Appendix E

T_c equations from the pseudogap theory

The pseudogap equations derived in Chapter 8 are presented here in a more clear and explicit form. The Matsubara summations are performed and the expansion coefficients of Eq. (8.54) are clearly listed. A detailed discussion of their derivation is given in the text. They are valid for $T \leq T_c$ and may be thought of as an extrapolation for a small region above T_c.

E.1 Gap equation

The Gap equation is given by

$$1 = -g_{eff}(0) \sum_k \frac{1 - 2f(E_k)}{2E_k} \varphi_k^2, \tag{E.1}$$

where $E_k = \sqrt{\varepsilon_k^2 + \Delta^2 \varphi_k^2}$ is the gapped energy spectrum, $\varepsilon_k = k^2/2m - \mu$, and the gap is defined in terms of the superconducting gap Δ_{sc} and the pseudogap Δ_{pg}:

$$\Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2. \tag{E.2}$$

The Fermi distribution function is defined as $f(x) = 1/(\exp(\beta x)+1)$. The resonant interaction contribution is given as $g_{eff}(0) = U - \frac{g^2}{\nu - 2\mu}$ and the function $\varphi_k^2 = \exp[-(k/Kc)^2]$ is introduced to impose a cutoff K_c upon the integral. For this regularization to work we must define the renormalized parameters as in Appendix A.3.

E.2 Number equation

The equation for the total number of particles in the system is

$$\begin{aligned} N &= N_f + 2N_b + 2N_b^0 \\ &= 2\sum_k \left[f(E_k) + \frac{1}{2}(1 - \frac{\varepsilon_k}{E_k})(1 - 2f(E_k)) \right] + \frac{2}{1+C} \sum_q n\left(q^2 \frac{\frac{1}{2M} + BC}{1+C} \right) + 2N_b^0 \end{aligned} \tag{E.3}$$

The Bose distribution function is introduced as $n(x) = 1/(\exp(\beta x) - 1)$. Here we have also introduced the following constants a_0, B, C. The first two, a_0 and B, will be defined in a following section; the third:

$$C = \frac{g^2 a_0}{(1 - U g_{eff}^{-1}(0))^2}. \tag{E.4}$$

E.3 Pseudogap equation

The equation for the pseudogap is given by

$$\Delta_{pg}^2 = \frac{1}{a_0(1 + 1/C)} \sum_q n\left(q^2 \frac{\frac{1}{2M} + BC}{1+C} \right). \tag{E.5}$$

E.4 Coefficients a_0,B

We now define the expansion coefficients coming from Eq. (8.54) and appearing in the above equations. To begin

$$a_0 = \frac{1}{2\Delta^2} \sum_k \left[(1 - 2f(\varepsilon_k)) - \frac{\varepsilon_k}{E_k}(1 - 2f(E_k)) \right]. \tag{E.6}$$

For the Gaussian potential

$$\vec{\nabla}_k^2 \varphi_k^2 = \tfrac{2\varphi_k^2}{K_c^2}(2(\tfrac{k}{K_c})^2 - 3) \quad \text{and} \quad (\vec{\nabla}_k \varepsilon_k) \cdot (\vec{\nabla}_k \varphi_k^2) = -\tfrac{2}{m}(\tfrac{k}{K_c})^2 \varphi_k^2, \tag{E.7}$$

so

$$\begin{aligned} B &= -\frac{1}{6a_0\Delta^2}\sum_k \Big\{ \Big[\frac{4}{m}(\epsilon_k+\mu)f'(\epsilon_k) \\ &+ \frac{2}{m}\frac{E_k(\varepsilon_k+\mu)}{\Delta^2\varphi_k^2}\Big[(1+\frac{\varepsilon_k^2}{E_k^2})(1-2f(E_k)) - 2\frac{\varepsilon_k}{E_k}(1-2f(\varepsilon_k))\Big] \\ &- \frac{1}{2m}\Big[(1-2f(\varepsilon_k)) - \frac{\varepsilon_k}{E_k}(1-2f(E_k))\Big]\Big[3-2(\frac{k}{K_c})^2\Big] \\ &- \frac{\Delta^2}{2K_c^2}\frac{1-2f(E_k)}{2E_k}\varphi_k^2\Big[3-2(\frac{k}{K_c})^2\Big]\Big\}, \end{aligned} \tag{E.8}$$

where we have introduced the shorthand notation $f'(x) = -f(x)(1-f(x))\beta$.