

Appendix A

Renormalization of model potentials

Throughout this thesis we have made use of contact and separable pseudo-potentials to simplify the evaluation of the resonant field theories. This has introduced a need to renormalize the resulting equations. Equation (2.41), which defines our renormalization scheme, will now be solved for three different pseudo-potentials: contact, Lorentzian, and Gaussian.

A.1 Contact

The contact potential has the Fourier representation: $\lambda(k) = \sqrt{U}$ and $g(k) = g$. To prevent Eq. (2.41) from diverging we must impose a cutoff K bounding the integral. After performing the integrals, Eq. (2.41) may be solved for $T(k)$. The Feshbach theory, as given by Eqs. (2.20) and (2.25), may be compared to this result by series expanding to order $\mathcal{O}(k)$. Matching terms, order by order, we get a series of algebraic equations which may be solved to find the following relationships between U, g, ν and the physical quantities $\bar{U}, \bar{g}, \bar{\nu}$:

$$\begin{aligned} U &= \bar{U}\Gamma \\ g &= \bar{g}\Gamma \\ \nu &= \bar{\nu} + \alpha\Gamma\bar{g}^2, \end{aligned} \tag{A.1}$$

where we introduce the following definitions:

$$\Gamma = (1 - \alpha\bar{U})^{-1} \quad \text{and} \quad \alpha = mK/(2\pi^2\hbar^2). \quad (\text{A.2})$$

A.2 Lorentzian

In Fourier space the Lorentzian has the form: $\lambda(k) = \sqrt{U}(1 + (k/K)^2)^{-1}$ and $g(k) = g(1 + (k/K)^2)^{-1}$. The form of the pseudo-potentials regularizes the integrals so we do not need to impose a cutoff and the integrals may extend to infinity. The relations given in Eqs. (A.1) again hold and we find

$$\Gamma = (1 - \alpha\bar{U})^{-1} \quad \text{and} \quad \alpha = mK/(4\pi\hbar^2). \quad (\text{A.3})$$

A.3 Gaussian

In Fourier space we have: $\lambda(k) = \sqrt{U}e^{-(k/K)^2}$ and $g(k) = ge^{-(k/K)^2}$. Again the potentials regularize the integrals. Equations (A.1) still hold and we find

$$\Gamma = (1 - \alpha\bar{U})^{-1} \quad \text{and} \quad \alpha = mK/(4\pi^{3/2}\hbar^2). \quad (\text{A.4})$$