Chapter 1

Introduction

1.1 Degenerate atomic gases

Dilute atomic gases provide a fascinating setting in which to study an array of fundamental questions in many-body physics. The precise control with which these gases can be manipulated, combined with a detailed understanding of the atomic interactions, allows for an unprecedented level of comparison between theory and experiment. Since the first observations of Bose-Einstein condensation(BEC)[1, 2, 3], innumerable works have been published on the nature of quantum degenerate gases. With the achievement of degeneracy within a Fermi gas [4], the next big push is toward superfluidity, a contest analogous to the race toward condensation almost a decade ago.

The main ingredient to creating a degenerate atomic gas is to cool an ensemble of atoms until the de Broglie wavelength of the particles, defined as:

$$\lambda_{db} = \hbar/(2Mk_BT)^{1/2}, \tag{1.1}$$

becomes large compared to the average inter-particle spacing. For a system of bosons, cooling leads to a condensate of atoms where a macroscopic fraction occupies a single quantum state. For fermions, cooling will cause the atoms to fill each energy level, allowing for no more than one of each spin state at a given level. The result is a degenerate "Fermi sea" of atoms.

Unfortunately, the onset of quantum degeneracy will often be proceeded by a liquid or solid phase transition. In order to reach the quantum degenerate regime, the timescale for formation of molecules by 3-body collisions must be much longer than the time needed to reach degeneracy. Since equilibrium is reached through binary elastic collisions (a process which leads to cooling of the gas) at a rate proportional to the density $\sim n$, and 3-body inelastic collisions (which result in losses) occur at a rate proportional to $\sim n^2$, a window may open at extremely low densities in which degeneracy may be achieved. However, such low densities also depress the temperature requirement for quantum degeneracy into the nanokelvin range. As an illustration, current BEC experiments reach quantum degeneracy at temperatures between 500 nK and 2 μk at densities between 10^{14} and 10^{15} cm^{-3} [5].

The first observation of Bose-Einstein condensation(BEC) was made in a dilute alkali gas of ^{87}Rb at JILA [1], followed almost immediately by similar reports in ^{23}Na at MIT [2] and ^{7}Li at Rice [3]. This achievement was made possible by the rapid advances that had occurred in the field of laser cooling and trapping over the previous few decades. Not long after, quantum degeneracy was achieved at JILA within a two-component Fermi gas of ^{40}K atoms [4].

Although similar to the methods employed in achieving BEC, the techniques used to reach the degenerate regime within fermions had to be adapted to account for Pauli statistics. A major hurdle results from the fact that identical fermions tend to avoid each other. In this case, s-wave interactions, which dominate the low energy scattering of these gases, would vanish between a single species of fermions. To overcome this difficulty, the JILA group used a mixture of atoms within the two hyperfine levels: $|F = 9/2, m_F = 9/2\rangle$ and $|F = 9/2, m_F = 7/2\rangle$, where $F = 9/2$ is the total atomic spin and m_F is the magnetic quantum number. Runaway evaporation, by which the collision rates increase as the temperature decreases, was achieved by carefully removing equal populations of both spin states as the gas cooled. Another method for increasing elastic collisions is to sympathetically cool the fermions with an auxiliary, bosonic population.

This technique was soon employed to create degenerate mixtures of ^{7}Li and ^{6}Li at ENS [6], Rice [7], and Duke [8], a mixture of ^{6}Li and ^{23}Na at MIT [9], and a ^{40}K and ^{87}Rb mixture at Firenze [10].

1.2 Tunable interactions

One of the more unique qualities of cold atomic gases is the ability to finely tune the interatomic interactions. This control is achieved by making use of low-energy Feshbach resonances which can be precisely tuned by variation of a magnetic field. The observation of a Feshbach resonance within a dilute atomic gas is a relatively recent event, first attained within a Na BEC at MIT [11]. Quite soon after, observations of Feshbach resonances where reported in ^{85}Rb at U.T. Austin[12] and JILA[13], as well as in Cs at Stanford [14]. Feshbach resonances have also been observed in degenerate Fermi systems such as ^{40}K at JILA [15], and ^{6}Li at MIT [16], ENS [17], and Innsbruck [18], to name a few.

In cold alkali collisions, the atom-atom interactions are predominantly determined by the state of the valence electrons. If two atoms involved in a collision form a triplet, where "triplet" refers to the electronic spin configuration of the two valence electrons, the electrons will tend to avoid each other in order to preserve the overall antisymmetry of the wavefunction. This behavior acts to reduce the Coulomb repulsion felt by the two atoms resulting in a somewhat shallower potential well. For a singlet state, however, the atoms may sit upon one another resulting in a much stronger potential. Because of these considerations, singlet potentials are in general much deeper than corresponding triplet potentials.

The singlet and triplet spin states may couple to one another through hyperfine interactions with the spin of the nucleus. If two atoms are scattering within a triplet potential, for instance, the hyperfine coupling may flip the electronic and nuclear spins of one of the atoms bringing the collision into the singlet potential. Another spin flip may then bring the collision back to the triplet potential. For large internuclear separations we may define a quantity Δ which gives the energy shift between the scattering threshold of the singlet and triplet potentials. This relative separation is determined by the Zeeman shifts of the internal states of the two atoms and may, therefore, be adjusted by variation of a magnetic field.

Since the threshold of the singlet potential generally appears above the threshold of the triplet potential it becomes energetically unfavorable for atoms to scatter out of the singlet potential. The singlet is referred to then as a closed channel potential and the triplet as an open channel potential. Of course, for real alkali atoms, the atoms appear as a linear combination of a singlet and triplet states so it is best to talk in terms of open and closed channels rather than singlet and triplet channels.

If a closed channel Q supports a bound state, we may form a Feshbach resonance by tuning this bound state near the threshold of the open channel P (see Fig. 1.1). The energy between the bound state in channel Q and the threshold of channel P is referred to as the detuning ν. Two atoms scattering within the P channel may collide to form a quasi-bound molecular state within the Q channel with a different internal spin arrangement. The molecular state is only considered to be quasi-bound since it remains coupled to the continuum states resulting in a finite lifetime for the molecular state. Another spin flip breaks apart the molecule and the system is returned to the initial P channel.

The detuning of the two channels has a dramatic impact on the scattering properties of the system and results in a scattering length which depends on the external magnetic field as:

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right). \tag{1.2}$$

Here a_{bg} would be the background scattering length of the open channel if the closed channel was not accessible, ΔB is a measure of the width of the resonance, B_0 is the value of the magnetic field B just on resonance, and $B - B_0 \sim \nu$. We will discuss more the details of Feshbach resonance theory in the next chapter.

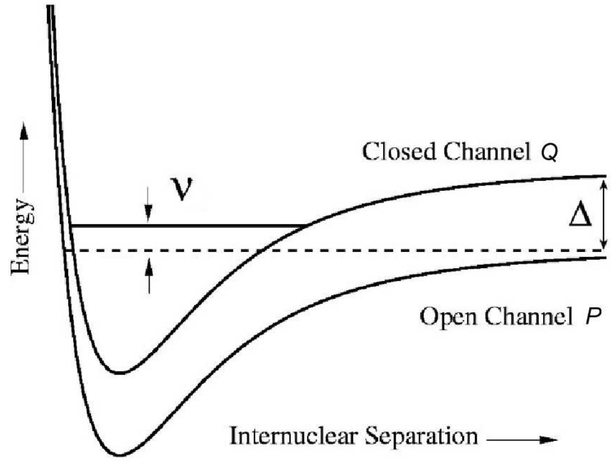

Figure 1.1: Born-Oppenheimer curves illustrating the mechanism of a Feshbach resonance. A Feshbach resonance results when a closed channel potential possesses a bound state in proximity to the scattering threshold of an open channel potential. The detuning of the bound state from the edge of the collision continuum is denoted by ν. Δ represents the energy shift between the two channels at large, relative separation.

1.3 Molecular superfluids and beyond

The introduction of a Feshbach resonance not only allows direct control over the atom-atom interactions, but inherently introduces a process of molecular formation and disassociation. The presence of molecules greatly increases the richness and utility of these systems. Molecular BECs, for instance, have remained out of reach to experiment due to the difficulty involved in cooling molecules as compared to alkali atoms. One way of overcoming this difficulty would be to create an atomic condensate through traditional laser trapping and cooling techniques and to then ramp across a Feshbach resonance converting the atomic condensate into a molecular condensate. This technique was used in the JILA ^{85}Rb experiment by Donley *et al.* [19] to create a superposition of atoms and molecules–although the question of whether a true molecular condensate was formed is still being debated. A similar experiment was performed by Dürr *et al.* [20] where they were able to spatially separate the molecular component from the atomic population by performing a Stern-Gerlach experiment.

Surprisingly enough, degenerate Fermi gases proved more adept at forming molecules than Bose gases. Due to Pauli blocking of the available decay channels, these molecules showed extremely long lifetimes, some remaining for up to several seconds [21, 22, 23, 24]. With such long lived molecules available, reports of molecular condensates quickly appeared at JILA [25], then MIT [26] and Innsbruck [27].

The production of a molecular condensate from a Fermi gas of atoms was the first step in experimentally studying the “crossover problem” of moving between Bardeen-Cooper-Schrieffer (BCS) superfluidity and Bose-Einstein condensation (BEC). Soon after these initial observations, reports of condensate formation above the resonance, well within the crossover regime between the extremes of BCS and BEC, where made first at JILA [28] and then at MIT [29]. These reports may be the first observations of “resonance superfluidity” within Fermi gases.

1.4 Outlook and overview

The structure of this thesis will be as follows. Chapter 2 will present the formalism of Feshbach resonant scattering. In Chapter 3 we will use the ideas discussed in the previous chapter to develop a field theory for bosons and apply it to the collapse of a condensate. We will review some of the fundamental notions behind BCS theory in Chapter 4 and go on to extend these ideas to form a theory of resonance superfluidity in Chapter 5. This theory will be used to determine signatures of a superfluid phase transition in a degenerate Fermi gas. Chapter 6 will discuss the crossover problem, briefly mentioned in the previous section of this introduction, in greater detail and in Chapter 7 we will treat the crossover by extending a lowest order theory to account for the production of molecules. Chapter 8 will extend the theory of the previous chapter to formulate a nonperturbative theory of the crossover. In Chapter 9 we introduce the fractional quantum Hall effect, properties of which are predicted to occur in a rapidly rotating Bose gas, and, in Chapter 10, discuss how a resonance may alter these properties focusing on the ground state of such a system.