Chapter 3

Driven two-level atom

The goal of this chapter is to describe how a two-level atom is affected by a coupling radiation field and a specific case of two hyperfine levels in ^{87}Rb coupled by a two-photon field. This chapter includes a basic description of Rabi flopping and Ramsey's method of separated oscillatory fields, which will be used as the method of spectroscopy for the following experiments. The goal of both of these methods of spectroscopy is to measure the energy difference between two atomic states. The choice of the two specific hyperfine levels and method of coupling them will also be discussed.

3.1 Rabi flopping

A two-level atom in an oscillating field can be described by a time dependent Hamiltonian, which is given by

$$H = \begin{pmatrix} \frac{\hbar\omega_0}{2} & \Omega_R e^{i\omega t/2} \\ \Omega_R e^{-i\omega t/2} & \frac{-\hbar\omega_0}{2} \end{pmatrix}, \tag{3.1}$$

where ω_0 is the the frequency difference between the two states, ω is the frequency of the driving field, Ω_R is the Rabi frequency, and t is time. The time-dependent Schrödinger equation with this Hamiltonian can be solved using the rotating wave approximation and the initial condition that the atom is in the ground state. The probability of being found in the excited state as a function of time is

$$P(t) = \left(\frac{\Omega_R}{\Omega'_R}\right)^2 \sin^2\left(\frac{\Omega'_R}{2}t\right), \tag{3.2}$$

where the effective Rabi frequency is

$$\Omega'_R = \sqrt{\Omega_R^2 + (\omega_0 - \omega)^2} \tag{3.3}$$

$$= \sqrt{\Omega_R^2 + \delta^2}, \tag{3.4}$$

and δ is the detuning from resonance. An atom in the field oscillates between the two states at the effective Rabi frequency, where the contrast of the oscillation is determined by the detuning. This sinusoidal population transfer is referred to as Rabi flopping.

Rabi's resonance method can be used to measure the energy splitting between two states. However, it has some limitations. Rabi's method probes the transition frequency while an atom is interacting with the coupling field, which can lead to frequency shifts from the AC Stark effect or additional phase noise. The frequency of the Rabi fringes depends not only on the frequency between the two states but also on the power of the driving field. Originally, in molecular beam experiments, the molecules would pass through a single region where they were illuminated by radiation and then be detected. It is reasonable to think that increasing the length of the interaction region l, would decrease the width of the spectral lines. However if the field is not uniform throughout the interaction region, increasing l will, at some point, lead to a broadening of the spectral lines. This broadening limits the resolution of Rabi spectroscopy. Ramsey's method of separated oscillatory fields was developed to overcome these limitations.

3.2 Ramsey spectroscopy

Ramsey's method uses two interaction regions separated by free propagation as seen in Fig. 3.1. Following a two-pulse Ramsey sequence, the probability of being in the excited state as a function of time, in the short intense pulse limit, with the approximations that $\delta \ll \Omega_R$ and $\tau \ll$T is

$$P_t(T, \delta) = \frac{1}{2} + \frac{1}{2} \cos(\delta\ T), \tag{3.5}$$

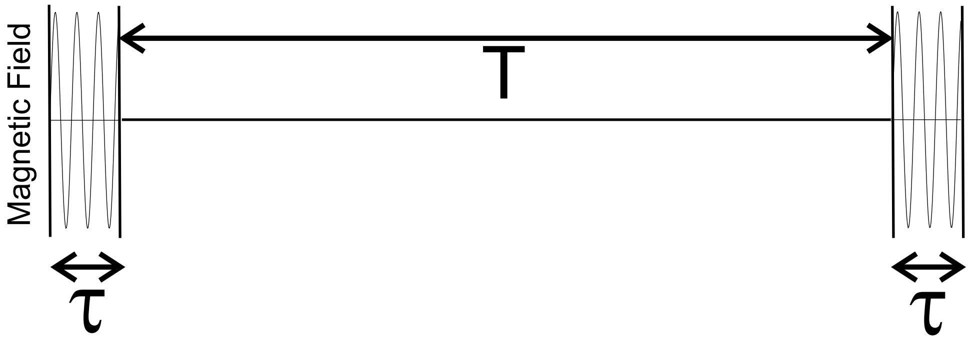

Figure 3.1: Ramsey's method of separated oscillatory fields, where two pulses length τ are separated by free propagation length T.

where T is the free propagation time. A π pulse corresponds to 100% transfer between states, and a $\pi/2$ pulse creates an equal superposition of the two states. The two pulses do not need to be $\pi/2$ pulses. However this condition gives the highest signal-to-noise ratio and full contrast fringes. Note that the Ramsey fringe frequency only depends on the field detuning from resonance and thus on the energy difference of the atomic states and not on the power of the field, as long as $\tau\Omega_R$ is constant. The driving field must still be stable during the pulses; this is a shorter time and the constraint on field amplitude stability does not increase with increasing free propagation time.

Ramsey's method can be thought of as an interferometer in the following way. If an atom is initially in the ground state, the first $\pi/2$ pulse places the atom in an equal coherent superposition of the ground and excited states. The splitting is identical to the splitting by a beamsplitter in a optical interferometer. Next, during the propagation time, the relative phase of the two states evolves at a rate proportional to the energy difference between the states, and the coupling drive accumulates a phase ωT. The second $\pi/2$ pulse recombines the two states, interferometrically comparing the relative phase accumulated.

3.3 Two-photon transition in ^{87}Rb

We want to measure the phase properties of an ensemble of atoms to gain understanding of phase coherence and mechanisms that cause decoherence; these measurements will be described in depth in chapter 4. The absolute phase of an atom can not be measured but rather only the phase relative to another system. We study the phase properties of a system that is made up of two hyperfine ground states in ^{87}Rb. These states must have the same magnetic moment to first order to be trapped, overlapped spatially within the trap, and have nearly identical responses to magnetic field for spectroscopy.

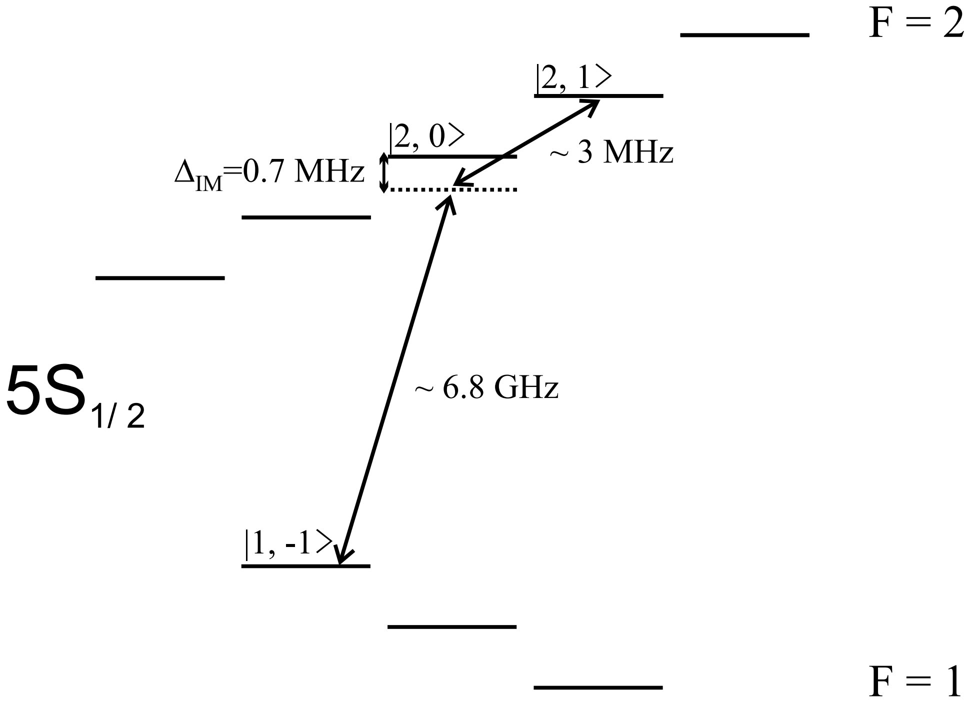

Figure 3.2: Energy level diagram for the ground state of ^{87}Rb in a field of 3.2 G showing the relevant two-photon transition.

The two states we use are the $|1\rangle \equiv |F=1, m_f=-1\rangle$ and $|2\rangle \equiv |F=2, m_f=1\rangle$ states in the $5S_{1/2}$ manifold. They satisfy the requirement of having nearly identical magnetic moments. We can coherently transfer population between the two states via

$\frac{\mu_b}{h}$	1.399624624 (56) MHz/G
n_i	3/2
g_i	0.9951414(10) $\times 10^{-3}$ [28]
g_j	2.00233113(20)
ν_{hf}	6834.68261090434(3) MHz [29]

Table 3.1: Breit-Rabi equation constants for ^{87}Rb

a two-photon transition. We need two photons because the states differ by two units of angular momentum. We use a microwave photon ($\sim$ 6.8 GHz) and a rf photon ($\sim$ 3 MHz) to couple the two states as shown in Fig. 3.2. When the detuning from the intermediate state $|2,0\rangle$ is large compared to the Rabi frequency for the one photon transition, the probability of making a transition to the intermediate state is small. In this limit the system can be thought of as just a two-level system with effective coupling given by the two-photon Rabi frequency, which is

$$\Omega = \frac{\Omega_{\mu wave}\ \Omega_{rf}}{2\ \Delta_{IM}}, \tag{3.6}$$

where $\Omega_{\mu wave}$ is the Rabi frequency for the $|1,-1\rangle \rightarrow |2,0\rangle$ transition, Ω_{rf} is the Rabi frequency for the $|2,1\rangle \rightarrow |2,0\rangle$ transition, and Δ_{IM} is the intermediate state detuning. The magnetic field dependence of this transition can be calculated using the Breit-Rabi formula, which is

$$\nu(B) = -\frac{\nu_{hf}}{2(2n_i+1)} - g_i\ \mu_B\ m_f\ B \pm \frac{\nu_{hf}}{2}\sqrt{1+\frac{4\ m_f}{2\ n_i+1}x(B)+x^2(B)}, \tag{3.7}$$

where

$$x(B) = \frac{(g_j+g_i)\mu_B\ B}{\nu_{hf}}. \tag{3.8}$$

B is the magnitude of the magnetic field, and the constants for the equation are given in Table 3.1.