Appendix D

Glossary of Symbols

A prime ($'$) generally denotes interaction-picture operators.

V_0	rf voltage amplitude
Ω_T	rf drive frequency
d_0	characteristic trap dimension
Q	ion charge
m	ion mass
z_{tot}	ion position, including secular motion and micromotion
z	secular motion
z_μ	micromotion
$x_{\mu 0,j}$	micromotion amplitude of j^{th} ion in x-direction (2-ion entanglement)
ω_m	secular frequency of mode m
ω_z	z secular frequency $\approx \frac{2\sqrt{2}QV_0}{md_0^2\Omega_T}$
ω_r	radial secular frequency (spherical quadrupole trap) $\omega_r = \omega_z/2$
$\overline{D}_z$	pseudopotential well depth (z-direction) $\approx \frac{QV_0^2}{md_0^2\Omega_T^2}$
U_0	static quadrupole voltage on trap electrodes
q	Mathieu q-parameter
a	Mathieu a-parameter

β	$\frac{2\omega_m}{\Omega_T} \approx \sqrt{a + \frac{q^2}{2}}$
	also displacement parameter due to noisy electric fields
	also qubit coefficient
$\hat{a}$	harmonic oscillator lowering operator $= \sqrt{\frac{m\omega_z}{2\hbar}}(\hat{z} + \frac{i}{m\omega_z}\hat{p})$
$\hat{a}^\dagger$	harmonic oscillator raising operator $= \sqrt{\frac{m\omega_z}{2\hbar}}(\hat{z} - \frac{i}{m\omega_z}\hat{p})$
$\hat{n}$	harmonic oscillator number operator $= \hat{a}^\dagger\hat{a}$
$H.C.$	Hermitian conjugate
z_0	ground state wave packet spread $= \sqrt{\frac{\hbar}{2m\omega_z}}$
$\mathcal{Q}$	resonator quality factor
Z_0	resonator characteristic impedance
α	atomic polarizability of neutral background gas atoms
	also coherent state amplitude
	also qubit coefficient
μ_{red}	reduced mass for ion/neutral collisions
ν	relative velocity in ion/neutral collisions
b	impact parameter for ion/neutral collisions
b_{crit}	critical impact parameter for spiralling ion/neutral collisions
k_{Langevin}	Langevin rate constant
γ_{Langevin}	reaction rate due to background neutral gas particles/spiralling collisions
γ_{el}	background gas reaction rate based on total elastic cross sectioin
ρ	background gas density
$\hat{\rho}, \rho_{n,m}$	density matrix, matrix elements
σ_{el}	total elastic collision cross section
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	unit vectors in the x-, y-, and z- directions

e	electron charge
$\mid\downarrow\rangle$	$^9Be^+,\ 2s\ ^2S_{1/2},\ \lvert F=2, m_F=-2\rangle$ state
	(sometimes $\lvert F=2, m_F=2\rangle$)
$\mid\uparrow\rangle$	$^9Be^+,\ 2s\ ^2S_{1/2},\ \lvert F=1, m_F=-1\rangle$ state
	(sometimes $\lvert F=1, m_F=1\rangle$)
$\lvert v\rangle$	"virtual level" for Raman transitions: $^9Be^+,\ 2s\ ^2S_{1/2}$ level
γ	excited state ($2p$) linewidth ≈ 19.4 MHz for $^9Be^+$
	also Berry's (geometrical) phase
δ_{FS}	$^9Be^+$ fine-structure splitting ≈ 197 GHz
τ	excited state lifetime ($= 1/2\pi\gamma$)
$\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3$	Pauli matrices (one also sees the notation $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$)
$\hat{S}_1, \hat{S}_2, \hat{S}_3$	atomic spin operators $= \frac{1}{2}\hbar\hat{\sigma}_j$
$\hat{S}_+, \hat{S}_-$	$\hat{S}_\pm = \frac{1}{2}(\hat{S}_1 \pm \hat{S}_2)$
$\hat{\boldsymbol{\mu}}$	electric dipole moment of the atom $= e\hat{\mathbf{r}}_{el}$
$\mathbf{k}, k$	laser wave vector, magnitude
$\boldsymbol{\Delta k}$	laser wave vector difference (Raman transitions)
ω_L	laser frequency
$\Delta\omega_L$	laser frequency difference
ϕ	laser phase
	also general phase
ω_0	carrier resonance frequency
	($^9Be^+$ hyperfine splitting plus Zeeman shift difference)
δ	laser detuning from resonance
Δ_R	Raman laser detunings from virtual level v

Ω	carrier Rabi frequency
$\Omega_{n,m}$	sideband Rabi frequency
Ω_d	effective displacement amplitude for applied electric fields
$\hat{g}_i$	single-photon coupling operators (Raman transitions)
	$= E_i e\, e^{-i\phi_i} \boldsymbol{\epsilon}_i \cdot \hat{\mathbf{x}}/2\hbar$
g_1	single-photon coupling strength (Raman transitions)
	$= \langle\downarrow\|\hat{g}_1\|v\rangle$
g_2	single-photon coupling strength (Raman transitions)
	$= \langle\uparrow\|\hat{g}_2\|v\rangle$
g	general Rabi frequency (e.g. if $g_1 = g_2 = g$)
	also red sideband coupling strength for engineered
	$T = 0$ reservoir $= i\eta\hbar\Omega e^{i\phi}$
η	Lamb-Dicke parameter ($= kz_0$, 1D)
$\overline{n}$	average phonon number
ζ	overall detection efficiency
	also dimensionless time in Mathieu equation
t_{DG}	length of Detection Gate
t_{pr}	length of probe pulse
T_R	length of free evolution time in Ramsey experiment
I	laser intensity
I_S	saturation intensity ($\approx$85 mW/m^2 for $^9Be^+$)
$\overline{m}$	avg. number of photons detected
$\hat{D}(\alpha)$	displacement operatore $= e^{(\alpha\hat{a}^\dagger - \alpha^*\hat{a})}$
L_n^m	associated Laguerre polynomial
$P_\downarrow$	probability that the ion is in $\|\downarrow\rangle$

$C_n, C_{\downarrow,n}, C_{\uparrow,n}$	motional state probability amplitude
$C_i^{(k)}(t)$	k^{th}-order perturbative expansion of $C_i(t)$
P_n	prob. that the ion is in motional Fock state $\|n\rangle$
r	ratio of red to blue sideband sizes
	also squeeze amplitude in $\hat{K}_1$, $\hat{K}_2$ formalism
p_r	recoil momentum $= \hbar k$
E_r	recoil energy $= \frac{p_r^2}{2m}$
$\mathcal{B}$	background counts
$S_E(\omega)$	electric field noise spectral density (V/m^2)
Γ_0	rate at which ion is heated out of ground state
	of motion
k_B	Boltzmann's constant $\approx 1.38 \times 10^{-23}$ J/K
$g(t)$	time-dependent laser coupling strength ("walking standing
	wave" generation of coherent states)
	$= \langle\downarrow\|\mathbf{e_x}\cdot\mathbf{r}_{el}\frac{eE}{2\hbar}\cos[\frac{1}{2}(\Delta kz - \delta t + \delta\phi)]\|v\rangle$
$\hat{\mathcal{S}}(\epsilon)$	squeeze operator $= \exp[\frac{1}{2}(\epsilon^*\hat{a}^2 - \epsilon(\hat{a}^\dagger)^2]$
ϵ	squeeze parameter $\epsilon = Re^{2i\phi}$
	also energy
$\hat{\boldsymbol{\epsilon}}$	laser polarization vector
$\Omega_\downarrow$	Rabi frequency for $\|\downarrow\rangle$ (including laser
	polarization/matrix elements)
$\Omega_\uparrow$	Rabi frequency for $\|\uparrow\rangle$ (including laser
	polarization/matrix elements)
$W(\alpha) \equiv W(z, ip)$	motional Wigner function
$Q_n(\alpha) \equiv \|\mathcal{C}_n(\alpha)\|^2$	Fock state probabilities for displaced motional state (density
	matrix reconstruction) $= \langle n\|\hat{D}^\dagger(\alpha)\hat{\rho}\hat{D}(\alpha)\|n\rangle$

$\lvert\psi_B^{\pm}\rangle$	Bell states $= \frac{1}{\sqrt{2}}(\lvert\downarrow,\uparrow\rangle \pm \lvert\uparrow,\downarrow\rangle)$
$\lvert\psi_e(\phi)\rangle$	approximate Bell state $= \left(\frac{3}{5}\lvert\downarrow,\uparrow\rangle - e^{i\phi}\frac{4}{5}\lvert\uparrow,\downarrow\rangle\right)\lvert n=0\rangle$
Ω_i	Rabi frequency of ion i (Bell state production)
ε	detuning in “Sørensen and Mølmer” entangled-state production scheme
	also small error in Berry's phase squeeze operators
	also error
$\oplus$	Exclusive-OR operator (addition modulo 2)
	also Hilbert space direct sum
$\lvert\mathcal{R}\rangle$	state of qubit register
$\mathcal{R}$	parameters upon which Berry's phase depends
	also reservoir
$\hat{\mathcal{H}}, \hat{\mathcal{H}}^{(N)}$	Hadamard transform
$\lvert aux\rangle$	auxilliary level for Controlled-NOT logic gate
	$= \lvert 2s\ ^2S_{1/2}, F=2, m_F=0\rangle$
$\hat{s}_i$	system operator (reservoir interactions)
$\hat{\Gamma}_i$	reservoir operator of mode i (reservoir interactions)
$\hat{\chi}$	complete density matrix of system + reservoir
$\hat{R}_0$	initial reservoir density matrix
$\mathcal{M}$	superoperator
$\mathbb{1}$	identity operator
$\hat{b}_k, \hat{b}_k^{\dagger}$	lowering and raising operators for reservoir (harmonic oscillator) mode k
$\overline{N}$	average reservoir occupation number (for given mode or frequency)
κ	coupling strength (to reservoir mode)

$\mathcal{V}$	rms voltage noise
Γ	effective decay rate ($T = 0$ reservoir engineering)
Ω_p	Rabi frequency of Red Doppler beam ($T = 0$ reservoir engineering)
ξ	dynamical phase
$\hat{U}(t)$	(Ch. 9) unitary operator which changes quantum state
	to produce Berry's phase
$\hat{G}(t)$	(Ch. 9) operatory which generates $\hat{U}(t)$
$\hat{S}(t)$	spin operator for spin-$\frac{1}{2}$ Berry's phase
$\mathbf{f}_i$	i^{th} frame field unit vector
$\omega_{i,j}$	differential "one form" which measures rotation of frame field
$\hat{K}_1$	generator of squeezes along z
	$= -\frac{i}{4}[\hat{a}^2 - (\hat{a}^\dagger)^2]$
$\hat{K}_2$	generator of squeezes along $z + ip$
	$= \frac{1}{4}[\hat{a}^2 + (\hat{a}^\dagger)^2]$
$\hat{J}_3 \equiv \hat{K}_3$	generator of rotations in (z, ip)-plane $= -\frac{1}{2}(\hat{n} + \frac{1}{2})$
$\hat{K}(t)$	state operator for SU(1,1) Berry's phase
$\widetilde{\times}$	Minkowski cross product (see Eq. (9.44))
$\tilde{\cdot}$	Minkowski dot product (see Eq. (9.45))
$\widetilde{\varepsilon}_{ijk}$	$\doteq (-1)^{\delta_{k,3}} \varepsilon_{ijk}$