Appendix B

Electro-Optic Effect

This appendix presents the mathematical background and derivations relating to the electro-optic effect relevant to our experiment. In particular, our experiment utilizes a lithium niobate ($LiNbO_3$) crystal as our electro-optic (EO) device. We apply a voltage across the crystal in a plane perpendicular to the direction of propagation of our laser light. The electro-optic effect creates a phase modulation of the laser beam directly related to the applied voltage. By modulating the applied voltage we can easily transmit information with the laser.

B.1 Electro-Optic Phase Modulation

As is well known in electromagnetic theory, the phase velocity of a propagating wave in a crystal is dependent on the direction of its polarization. The problem is normally set up using the so-called index ellipsoid, given by

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1, \tag{B.1}$$

where n_x, n_y, n_z, are the indices of refraction in the x, y, and z directions respectively. When an electric field is applied to the crystal the indices of refraction change. In general, cross terms will appear in the index ellipsoid equation, signifying that the principal axes of the crystal have changed. We follow the convention in [107] and write the general index ellipsoid as

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1. \tag{B.2}$$

In the case of no applied electric field the cross term indices of refraction, $(n^{-2})_{4-6}$, are zero.

The changes in the indices of refraction are given by

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^{3} r_{ij} E_j \tag{B.3}$$

where r_{ij} are the electro-optic coefficients of the crystal. Lithium niobate has a trigonal crystal structure and is a member of the $3m$ symmetry group. The non-zero electro-optic coefficients of lithium niobate in units of pm/V are $r_{22} = -r_{12} = -r_{61} = 3.4$, $r_{51} = r_{42} = 28$, $r_{13} = r_{23} = 8.6$, and $r_{33} = 30.8$ [107]. With the electric field, E, applied across the z-axis of the crystal we have the following index ellipsoid

$$\left(\frac{1}{n_o^2} + r_{13}E\right) x^2 + \left(\frac{1}{n_o^2} + r_{13}E\right) y^2 + \left(\frac{1}{n_e^2} + r_{33}E\right) z^2 = 1. \tag{B.4}$$

The unstrained ordinary (x- and y-axes) and extraordinary (z-axis) indices of refraction are $n_o = 2.232$ and $n_e = 2.156$ respectively [107]. The new ordinary and extraordinary indices of refraction are, assuming $r_{13}E \ll n_o^{-2}$ and $r_{33} \ll n_e^{-2}$, given by

$$n_{o'} = n_o - \tfrac{1}{2} n_o^3 \, r_{13} \, E \tag{B.5a}$$

$$n_{e'} = n_e - \tfrac{1}{2} n_e^3 \, r_{33} \, E \tag{B.5b}$$

For our setup described in Chapter 5 the incident light is polarized along the z-direction, so that it is purely extraordinary. The phase retardation, Γ, of the light is directly related to the change in the index of refraction of the crystal. Using the above relations we obtain

$$\Gamma = \frac{2\pi}{\lambda}\, \Delta n \, L = \frac{\pi}{\lambda}\, n_e^3 \, r_{33} \, EL = \frac{\pi}{\lambda}\, n_e^3 \, r_{33} \, V_z \, \frac{L}{d} \tag{B.6}$$

where L is the length of the crystal, $\lambda = 1064\,\text{nm}$, and $E = V_z/d$ the applied voltage divided by the thickness of crystal. Our setup has $L = 20\,\text{mm}$ and $d = 5\,\text{mm}$. Using these values $\Gamma = 3.65\, V_z$, with V_z in kilovolts. The voltage then required for a phase-shift of π is $V_\pi = 0.862\,\text{kV}$. The modulation depth is equal to the phase retardation. This result is mentioned near the beginning of §5.1.1.

B.2 Electro-Optic Amplitude Modulation

Placing a polarizer on either side of an EOM will transform the phase modulation into an amplitude modulation of the laser beam. In this section we derive the amplitude modulation for our alignment.

If the laser polarization is along the z-axis of the crystal, which is the same axis as the applied voltage, then no amplitude modulation can occur. We use a square-law detector to observe the laser light. If we did not heterodyne our laser light with another beam then we would not be able to observe the phase modulation directly. Therefore we might convert the phase modulation of the EO crystal into amplitude modulation. This conversion from phase modulation to amplitude modulation will happen if there is any angle difference between the axis of the crystal and the polarization of the light.

In this section we examine the effect of tilting the axis of the EO crystal slightly with respect to the laser polarization. The axes of the EO crystal will be denoted x' and z', and the axes of the laser plane polarization x and z. The propagation of laser light is in the $y = y'$ direction. We start with linear polarized light aligned along the z direction with amplitude A. The angle between the z and z' axes is ϵ and is assumed to be small. Therefore we can write the projections of the incident light in the EO coordinate system as

$$E_{x'} = \epsilon A\, e^{i(\omega t - \omega n_{x'}\, y/c)} \tag{B.7}$$

$$E_{z'} = \left(1 - \frac{1}{2}\epsilon^2\right) A\, e^{i(\omega t - \omega n_{z'}\, y/c)}, \tag{B.8}$$

where $\omega = 2\pi c/\lambda$ is the angular frequency of the light. Passing through the EO crystal (with length $y = L$) creates a phase retardation in both components of the incident light. We time advance our expressions so that we are able write the two components with one phase difference, Φ, between them:

$$E_{x'} = \epsilon A\, e^{i\omega t} e^{i\Phi} \tag{B.9a}$$

$$E_{z'} = (1 - \frac{1}{2}\epsilon^2) A\, e^{i\omega t} \tag{B.9b}$$

$$\Phi = \frac{\omega}{c}(n_{z'} - n_{x'})L. \tag{B.9c}$$

Using the expressions for the indices of refraction in (B.5), the phase difference becomes

$$\Phi = \frac{\pi L}{\lambda}\left[2\,(n_e - n_o) + \frac{V_z}{d}(n_o^3\, r_{13} - n_e^3\, r_{33})\right] = -8.97 \times 10^6 - 2.52\, V_z \equiv 3.59 - 2.52\, V_z. \tag{B.10}$$

The equivalence is mediated by modulo 2π. The voltage required for a relative phase shift of π between the two components is $V_\pi = 178\,\mathrm{V}$

In our experimental setup the light then passes through a quarter-wave plate, reflects off a spherical mirror, passes through the same quarter-wave plate, and then through a polarizing beamsplitter. This process preserves the z-plane polarized light at the deflected port of the PBS. The z-component of light emanating from the EO crystal is

$$E_z = Ae^{i\omega t}\left[1 + \epsilon^2(e^{i\Phi} - 1)\right]. \tag{B.11}$$

The intensity measured at our photoreceiver from just this beam is proportional to $E_z E_z^*$. The ratio of the output intensity to the input intensity is, to second order in ϵ, given by

$$\frac{I_o}{I_i} = 1 + 2\epsilon^2(\cos\Phi - 1) = 1 - 4\epsilon^2 \sin^2\frac{\Phi}{2} \approx 1 - 3.80\,\epsilon^2 - 2.18\,\epsilon^2\ V_z. \tag{B.12}$$

The modulation depth, δ, is the amplitude of the modulation divided by the mean intensity level

$$\delta = \frac{2.18\,\epsilon^2\, V_z}{1 - 3.80\,\epsilon^2} \approx 2.18\,\epsilon^2\, V_z. \tag{B.13}$$

The more general equation, with an arbitrarily large rotation angle, ϵ, of the z' axis of the crystal, is

$$\delta = 0.545\,\frac{1 - \cos 4\epsilon}{1 + \cos 4\epsilon}\, V_z, \tag{B.14}$$

which reduces to (B.13) in the small ϵ limit.

B.3 Experimental Results

The light emanating from the EO crystal is heterodyned with a portion of light from the laser. This light has passed through an AOM and has received the same frequency shift as the light in the EO arm. The electric field of this light is $E_0 = Ae^{i\omega t}$. Using similar equations to (B.9) for the light emanating from the EO crystal we obtain the following electric field at the photodetector

$$\frac{E_{PD}}{E_0} = 1 + \epsilon^2 e^{-i\,\phi_x} + \left(1 - \frac{1}{2}\epsilon^2\right) e^{-i\,\phi_z}. \tag{B.15}$$

where $\phi_x = \omega n_{x'} L/c$ and $\phi_z = \omega n_{z'} L/c = \phi_x - \Phi$. The received power at the photodetector is proportional to $E_{PD}E^*_{PD}$. Using the expressions for the indices of refraction in (B.5), the ratio of the output intensity to the input intensity is, to second order in ϵ, given by

$$\frac{I_o}{2I_i} = 1 + \cos\phi_z + \epsilon^2\left(1 + \cos\phi_z + \cos\phi_x + \cos\Phi\right) \approx 1.25 + 3.53\,V_z + \epsilon^2\left(0.544 + 3.52\,V_z\right). \tag{B.16}$$

The modulation depth is then

$$\delta = 2.84\,\frac{1 + 0.996\,\epsilon^2}{1 + 0.437\,\epsilon^2}\,V_z, \tag{B.17}$$

where again V_z is measured in kilovolts. This is equation we use for the modulation depth mentioned in §5.1.1.

B.4 Modulation Bandwidth

The parallel-plate capacitance of the EO crystal is $C = \epsilon_3 A/d$. We utilize a 5 x 5 x 20 mm crystal so that $A/d = L = 20\,\text{mm}$. The dielectric constants of lithium niobate are $\epsilon_1 = \epsilon_2 = 48$ and $\epsilon_3 = 28$ [107]. The modulation bandwidth, $\Delta\nu$, of the EO crystal is approximated by the equivalent circuit containing the EO as a capacitor and a load resistance, R_L, of $50\,\Omega$:

$$\Delta\nu \sim \frac{1}{2\pi R_L C} \approx 0.64\,\text{GHz}. \tag{B.18}$$

This shows that electro-optic modulators are well-equipped to be used for fast modulation rates as needed for laser data communications.