From Cooper Pairs to Molecules: Effective field theories for ultra-cold atomic gases near Feshbach resonances

by

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B.A., Northern Illinois University, 1996

A thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

Department of Physics

2004

This thesis entitled:
From Cooper Pairs to Molecules: Effective field theories for ultra-cold atomic gases near
Feshbach resonances
written by J. N. Milstein
has been approved for the Department of Physics

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Date ____________

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

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Thesis directed by Prof. Murray Holland

Feshbach resonances in dilute atomic gases are a powerful tool used to control the strength of atom-atom interactions. In practice, the tuning is accomplished by varying a magnetic field, affording experiments on dilute atomic gases a knob with which they can arbitrarily adjust the interactions. This precision control makes atomic gases an ideal place to study many-body phenomena. The resonance works by introducing a closed channel containing a bound, molecular state within the open channel of continuum scattering states. The molecular state greatly modifies the scattering responsible for the interactions, as it is tuned near resonance, introducing pair correlations throughout the sample. The size of these correlations may range from either very small, where they appear as molecules, to very large, where they resemble Cooper pairs. This leads to a "crossover" problem of connecting the Bardeen-Cooper-Schrieffer theory of Cooper pairing, which describes conventional superconductors, to the process of Bose-Einstein condensation of molecules. To answer this question, it is necessary to develop an appropriate field theory for both Bosons and Fermions that can account for the Feshbach processes and, therefore, properly describe resonant, ultra-cold atomic gases.

Dedication

This work is dedicated to Jack Shear for getting me to think about the world and to Terry Robinson for not letting me stop.

Acknowledgements

I should express my deepest gratitude to Murray Holland for all his support during the formation of this dissertation work. The environment in which I spent my graduate career could not have been more ideal. The independence which I was granted helped to both nurture and stimulate a variety of interests within me as can be seen in the broad range of topics present within this work.

In fact, an innumerable number of people were instrumental in the development of this dissertation such as the various members of my research group, many of the experimentalists and staff of JILA, as well as all the collaborators I have had the chance to work with on the projects discussed within. I would particulary like to acknowledge the collaboration and friendship I have had with Servaas Kokkelmans and would like to thank Christophe Salomon for hosting me at ENS in Paris where I was fortunate enough to be during the final stages of this work.

If I look back at those who helped me achieve this doctorate I should thank both George Crabtree and Art Fedro for all their encouragement. If it had not been for the kind reception I received as I entered the physical sciences with a degree in the social sciences it is not certain that I would ever have written this work.

This dissertation is dedicated, in part, to my grandfather, Jack Shear, who was not able to see it completed. I'm sure he hardly suspected that all those broken appliances he used to have me fetch from the dumpsters when I was a kid were worth anything, but they were.

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