

**Cone emission from single-filaments**

**Theory and Experiment**

by

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Date \_\_\_\_\_

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Paul, Bradley David (Ph. D.)

Cone emission from single-filaments

Theory and Experiment

Thesis directed by Dr. Alan Gallagher

Observations of conical emission induced by a 5 ns pulsed laser beam will be presented. The incident laser power, blue detuning, and beam diameter are matched to steady state single filament conditions (for which the beam propagates with nearly constant diameter and with a constant, known phase velocity) through 5 cm of strontium (Sr) vapor. As in experiments where the laser beam breaks up into multiple filaments, forward emission at the blue-detuned Rabi side-band is much weaker than at the red (cone) side-band. Part of the spectrally-broad, red-detuned cone emission emerges at angles that appear to be related to the index-of-refraction in the unsaturated medium, but a constant angle occurs for the larger detunings. These emission angles are largely independent of the phase propagation velocity of the self-focused laser beam, indicating that the full radial intensity profile and diffraction must be considered to explain cone emission. The cone intensity versus laser intensity in the filament indicates a threshold at low intensity and incomplete saturation at high intensity. Also a time dependent model which includes diffraction for the propagation of an intense laser pulse through a medium of Doppler broadened two-level atoms is presented. The model employs an expansion of the Maxwell-Bloch equations in the dressed atom frame which is valid to order  $\gamma/\mathcal{R}$ , where  $\gamma$  is the dephasing rate and  $\mathcal{R}$  is the generalized Rabi frequency. Non-saturating side-bands are included to first order in the side-band field, provided the frequency widths of the side-bands are much narrower than their frequency separation from the laser. It is shown that even though Doppler averaging at a detuning much larger than the Doppler width can be insignificant for the propagation at the laser frequency, it significantly modifies the gain profile for the side-bands. Transient laser pulse propagation has shown that there are regions in parameter space for which the pulse will be largely self-focused, but unlike steady state propagation, no stationary filaments (not oscillating in radius) have been found. The implication to cone emission will be discussed.

## **Dedication**

To the reader.

## Acknowledgements

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My writing skills are not the best. I write for a while, then stop and type a decimal point after which I use a capital letter, and occasionally I indent. My wife, Julie, has spent many hours reading through this thesis and cleaning up the English. Thank you Julie. The other people I must thank for editing help are Jinx, Alan, and Michelle Silva.

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