Chapter 1

Introduction

1.1 Overview

Bose-Condensation is formed when a macroscopic population of bosons occupy the same quantum state at non-zero temperature. Our experiment can be thought of as a series of steps that make all degrees of freedom for a large number Rubidium atoms exactly the same. Luckily, most of the steps are simply selecting out atoms which are already in the same state. The methods we go through follow a progression from higher to lower energy scales, becoming more difficult as the energy lowers. This of course begins by selecting Rb to put in our vacuum chamber (this part is simple, because our living at room temperature means we find Rb is already non-degenerate with other elements – only chemically combined). Next we select out the isotope of ^{87}Rb from ^{85}Rb by tuning our magneto-optical trapping (MOT) lasers to the proper wavelength so that only ^{87}Rb is captured in our trap. The only remaining degrees of freedom are the internal energy levels of ^{87}Rb (the hyperfine and Zeeman levels) and the external, motional degree of freedom (the harmonic oscillator energy level). With optical pumping techniques we can force most of the atoms into the same internal level. Those that we don't get are effectively removed from the sample by our magnetic trap's selectivity – only specific internal levels are confined. The last step is the most difficult. At this point in the experiment the atoms are confined in a harmonic, magnetic trap, but their energy is much larger than the spacing of the oscillator levels, so the probability of even a single atom existing in a given state is small. Forced evaporation [61] is used to cool the sample until nearly all atoms exist in the same ground state; the zero-point energy of our trap.

In principle BEC can occur in states other than the ground state, but the large state space available to particles makes arbitrary-state BEC creation unlikely. The ground state posses this one advantage; it is the end of the road for a system being cooled. Atoms cooled in a harmonic trap must eventually run into the ground state, piling up as they run out of energy levels to occupy. Of course, BEC is much more than a simple state-space argument. The symmetry of bosons brings about a stimulated process whereby atoms have a preference to scatter into a state already occupied. There are, however, so many states available for an atom to occupy when at higher temperatures, that the enhancement is generally immaterial. The number of accessible states can be limited by colder temperatures (*i.e.* narrowing of the distribution and truncation of the state space by the zero point). Only when the atomic wavefunctions overlap and when there is significant probability to exist in the same state does the Bosonic symmetry become important. This brings about the temperature and density requirements for condensation. The small enhancement factor also grows as the number of atoms in the ground state becomes larger. The wavefunction for N atoms in the ground state is the product wavefunction

$$\psi_{\mathrm{BEC}} = \psi_0(1)\psi_0(2)...\psi_0(N). \tag{1.1}$$

Eq. 1.1 based on a harmonic oscillator potential is not quite a complete description of the multiparticle ground state. Two colliding atoms experience repulsive (in the case of ^{87}Rb) potentials which can be well modeled by a hard sphere interaction. The size of this sphere is given by the s-wave scattering length

a, and leads to a nonlinear term in the wave equation $\psi_{\rm BEC}$ for a condensate, proportional to $aN|\psi_{\rm BEC}|^2$. For ^{87}Rb the positive scattering length imposes a resistance to compression on the condensate. Condensates with many atoms (like current experiments with $\sim 10^6$) are many times larger in extent, and correspondingly less dense than if $a = 0$. Most of the original BEC experiments over the first two years after 1995 centered on validating the non-linear wave equation, and studying the added richness of the system from the standpoint of the density distribution $|\psi|^2$.

Another interesting trait of BEC is its phase. A phase factor may be written in front of Eq. 1.1, but of course, it takes another condensate in order for the phase to be meaningful. Double condensates were first produced by Myatt *et al.* [108] through poor optical pumping which led to creation of condensates in the two long-lived trappable states of ^{87}Rb (the $|2,2\rangle$ and $|1,-1\rangle$). These two states are not ideal for most double condensate studies since they feel a factor of two difference in trapping potential, and sit in quite distinct positions in the trap due to gravity. ^{87}Rb has another trapped state, the $|2,1\rangle$, which is confined with the same strength as the $|1,-1\rangle$. Thus started our studies on double condensates. It is not trivial to cool these two states simultaneously into condensation since the $|2,1\rangle$ state has a relatively large spin-exchange loss rate. This means it is much easier to create the $|2,1\rangle$ state from an existing $|1,-1\rangle$ condensate, which has the added advantage of starting the system with a well defined relative phase between the two condensates.

This is accomplished through a two photon-transition. The energy splitting between the $|1,-1\rangle$ and $|2,1\rangle$ is second order in magnetic field, centered about the "magic" field value of 3.24 G. We try to operate as close to this field as possible so that the effect of noise from various sources is small. It turns out there is an interesting (and useful) side effect of the TOP trap [114] (our implementation of a harmonic, magnetic trap) on the two-photon transition. The time-dependent rotating field of the TOP trap is equivalent to a fictitious, static magnetic field which acts oppositely on the two states. This translates into linear offsets in the relative position of the states in the trap, and effects the two-photon transition energy. As it turns out, we can use this effect to cancel another offset imposed by the differing magnetic moments (from the higher-order B dependence). This allows for unique control over the separation of the condensates in the TOP trap.

The addition of the new condensate state requires modification of the wave-function describing each condensate. An extra nonlinear term in each state's wave equation represents the pressure of one condensate on the other. Interestingly, all the scattering lengths involved in the double condensate system are very similar in magnitude. This provides for subtle behavior: each component in a mixture of the two states is able to rearrange its density distribution without much energy cost, as long as it does not affect the overall density distribution.

When the two-photon drive is on, there can be coherent transfer of population between the two states. This can make the dynamics very complicated given the nonlinear interaction between the states. However, when the time scale to make coherent transfer is short compared to the trap oscillation period and the time needed for the interactions kick in, the system can be modeled simply. The atoms can then be treated as discrete, non-interacting two-level systems, and the equations become the two-level Bloch equations applied locally to each part of the condensate. We can then think in terms of familiar concepts to atomic physicists: things like the Bloch sphere, Rabi oscillations, and π-pulses. As the first demonstration of these concepts we placed a condensate in a superposition of the two spin states, and at a later time, read out the relative phase, much like a standard atomic or optical interferometer.

There is of course more to a condensate than just single atom spectroscopy.

In fact, we have an interesting new piece of physics here that is not so accessible in atomic physics. This is a wavefunction which is macroscopic in size, so that we can affect the relative phase and population in a spatially selective manner. It becomes possible to think of multiply connected phase structures such as vortices. Interestingly these experiments can also occur quickly compared to trap oscillations or density-changing behaviour, which means we can apply our two-level model locally to create phase and population structures. Experimentally we can adjust the local detuning and Rabi frequency of the two-photon drive to control the evolution of each discrete point. Of course our ability to implement arbitrary control is finite, and we have only succeeded in creating large scale (order of the condensate) structures.

The first of these was a simple linear phase gradient across the condensate. When the two-photon drive is instantly turned on the condensate oscillates between the two spin states. A continuous spatial variation in the detuning and/or Rabi frequency of the coupling drive means the local phase accumulates at different rates for different positions.

Experimentally what we see corresponds exactly to our local view of discrete two-level systems (at least for short times, before mean-field has much effect). For a vertical gradient in the Rabi frequency, the population oscillations eventually get out of phase between the top and bottom of the condensate. As more time goes by, other parts of the condensate get further out of phase and the condensates appear to be striped, alternating between population of $|1,-1\rangle$ and $|2,1\rangle$ along the vertical axis. Some of these structures actually have quite good overlap with the harmonic oscillator states, which makes this the first quantum-state engineering done with a condensate. For longer times with the two-photon drive on, the phase winds up tighter and tighter.

There turns out to be a subtle condensed matter analogy in this experiment.

If we don't ignore the mean-field and potential terms in the wave equation (or we run the experiment for longer time), then we find a peculiar behavior. The phase which has wound up begins to unwind, even with the two-photon drive still present. In fact it unwinds back to its initial condition of all atoms in the $|1,-1\rangle$ state. An analogy may be drawn to spin-echo [74], except that this recurrence occurs in an inhomogeneous system *without* the application of additional unique pulses to reverse the spin evolution. In this case there is an energy cost associated with the spatial change in phase, and it is the hydrodynamics which drive the atoms to change their superposition of the two spin states. The two-dimensional vector that defines the geometric phase actually has access to a higher dimension – the relative phase between the two states (or equivalently, the relative populations of the dressed states). The fact that it is able to use its extra spin degree of freedom to minimize the energy is a unique feature of our two-component condensate.

One might ask why the phase, instead of unwinding, doesn't just break at some point in order to remove windings. The answer is again the energy cost. Since the condensate is *not* simply the sum of the parts from our local model, there is a large amount of energy required to support a large gradient in phase. If the phase were to become discontinuous at a point, then density must go to zero at that point. This is energetically unfavorable since the pressure from the repulsive interactions opposes that. There is a kind of threshold predicted in this qualitative model. If the winding of the phase is slow enough, then the condensate should have enough energy to continuously remove the phase gradient. We have seen this kind of behavior; for very small gradients in Rabi frequency the condensate never develops stripes, even though it should if a simple scaling applied from the cases with larger gradients. This has only been qualitatively explored.

Given this kind of interpretation of our technique, Williams and Holland [6] proposed another scheme. Their motivation was simple – if a linear phase gradient can be applied in the vertical direction, is it possible to apply the gradient *around* the condensate, in such a way to create a vortex state? The lowest angular momentum state is one with a single quanta of rotation, in which the phase winds by 2π around condensates center. A simple angular dependent phase describes the new wavefunction:

$$\Phi(\vec{r}) = e^{i\phi}\Phi(r). \tag{1.2}$$

One intriguing aspect of superfluid vortices is their metastability. This comes from the same energy argument discussed earlier. Since the phase in a vortex state winds continuously by 2π around the core, it can only unwind by creating a discontinuity – "breaking" the phase. This discontinuity must force the density to zero at that point, creating a large slope in the wavefunction, equivalent to a large kinetic energy. If this energy is not available, the state is metastable.

Numerically solving the nonlinear wave equation, and including an arbitrary mechanism for offsetting the confining potentials for the two spin states (one way to make the phase gradient), Williams and Holland found that such a state could be generated. They proposed applying the two-photon drive to the initial $|1,-1\rangle$ state, while applying a rotating offset. The offset instantaneously produces a gradient in detuning across the cloud, but it is rotated about the cloud's center at a rate comparable to the Rabi frequency. The result is a vortex state in $|2,1\rangle$ with the core region occupied by the $|1,-1\rangle$ state.

This method gets around one of the concepts stated earlier in a unique way. The problem with creating a vortex in an existing, single-state condensate is that the uniform phase of the condensate must be broken, then twisted, then put back together in order to achieve the 2π winding. We found earlier that a condensate resists this breaking (or even tightly twisting) of the phase since it requires a large amount of energy. Using a two-state condensate, we write on the phase *as* the population is transferred into the $|2,1\rangle$ state. Thus we are not bound by the condensates stiffness since there is not yet a condensate in the target state. There is yet another attractive feature of this technique. Since the central region of the condensate is in the $|1,-1\rangle$ state when the vortex is created in the $|2,1\rangle$ state, the states partially overlap. The presence of the $|1,-1\rangle$ enables us to use standard atomic interferometry to read out the relative phase between the two states. What we find is exactly the 2π winding in the phase of the ring shaped $|2,1\rangle$ state.

1.2 Who cares?

These new experiments dealing with and manipulating the condensate's phase, and related work going on in other groups, will be the main focus of BEC physics for many years. So why is this interesting and who cares? ("who" refers to Joe Scientist, not Joe Six-pack). Before BEC was observed in a dilute atomic system, every well-rounded physicist certainly had some interest in the creation of an ideal Bose-Einstein condensate. Not because the theory was thought lacking, but because it was a regime in phase-space which had never been accessible experimentally. Of course physicists in condensed matter and liquid helium have long been able to produce super-conductors and fluids. Unfortunately these are far from an ideal realization of BEC, and required heroic efforts in probing the system on the microscopic level. The thing that draws atomic physicists to the idea of a dilute system is twofold – the ability to have nearly 100% of the particles in the condensate, and the ability to do atomic spectroscopy on them.

The first reason is obvious. Nearly any measurement in atomic physics must take into account broadening from some source, almost always including thermal. With a million or so atoms in the exact same state, the prospects for precision measurements are phenomenal (well, almost... see below), since the broadening is erased, and the signal to noise that much better. Schooled in the atomic clock techniques of magnetic resonance and precision spectroscopy, I find this a strong motivation for creating long lived, unique superpositions and entangled states with the ability to test fundamental physics. Most physicists surely find it extremely appealing to measure quantities with far reaching implications such as parity non-conservation, or the electric dipole moment of the electron, on the top of a small table, in a small room, with only a handful of money and scientists. It is somehow more elegant, or at least, complimentary to high energy experiments.

Unfortunately, some of the experiments in this thesis show precision measurement with condensates still has some hurdles to clear. In the most straightforward form, a precision measurement utilizes the relative phase accumulation between two states which exist in superposition. This phase is easily read using many well developed techniques. It is the amount of relative phase gained by the states while in superposition that is proportional to some energy difference between the states. The real trick then, is to arrange the superposition so that the energy involves the quantity to be measured, and affects the two states differently. Unfortunately, the interactions can produce very unpleasant results. When we create the two-state superposition, the interactions drive some very complex spatial rearrangement of the states. This is unfortunate not only because the state separation makes a relative phase measurement difficult, but also since there is another source of energy which affects the two states differently. There are some possible solutions to this, like optically (or magnetically if the energy of the states can be insensitive to fields) turning off the interactions, or by going to a beam type experiment. The later is one very real application for coherently removing atoms from the condensate (the so-called "atom-laser" [97]). Things like continuous BEC production will be a critical step in this and many other applications.

There is another area I have learned to appreciate and take an interest in, mostly because of Eric's motivation. These are the experiments to make connections with condensed matter physics. Coming into this mostly ignorant of superfluid and superconductor physics, partly because of their theoretical complexity, my real motivation was exploring the quantum nature of BEC, similar to old atomic physics work. And some of our original work with collective modes and other "fluid dynamics" type behavior I found somewhat dry and unenlightening. However there are some great experiments on liquid helium that we have been able to draw strong analogies to, which we can also talk about in atomic physics terms. Plus we have the luxury of superior imaging and probing to get at the important quantities like phase and state distribution. I think atomic physicists will be able to add a new perspective on many helium experiments in the near future.

This thesis focuses mainly on double condensate experiments, including mean-field interactions and relative phase measurements. Chapter 2 begins with finite temperature excitations, the only remnant of the first BEC apparatus not covered by Ensher [61]. Chapters 3 and 4 focus on two-photon and imaging technology respectively, with emphasis on double condensate experiments. The remaining chapters cover mean-field interactions (Chapter 5), the measurement of relative phase (Chapter 6), creation and evolution due to a gradient in phase across the condensate (Chapter 7), and creation of a vortex state (Chapter 8).