Chapter 6

Feshbach Resonant Crossover Physics

6.1 Historical background

In the early stages of developing a theory for superconductivity it was suggested that superconductivity was a result of a Bose-condensation of pairs of electrons into localized bound states. This mechanism came to be known as "Schafroth condensation" and was extended upon in the work of Schafroth, Blatt, and Butler in, what they had coined, "quasi-chemical equilibrium theory" [77, 78]. In this theory, they considered the size of the bound pairs to be small compared to the inter-particle spacing. Superconductivity would result as a continuum of Bose-Einstein excitations above a ground state without an energy gap. Due to the complexity of this theory, and the extreme success of the soon to be unveiled theory of Bardeen, Cooper, and Schrieffer (BCS), the quasi-chemical equilibrium theory fell into obscurity. Later, however, it was shown by Blatt and coworkers [79] that their Bose-condensation approach could be extended to give the same results as the BCS pairing theory.

At the time, the differences between these two approaches were highly emphasized and it was thought that superfluidity should be understood in terms of momentum-space pairing and not of real-space pairing. However, the similarities between the two approaches remain so the question may be asked: How does one reconcile the difference between BCS superfluidity of highly-overlapping, long-range fermion pairs and that of Bose-Einstein condensation of non-overlapping, localized pairs. These two cases are the extremes and the problem of how to move from one to the other is known as the "crossover problem" (see Fig. 6.1).

One of the first attempts to understand this crossover was put forth by Eagles in a 1969 paper on pairing in superconducting semiconductors [80]. He proposed moving between these two limits by doping samples, in this case, by decreasing the carrier density in systems of $SrTiO_3$ doped with Zr. In a 1980 paper by Leggett [81], motivated by the early ideas of quasi-chemical equilibrium theory, he modelled the crossover at zero temperature by way of a variational wavefunction:

$$\psi_{BCS} = \prod_k (u_k + v_k a_k^\dagger a_{-k}^\dagger)|0\rangle. \tag{6.1}$$

This wavefunction is simply the BCS wavefunction and assumes that at $T = 0$ all the fermions form into Cooper pairs. What Leggett was able to show was that he could smoothly interpolate between conventional BCS theory and the process of BEC.

In 1985, Nozières and Schmitt-Rink (NSR) extended this theory to finite temperatures in order to calculate the critical temperature T_c [82]. NSR derived the conventional BCS gap and number equation, but introduced into the number equation the self-energy associated with the particle-particle ladder diagram (or scattering T-matrix) to lowest order (see Fig. 6.2). This very influential paper would be built upon by many other groups and was transformed into a functional form by Randeria et *al.* [83].

A compelling reason for understanding the crossover problem comes from the fact that many high-T_c superconductors seem to fall within the region intermediate between BCS and BEC. In the copper oxides, for instance, the coherence length of the Cooper pairs has been measured to be only a few times the lattice spacing. In contrast, in conventional superconductors, the coherence lengths are usually much greater than the lattice spacings. An understanding of the crossover may be one of the keys to understanding and manipulating high-T_c materials. In Chapter 7 we will extend the NSR model of Randeria to account for Feshbach resonant interactions and the formation of molecules. In Chapter 8 we will attempt to go beyond this formalism by adapting a dynamical Green's function approach, originally designed to model high-T_c systems, to the case of Feshbach resonant interactions.

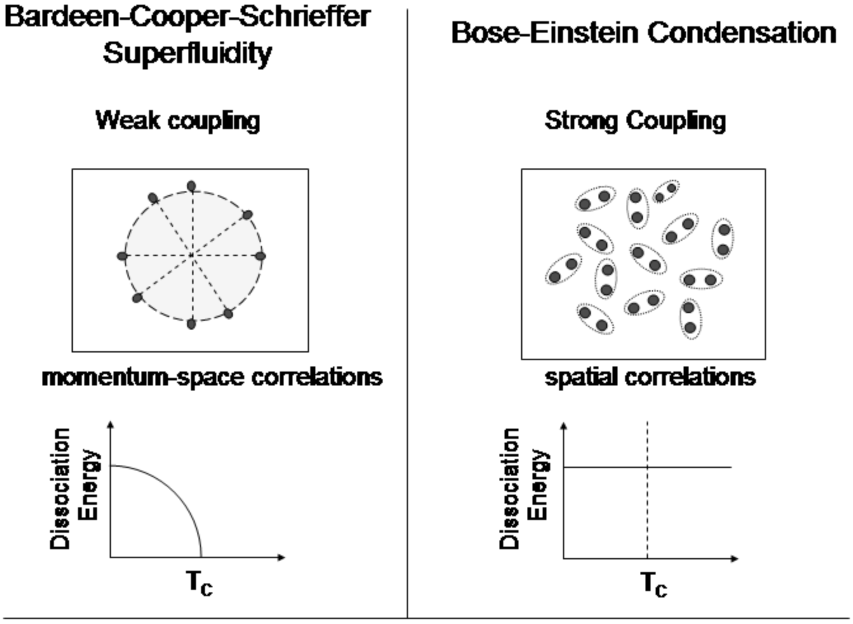

Figure 6.1: An illustration of the differences between Bardeen-Cooper-Schreiffer (BCS) superfluidity and Bose-Einstein condensation (BEC) of molecules in a Fermi gas. BCS theory describes weakly-bound fermions which are paired along the Fermi surface in k-space so may be highly overlapping in real space. The Cooper pairs break up at the critical temperature T_c signifying the phase transition. BEC results from from tightly-bound, real-space fermion pairs. Above the critical temperature T_c, these bosonic pairs loose their mutual phase coherence, but may remain bound into pairs.

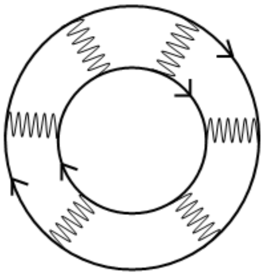

Figure 6.2: Diagrammatic contribution of pair fluctuations to the thermodynamic potential included by NSR. The solid lines are free propagators and the wavy lines are interactions.