Chapter 1

Introduction

When we look out into the night sky we see a universe filled with matter. We see moons, planets, stars, galaxies, and even larger structures such as clusters of galaxies, and clusters of clusters, or superclusters, of galaxies. The force that keeps these objects together and the force that appears to rule the Universe on large scales is gravity. The enormous influence of gravity, and the understanding of its properties, effects, and consequences, such as the observed structure of the Universe, are the underlying driving forces behind this research.

As the title suggests, this thesis presents a composite of both analytical and experimental work. This work has been tailored towards the Laser Interferometer Space Antenna (LISA), which is described in Chapter 2. Some motivation for this particular gravitational wave antenna and some background on gravitational radiation are discussed in this chapter. Chapter 3 presents an experiment which demonstrates the LISA phase measurement principle. Chapter 4 presents the bulk of the completed experimental work, which relates to a table-top interferometer that produces realistic LISA-like fringes which can be used for testing LISA technologies, such as phase measurement techniques. Chapter 5 modifies this fringe generator by adding a method of communicating with the laser beam, while maintaining the LISA phase measurement sensitivity. Chapter 6 presents the bulk of the analytical work that has been completed, which relates to an investigation into environmental disturbances of the LISA constellation and their relation to the science output. Chapter 7 summarizes the experimental and theoretical results and presents directions for future related research. The appendices contain supplementary information which the reader may find beneficial.

1.1 Gravitational Astrophysics

We can obtain a large amount of information about our Universe by observing it using electromagnetic radiation. However, electromagnetic radiation, or light, has its limitations. New windows into the Universe allow us to bypass these limitations (although the new windows themselves possess other limitations) and observe the Universe in ways which we have never been able to before. Gravitational radiation is such a window, and it offers unique, complementary, and likely fundamentally new information about the Universe in which we live. In the following paragraphs we summarize some of the advantages observing with gravitational radiation has over electromagnetic radiation.

Electromagnetic radiation is coherently emitted by groups of discrete charged particles. Gravitational radiation (as explained in §1.2.1 below) is emitted by large conglomerations of mass, typically astrophysical binary systems involving massive compact objects. Whereas electromagnetic radiation can easily tell us the composition of the electromagnetically emitting object by means of atomic, molecular, and nuclear transition lines, gravitational radiation can easily tell us the mass of the gravitationally emitting object. While electromagnetic radiation can inform us about the nature of the emitting object, such as its temperature, gravitational radiation can inform us about the nature of the spacetime in which the emitting object resides.

Astronomers use spectral lines in observed light to determine distances to cosmological objects. This is accomplished by observing the same spectral lines in a laboratory setting and determining the "redshift" of the astronomical data. The redshift corresponds to a Doppler shift of the source, indicating that the source is moving away from us. Assigning that motion to the overall Hubble flow of the expanding Universe determines the distance to the object.

The amplitude of gross electromagnetic radiation from a cosmological source, such as a quasar, will be largely dependent on the temperature of the emitting gas. In contrast, the amplitude of gravitational radiation is dependent on the following quantities: inclination of the plane of the binary emitter, a combination of the masses of the binary system, the distance to the source, and the frequency. The frequency of gravitational radiation depends only on the same combination of the masses of the binary system, and on the time until the binary coalesces. Observation of the two polarizations of gravitational radiation (which can be accomplished with two antennae, such as the three arms of LISA) allows one to determine the inclination of the orbit. The frequency of the gravitational radiation determines a combination of the masses of the binary system, which then can be used, along with the rate of change of the frequency, to determine the redshifted luminosity distance to the source. The determination of the distance from gravitational wave data is straightforward and can be accomplished quite readily. This provides a complementary and perhaps more robust determination of the distances to cosmological objects.

Although what we see are many stars in the night sky, galaxies are made up of large collections of dust as well. In fact, dust is a major constituent of galaxies. Light from a distant object, such as a star in our own galaxy or in another galaxy, can be absorbed by intervening gas. This gas is gravitationally bound and is called an absorption nebula. Intervening gas between us and the objects in which we are interested also can re-emit light that it absorbed from some other light source. This is a so-called reflection nebula. Although interesting astrophysical objects in their own right, these nebulae scramble the light emitted by the original objects, making determinations of the astrophysics difficult.

Gravitational radiation also can be absorbed and scattered, but the amount of absorption and scattering is small for almost all known objects in the Universe. In this way gravitational waves travel unimpeded from source to observer. The sources of gravitational radiation themselves can be far back in time, at the edge of the observable Universe, at nearly the beginning of time as we know it.

Objects which are "electromagnetically dim" may be "gravitationally bright." For example, black holes do not emit electromagnetic radiation, but two black holes orbiting each other will emit, and possibly will emit very strong, gravitational radiation. In this way many of the sources we might like to observe with gravitational radiation are very hard, if not impossible, to observe directly with electromagnetic radiation.

One of the aspects of astrophysics we would like to explore using observations of gravitational radiation is the distribution of compact binary systems throughout the galaxy. Aside from a few nearby binary systems, most galactic binary systems having small distances between the constituents — white dwarfs, neutron stars, and even stellar-mass black holes — are very dim and currently are hard to observe by electromagnetic techniques. It is thought that binary systems such as these should be prevalent throughout the galaxy [63, 62]. Constructing a map of such objects would be desirable for several reasons, including determining the history of star formation in our galaxy. Massive black holes residing in the interiors of colliding galaxies will emit gravitational radiation. Signals from such events contain a wealth of information including indications on galaxy formation and evolution. The observation of gravitational radiation from astrophysical objects should yield a large amount of information on the structure and evolution of gravitationally-reacting objects, such as galaxies, in the Universe.

Aside from astrophysical information, gravitational waves are a new window into fundamental physics. Large accelerators have allowed us to probe smaller and smaller scales where quantum effects and short range forces, such as the strong nuclear force, are dominant. These tools have mapped out a good portion of our knowledge of fundamental physics. In particular, experiments using large accelerator-colliders have verified much of the Standard Model of particle physics, e.g., verifying the theories of the electroweak phenomena and quantum chromodynamics.

Our knowledge of fundamental particle physics continues to expand as we venture into the age of neutrino physics and astrophysics with large underground neutrino detectors [64, 65, 87]. Another new venue of discovery is gravitation. The theory of gravitation as surmised by Einstein contains specific predictions on the nature of gravitational radiation. Verifying that gravitational radiation exists, as well as the nature of that gravitational radiation, i.e., how well observed waves fit with theory, are cornerstones of physics and our understanding of the fundamental laws that govern the Universe.

1.2 Gravitational Waves

The Universe appears to be quantum mechanical in nature. That is, all fundamental particles, composite particles, and force mediators, obey the laws of quantum mechanics. Although we currently do not have a self-consistent quantum mechanical theory of gravitation, most researchers believe that gravity is mediated by a quantum mechanical particle, the *graviton*. An important question, which has yet to be definitively answered [106], is "At what speed do gravitons travel?"

If gravitons are massless, then Einstein's theory of special relativity implies that they travel at the speed of light. If gravitons travel faster than the speed of light, then one might ask what does a photon look like to a graviton? This is akin to Einstein's thought experiment that led him to one of his postulates of special relativity: that the speed of light is a universal constant, the same in every inertial reference frame. We therefore have a conundrum if gravity travels faster than light.

If gravitons are massive, they necessarily will travel slower than the speed of light. In fact, the speed of massive gravitons depends on their frequency [105]. Higher frequency gravitons will travel faster than lower frequency gravitons in a well predicted manner. In this way the arrival times of different frequency gravitons from a single source can be used to determine the mass of the graviton. Therefore the observations of gravitational radiation at different frequencies from the same astrophysical source can be used to constrain the propagation speed of gravity.

So long as the propagation speed of gravitation is finite, which it must be so long as we believe information cannot be transmitted instantaneously, there will be gravitons. A collection of gravitons constitutes a gravitational wave, just as a collection of photons constitutes an electromagnetic wave. Gravitational waves are the radiative form of the gravitational field.

1.2.1 On Gravitational Wave Theory

If we modify Newton's instantaneous theory of gravity to be consistent with special relativity, so that the speed of gravitation is finite, then gravitational waves will be manifest. Newton's theory is ideal for simple gravitational wave calculations [77]. The predicted gravitational wave amplitudes and frequencies from "back of the envelope" computations are of the same order of magnitude as those obtained from Einstein's relativistic theory of gravity.

Typically Einstein's theory of gravity is cast from the viewpoint of metric differential geometry. Numerous articles and books [58, 104, 36, 37, 93, 19] describe gravitational radiation using this formalism. The important point to remember is that gravitational waves are perturbations on whatever gravitational background that might be present. Gravitational waves realize themselves as tidal forces. In this way a fixed object will feel a strain when a gravitational wave passes by.

Aside from primordial gravitational waves (briefly discussed in §1.4 below), gravitational waves are produced through the motions of masses. Similar to electromagnetic radiation theory [39], gravitational waves typically are decomposed into multipole harmonics. Conservation of mass-energy prohibits the existence of monopole gravitational waves, just as the conservation of charge prohibits the existence of monopole electromagnetic radiation. Dipole gravitational radiation would violate momentum and/or angular momentum conservation. Therefore the first non-zero multipole harmonic of gravitational radiation is quadrupolar. This is in contrast to electromagnetism where dipolar radiation is the first non-zero harmonic. So gravitational radiation is dependent on the quadrupolar distribution of mass. In particular, the amplitude of gravitational waves will be dependent on the second time derivative of the second (i.e., quadrupolar) moment of the mass distribution.

1.3 Gravitational Wave Amplitudes and Frequencies

The frequency of gravitational radiation will be the frequency of the rate of change of the quadrupolar moment. For binary mass systems in circular orbits this is simply twice (since the radiation is quadrupolar) the orbital frequency,

$$f_{\mathrm{GW}} \approx 2\left(\frac{GM}{R^3}\right)^{1/2} = 1.2\,\mathrm{mHz}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{R}{R_\odot}\right)^{-3/2}. \tag{1.1}$$

Binaries in non-circular orbits may exhibit gravitational radiation at the fundamental orbital frequency as well as higher harmonics.

Of course the amplitude of gravitational radiation depends on the masses involved, but the general scale of such waves is computed quite readily. The amplitude of a gravitational wave is unitless. This reflects the strain nature of the radiation. The quadrupolar moment has units of mass times distance squared. Just like electromagnetic radiation, the field falls off linearly over distance. Using Newton's constant of gravitation, G, and the speed of light, c, we can write the following relation for the gravitational wave amplitude, h,

$$h \propto \frac{G}{c^4}\frac{\ddot{Q}}{r}, \qquad (1.2)$$

where $\ddot{Q}$ is the second time derivative of the quadrupolar moment. We can approximate $\ddot{Q}$ as the mass of the system multiplied by the square of the non-spherical velocity. The non-sphericity is due to the quadrupolar nature of the radiation. If we have a binary system containing two equal masses, M, separated by some distance R, so that $v^2 = GMR^{-1}$, then we can write the following

$$h \propto \frac{G}{c^4}\frac{Mv^2}{r} = \frac{G^2M^2}{c^4 rR} \propto \frac{r_s^2}{rR} = 2\times 10^{-23}\left(\frac{M}{M_\odot}\right)^2\left(\frac{1\,\mathrm{kpc}}{r}\right)\left(\frac{10\,\mathrm{AU}}{R}\right), \qquad (1.3)$$

where r_s is the Schwarschild radius. So we see the strain produced by gravitational radiation is quite small. In contrast, the amount of energy that is being transferred in a gravitational wave turns out to be rather large. Similar estimates show that the gravitational wave luminosity, L, can be given by

$$L \propto \frac{G^4M^5}{4\pi^2c^5R^5} \approx 10^6\,\mathrm{Watt}\left(\frac{M}{M_\odot}\frac{10\,\mathrm{AU}}{R}\right)^5. \qquad (1.4)$$

The fact that gravitational waves carry an enormous amount of energy, but only produce a very small strain indicates that the stiffness of the material through which the waves propagate, namely spacetime, is inherently quite high. A rudimentary calculation reveals that the stiffness of spacetime is $c^4/G \approx 10^{49}$ erg/cm. Over an interaction area of the size of a binary system, say 20 AU, this yields a rigidity on the order of 10^{19} Pa. For comparison, the Young's modulus of diamond is on the order of 10^{12} Pa.

1.4 Primordial Backgrounds

Just as the cosmic microwave background (CMB) radiation is left over from the Hot Big Bang, there should be a primordial background of gravitational waves. These waves will have been produced by quantum mechanical fluctuations in the graviton field. Around the time of the Planck Epoch these fluctuations will have been "frozen-in" and will appear to us today as a background of radiation, just like the CMB. Calculations show that the background from inflation should have a frequency dependent gravitational wave amplitude of [6]

$$h_{\mathrm{rms}}(f) = 5.5\times 10^{-25}\left(\frac{\Omega_{\mathrm{GW}}}{10^{-14}}\right)^{1/2}\left(\frac{1\,\mathrm{mHz}}{f}\right)^{3/2}\left(\frac{H_0}{75\,\mathrm{kms^{-1}Mpc^{-1}}}\right) \qquad (1.5)$$

The number chosen for the fraction of the closure density of the Universe attributable to gravitational waves, Ω_{GW}, is an estimate based on current cosmological assumptions and measurements [51]. This strain value will be undetectable by the first generation of space-based gravitational wave detectors, mainly LISA (see Figure 2.5 for LISA's gravitational wave sensitivity). However, it is possible that a subsequent follow-on mission might be able to detect such a background [12].

It should be mentioned that several other primordial backgrounds that may exist. These come about from such things as phase transitions in the early universe or vibrating cosmic strings. The predicted amplitudes of these backgrounds can vary by many orders of magnitude, and therefore the prospects for detection by LISA are very uncertain. [50, 12]

1.5 Measuring Gravitational Radiation

As mentioned above, gravitational waves realize themselves as tidal forces across objects — in other words, as strains in spacetime. There are several different techniques for measuring gravitational waves, such as resonant bars and other resonant structures or observations of the polarization of the cosmic microwave background radiation, but here we will focus on laser interferometers. An interferometer is an ideal instrument to use for the detection and observation of gravitational radiation. Laser interferometers have measurement sensitivities of small fractions of a wavelength of the laser light used. Measurements of microcycles of one micron light translates into distance measurements of picometers. If this distance change is measured over a large distance, such as one million kilometers, then the measured strain would be 10^{-21}, which just happens to be on the order of the strength of gravitational waves.

The size of an interferometric gravitational wave detector determines the frequencies to which it is sensitive. By examining the difference between the travel time of light down the two arms of an interferometer one determines that if the gravitational wavelength is sufficiently larger than the length of the arms of the interferometer then the gravitational wave transfer function is independent of the frequency of gravitational radiation.

At higher gravitational wave frequencies, the gravitational wave both extends and contracts the arms of the interferometer during the light travel time, effectively canceling out its effect on the strain measurement being performed. In this way the sensitivity to gravitational radiation becomes reduced. The crossover frequency between these regimes, i.e., the $1/e$ point in the transfer function, which in this case rolls-off as f, is at $f_c \approx (c/L)/e$. The interferometer armlengths should be chosen smaller than the wavelength of the gravitational waves being observed for optimum sensitivity.

Even though the transfer function response of an interferometer is relatively flat for frequencies below this cutoff, the gravitational wave sensitivity of an interferometric detector is not. At low frequencies acceleration noise of the end mirrors of the interferometer, begin to become important and reduce the sensitivity to gravitational waves. The displacement uncertainty of acceleration noise is proportional to f^{-2}, so the acceleration noise will be proportional to $f^{-2}L^{-1}$ in a strain measurement. Therefore to minimize the effect of acceleration noise in an interferometer, the armlengths should be chosen to be very large.

We now have two criteria to use in order to determine the size of an interferometric gravitational wave antenna. Equating these criteria at a certain frequency and sensitivity will determine the armlength of the interferometer. Acceleration noise typically is on the order of fm s^{-2} $\mathrm{Hz}^{-1/2}$ [6]. If we wished to observe gravitational waves from a close white dwarf binary system, with a gravitational wave frequency of 1 mHz and a gravitational wave amplitude of 10^{-21}, we would need an interferometer with armlengths of about one million kilometers. Historically, the armlengths of gravitational wave interferometers have been chosen as large as possible based on practical concerns such as cost. The high frequency roll-off due to the transfer function usually is of little concern since it only rolls-off as f.