Appendix F

Landau levels

The Hamiltonian for a charged particle moving in a transverse magnetic field may be written as

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 = \frac{1}{2m}\pi^2, \tag{F.1}$$

where we have introduced the operator π which satisfies the commutation relation $[\pi_x, \pi_y] = -i\hbar^2/l^2$, with $l = \sqrt{\hbar/|e|B}$ the magnetic length or Larmor radius. Equation (F.1) may be rewritten in terms of the ladder operators

$$\begin{aligned} a &= \frac{l}{\sqrt{2}\hbar}(\pi_x - i\pi_y), \\ a^\dagger &= \frac{l}{\sqrt{2}\hbar}(\pi_x + i\pi_y), \end{aligned} \tag{F.2}$$

which satisfy the commutation relation $[a, a^\dagger] = 1$. This substiturion results in the following Hamiltonian:

$$H = \hbar\omega_c\left(a^\dagger a + \frac{1}{2}\right), \tag{F.3}$$

where $\omega_c = |eB|/m$ is the cyclotron frequency. Equation (F.3) has eigenvalues $E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$, like those of the harmonic oscillator, each associated with a highly degenerate set of eigenvectors, known as Landau levels [104]. Often it is convenient to work within the lowest of these degenerate energy levels, referred to as the lowest Landau level (LLL), whose wavefunction may be written as

$$\psi_{0,m}(\mathbf{r}) = \frac{1}{\sqrt{2\pi 2^m m! l}} z^m e^{-|z|^2/4}, \tag{F.4}$$

where the quantum number $m = 0, \pm 1, \pm 2 \ldots$ and the complex position $z = x + iy$ is introduced.