probe laser, the DC Stark shift, and the blackbody-radiation (BBR) induced shift. The BBR shift arises simply from the integrated spectral response of the atoms to the blackbody radiation field. Although the overall BBR shift can be described in a power series of T^4, T^6, T^8, and so on, where T is the temperature, only the first two terms make significant contributions. The first term is the BBR static shift that scales as T^4, which represents the integration of the total BBR field with an effective DC electric field. Generally speaking, this term accounts for more than 90% of the total BBR shift. To reduce the uncertainty of this term, we need precise knowledge of the DC polarizability for both clock states. The second term, which scales as T^6, arises from the atomic resonance-weighted BBR spectrum for transitions that affect the two clock states.

For both Sr and Yb atoms, DC polarizabilities for the $^1S_0$ and $^3P_0$ clock states have now been precisely measured to sufficiently low uncertainties [18,19]. Hence the task of determining the BBR static frequency shift is reduced to the measurement of the overall BBR intensity experienced by the atoms, given by the thermal environment surrounding the atoms. Two distinct approaches have been developed to measure this environment precisely (see Fig. 2). For the JILA Sr optical lattice clock, two temperature probes are placed inside the vacuum system to radiatively probe the thermal environment [6,7]. The atomic vacuum chamber is placed inside a BBR shield to ensure thermal equilibrium. Even with little knowledge of the emissivity of different vacuum components (e.g., glass windows, metal chamber), a well-characterized and calibrated temperature probe, under excellent thermal equilibrium with minimal thermal gradient inside the chamber, would accurately measure the BBR spectrum corresponding to the frequency shift uncertainty at or below 1×10^{-18}. Furthermore, the second of the two temperature sensors is movable within the vacuum to probe any residual temperature gradients. We have used two types of temperature sensors. Factory-calibrated silicon diodes are extremely linear and have low self-heating, but can experience calibration drifts. Thin-film platinum-resistance-thermometers are long-term stable, but require careful calibration for effects of self-heating, vacuum pressure, axial gradients, and actual operating temperature range. Once calibrated against the NIST International Temperature Scale, these sensors show temperature readout stability and accuracy at the 1-mK level. We use two temperature sensors to directly verify extremely small temperature differences of a few

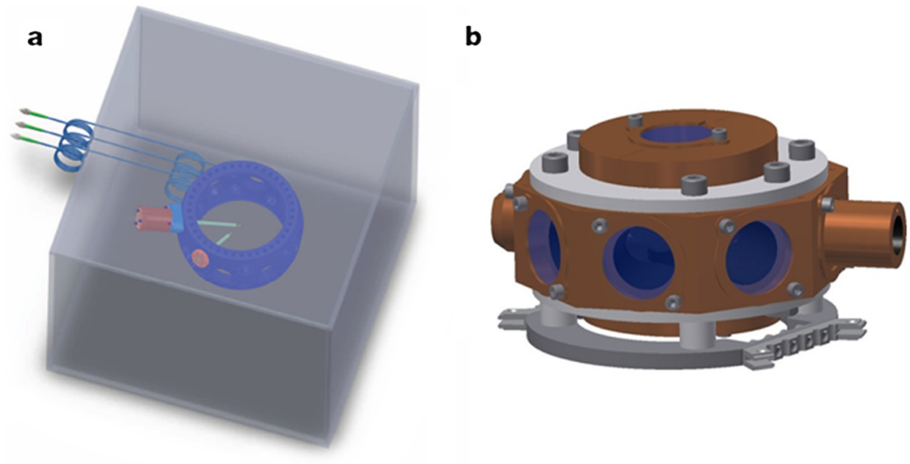

Fig. 2. (Color online.) Measuring the blackbody radiative environment. (a) In the JILA Sr lattice clock, temperature sensors sample the radiative environment inside the vacuum apparatus. An additional BBR shield around the vacuum system helps to maintain temperature uniformity. (b) In the NIST Yb lattice clock, a radiative enclosure (shown) is placed inside the vacuum system (not shown) and surrounds the lattice-trapped atoms.

milliKelvins over a few cm distances inside the atomic vacuum chamber. Using these thermal sensors we can determine the BBR temperature at the location of the atoms to a 5-mK uncertainty, corresponding to a frequency shift uncertainty of 3×10^{-19} for the static BBR effect [7].

In a different approach, Yb lattice-trapped atoms at NIST are surrounded by a radiative enclosure located inside vacuum [20]. The radiation enclosure offers entry and exit of a thermal atomic beam, as well as optical access for laser cooling, trapping in the optical lattice, atomic manipulation and readout. At the same time, it furnishes to the atoms a uniform, well-characterized radiative environment. The enclosure has a high-emissivity carbon-nanotube interior coating, helping to provide a well-known radiative environment by minimizing the influence of radiation entering the enclosure from the outside (through the apertures for the atomic beam). The enclosure is constructed of highly-thermally-conductive materials sitting isolated in vacuum, thus exhibiting exceptional temperature uniformity. The enclosure temperature is monitored in real-time with a series of platinum-wire resistance temperature devices (RTDs). These sensors have been calibrated at 5-mK accuracy, with care to account for parasitic heat flow, vacuum operation, RTD self-heating, and potential post-calibration shifts. In all, the radiative environment is sufficiently well-controlled and measured to yield a static BBR uncertainty of 5.5×10^{-19}. The shield temperature can be varied with a set of heaters, to directly measure the BBR shift temperature dependence. Together with independently measured atomic response coefficients [18,21], this provides a direct validation of the measured thermal environment.

For the dynamic BBR frequency shift, the uncertainty arises mainly from the inaccurate knowledge of the transition strength of $5s5p^3P_0$–$5s4d^3D_1$. This dynamic shift is related to the BBR spectral averaging of the frequency dependence of the differential polarizability between the two clock states. All transitions connecting $^1S_0$ to excited electronic states have their frequencies in the visible range and beyond. For $^3P_0$, only the low lying $^3D_1$ state has sufficiently strong oscillator strength with a transition frequency lying in the BBR spectrum with significant BBR photon occupancy. The transition wavelength is 2.6 μm for Sr and 1.39 μm for Yb. For both Yb and Sr, the $^3P_0$–$^3D_1$ transition dipole matrix element can be accurately determined from the $^3D_1$ lifetime. Thus, both Yb and Sr laboratories have now measured the $^3D_1$ lifetime [21,7] and precise ab-initio calculations also exist [22,23]. This has resulted in a reduction of the uncertainty for the BBR dynamic correction to 1.4×10^{-18} for Sr. Benefiting from a smaller dynamic correction, the corresponding uncertainty in Yb is $<1 \times 10^{-18}$ [20].

It is important to note another possible solution to the required control of the blackbody Stark effect: interrogation of the lattice trapped atoms in a cryogenic environment. This approach has the advantage of reducing the BBR shift through its dominant T^4 dependence, significantly relaxing the needed uncertainty on the differential scalar polarizability and the dynamic correction coefficients. The price for this benefit stems from the additional experimental complexity of cryogenic operation, often including transport of the atomic sample from a room-temperature environment where it is collected and measured, to the cryogenic environment where it is spectroscopically interrogated. Furthermore, shielding from any potential leakage of room temperature BBR background is very important with this approach. Substantial progress has been made both in the design of such a cryogenic apparatus [24] and its implementation [25], achieving a BBR uncertainty of 1×10^{-18}.

3.2. Cold collisions in the optical lattice clock

The principal advantage of the optical lattice clock with many atoms is the enhanced signal to noise ratio for spectroscopy and hence the improved clock stability. However, large atom numbers under tight spatial confinement result in high atomic densities, which could potentially give rise to non-zero collisional frequency shifts via atom–atom contact interactions. With atom–light coherence times reaching beyond one second, even very weak interactions (e.g., fractional energy level shifts of order $\sim 1 \times 10^{-16}$) can dominate the dynamics of these systems. Thus a key systematic effect that must be evaluated is the influence of atomic interactions on the clock transition frequency. This systematic uncertainty connects to many-body physics and is thus different from all other single-atom based effects. An interesting discovery in our push for ever increasing accuracy of the Sr and Yb lattice clocks is the interaction-induced frequency shift on the clock transition even with spin-polarized fermionic atoms prepared under ultralow temperatures.

At ultralow temperatures, atoms collide with a single or very few partial waves. For identical fermionic atoms, anti-symmetrization of the two-particle wave function forbids the s-wave interaction, and the p-wave is suppressed owing to the centrifugal potential arising from an angular momentum of $\hbar$. Higher partial wave collisions are further suppressed. In Boulder we have paid particular attention to the effect of atomic interactions on the clock transition frequency. Initial experiments on Sr with measurement precision of 10^{-15} revealed no atomic density-dependent frequency shift [26]. However, when the measurement precision improved further, we uncovered density-dependent frequency shifts with nuclear spin-polarized fermions for the JILA Sr clock [4] and the NIST Yb clock [5].

We first attributed the clock shifts to s-wave collisions allowed by inhomogeneous optical excitation, which would make the initially indistinguishable fermionic atoms distinguishable, permitting them to interact via s-wave [27–29]. It was considered that the inhomogeneous excitation, first demonstrated at 10% or higher values, would render the s-wave contribution larger than the suppressed p-wave interactions, especially since the Sr atoms are prepared at a temperature of $\sim$1 μK (Yb of $\sim$10 μK) in an array of two-dimensional optical traps. However, using Ramsey spectroscopy, a systematic study of atomic interaction-induced frequency shifts in the NIST Yb optical lattice clock revealed that the p-wave interaction played a dominant role in the frequency shift of the Yb clock transition [30]. Soon afterwards, we discovered evidences of inelastic p-wave interactions for both Sr [31] and Yb [32] atomic clocks. While in experiments of one dimension optical traps both s- and p-waves likely contributed to the observed frequency shifts in the case of Sr [33,34], in more recent measurements, we significantly improved the homogeneity of atom–laser coupling, reaching a 99% or better Rabi excitation ratio [13]. The role of s-wave interaction is thus greatly suppressed. Furthermore, theoretical work [35] has helped to elucidate the many-body nature of the interactions. Because we prepare our atomic sample all in the same internal state at the beginning of the clock probe cycle, the homogeneous p-wave interaction has a collective character that strengthens with the number of atoms in a single trap site, making its role further dominant over the inhomogeneous s-wave contribution, if any remains.

The theory model by Rey et al. is capable of describing full many-body spin–spin interaction dynamics well beyond a simple mean-field treatment. The effective spin degree of freedom relates to the two electronic states used for the optical clock transition in Sr and Yb. A master equation formulation has been developed to incorporate both the real and imaginary parts of the spin interaction, as well as single particle dephasing and loss mechanisms, along with an explicit inclusion of spin quantum noise. Compared to the mean-field approach, the master equation approach has much more accurately captured the spin evolution dynamics of a many-body, open quantum system. By including the nuclear spin degree of freedom in the problem, this formalism can successfully describe a many-body spin system with SU(N) symmetry [36].

The high resolution spectroscopy now possible in lattice clocks makes them well-suited to study these many-body spin interactions. When the spectral resolving power of the clock laser advances to better than 1 Hz, the seemingly weakly interacting spin system actually demonstrates strong correlations with complex excitation spectra, and SU(N) symmetry can now be directly explored to study complex quantum systems with high degeneracy [37–39]. This is an exciting development in addition to the advance of the frontier of the clock stability accuracy.

With the improved understanding of the roles played by atomic interactions in clock frequency shift, we have reconfigured both Sr and Yb systems to push the uncertainty of the atomic density related frequency shift to 1×10^{-18} or below. A cavity-enhanced optical lattice is implemented, allowing a lower atomic density even though the number of atoms interrogated for clock operation remains the same.

3.3. Other systematics

Enhanced clock stability has greatly helped our effort in evaluating systematic effects down to the 10^{-18} level. On the other hand, improved clock design has allowed us to make important advances on all three largest systematics: BBR induced frequency shifts, density-dependent shifts, and AC Stark shifts. The important role of the improved clock laser stability cannot be overstated. Overall, for the JILA Sr clock, a full evaluation reached overall systematic uncertainty of 6.4×10^{-18} [6], surpassing that of single trapped ion clocks for the first time [40]. More recently, the total uncertainty of the JILA Sr clock has been further advanced to 2.1×10^{-18} [7]. A full evaluation of the NIST Yb lattice clock is currently underway.

Atoms are confined in optical lattices with sufficiently deep trapping potentials to eliminate motional effects, including Doppler and recoil, during clock spectroscopy. The idea of a "magic" wavelength optical trap is to induce identical AC Stark energy level shifts for the two clock states, thus making the clock transition frequency unshifted inside the spatially inhomogeneous optical traps. The lattice induced AC Stark shift ranked just behind the BBR and density shifts in its significance for the early round of uncertainty evaluation [4]. For the one dimensional optical lattice geometry, it is advantageous to line up the bias magnetic field, lattice light polarization, and the clock laser polarization all in the same laboratory direction. Along with magnetic field stabilization, this can effectively remove the lattice vector shift by probing two opposite nuclear spin stretched states and taking their averages. The scalar and tensor shift can be grouped together and we can find an effective optical trap wavelength where the differential shift is zero. Typically we measure the lattice AC Stark shift at various optical trap depths and extrapolate the systematic effect at the zero trap intensity [6]. For Sr we have recently been able to operate at such a wavelength where the AC Stark shift is zero across the entire range of lattice depth from 50 to 200 units of photon recoil energy, with uncertainty reaching 1×10^{-18} [7].

Another important effect arises from the magnetic field-induced Zeeman energy shift. Similar to the vector shift of AC Stark effect, the first-order Zeeman shifts are canceled by alternately interrogating opposite nuclear spin stretched states. The second-order Zeeman shifts, caused by the presence of an applied quantization field determined by fast modulation of the bias magnetic field between high and low values. These effects can all be measured now to below 1×10^{-18}.

Other important effects that have been measured at the similar level of uncertainty include the Stark effect induced by residual DC electric fields [41], the AC Stark effect of the clock laser itself, residual Doppler shifts, frequency-pulling effects by neighboring resonances, background gas collisions, and optical phase transients during optical beam switching cycles [6,7].

4. Outlook

As has been demonstrated, the continued advances in the clock stability and clock accuracy go hand in hand. The better clock stability allows much faster evaluations of systematic uncertainties, helping discover previously unknown effects by pushing the measurement precision into new domains. Meanwhile, the relatively short averaging times required to reach a certain level of uncertainty will allow the clock to be used in sensory applications that demand increasingly large bandwidth for monitoring.

Systems with this unprecedentedly coherent matter-field interaction are also ideal stages for exploring quantum metrology where quantum correlations will be harnessed to advance the frontier of measurement precision beyond the standard quantum limit. In fact, Sr and Yb possess some of the best atomic structure for demonstrating spin squeezing for the optical clock application. This development is important not only for the further advances in clock frequency stability due to reduced quantum noise, but also for the preparation of cascaded and distributed quantum systems for the construction of a quantum network of clocks [42] ensuring a global accurate time that is robust against local perturbations.

Whether via a classical network or quantum network, comparisons among accurate atomic clocks play an important role for the demonstration and verification of these state-of-the-art systems. The availability of highly phase stabilized wide-bandwidth optical frequency combs make the inter-comparison of various clocks of different atomic species straightforward. This comparison will allow us to discover unforeseen systematic effects for a particular type of clock and enable rigorous evaluations of clock frequency uncertainties. Once the inter-comparison of different clocks has been accurately established, the size of the clock network can vary to accommodate different types of applications where interesting physical quantities can be measured via frequency metrology across the network.

Finally, frequency ratios of different types of clocks are related to some fundamental constants. Thus an accurate measurement record of a frequency ratio may serve as a highly sensitive search for possible time-dependent or space-dependent variation of these constants (e.g., [43–45]). The obvious motivation of using increasingly more stable and accurate clocks for this purpose lies with our desire of testing fundamental laws of nature ever more stringently than previous generations. Clocks are the classic example as useful scientific instrument for the test of general relativity. At the same time, we are also using clocks to study quantum physics and explore many-body correlations. It appears that clock measurements may also be useful for exploring dark matter (e.g., [46]). The spirit of building a large scale network of clocks in space, where we will push the measurement limits for both space and time, seems promising to extend our knowledge into unknown realms.

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