

Chapter 4

Experimental Results for Linking Optical and Microwave Frequencies with the Comb

Once the carrier-envelope offset frequency of a mode-locked laser is stabilized, all that is needed for the frequency comb produced by the laser to be used as a phase-coherent connection between microwave and optical frequencies is to stabilize the repetition frequency, f_{rep} , of the laser. The repetition frequency can either be stabilized directly to a microwave frequency standard, allowing the comb to be used for making optical frequency measurements referenced to the microwave standard, or an optical frequency standard can be used to stabilize f_{rep} , providing a microwave frequency that is phase-coherently linked to the optical standard. Both of these scenarios have been explained in detail in Section 2.3. In this chapter I will present experimental results for both methods of linking optical and microwave frequencies using the comb produced by a mode-locked Ti:sapphire laser. Optical transitions in two different atomic species are connected to microwave frequency references. The first is a transition in molecular iodine (I_2), contained in a vapor cell, and the second is one in ultracold, laser-cooled strontium-88 (^{88}Sr) atoms. From the experimental results, we will also see how the performance of a vapor cell compares to that of a laser-cooled sample of atoms.

4.1 Cell-based optical standard

An optical frequency standard based on atoms or molecules enclosed in a vapor cell is attractive for the compact size, reliability, and relative simplicity of the system. Molecular iodine provides several narrow transitions that can potentially serve as secondary, portable optical frequency standards. A system based on a frequency-doubled Nd:YAG laser stabilized to a transition near 532 nm of I₂ contained in a vapor cell has proven to be one of the most practical optical frequency standards, demonstrating an excellent stability for such a compact and simple system, with a fractional instability $< 5 \times 10^{-14}$ at 1 s [87]. To achieve even higher frequency stability, it is useful to explore I₂ transitions at wavelengths below 532 nm, where the natural linewidths of the transitions decrease at a rate faster than that for the transition strengths as molecular iodine approaches its dissociation limit [10]. There is a transition near 515 nm which may provide one of the better frequency standards based on I₂, but the transition frequency has previously only been measured using wavelength interferometry. Stabilizing the repetition frequency of a mode-locked laser to a microwave source referenced to the Cs standard allows the frequency comb to be used to measure the absolute frequency of this transition with respect to the primary standard defining the SI second [46].

For probing the I₂ transition, a cw Ti:sapphire laser operating near 1030 nm is first pre-stabilized to a mode of a medium-finesse, evacuated, vibration-isolated, thermally stabilized optical cavity. A more compact Yb:YAG laser has also been used for this experiment. The narrow-linewidth laser is frequency-doubled and stabilized to the transition of the I₂ contained in an 8-cm-long vapor cell using frequency-modulation (FM) saturated-absorption spectroscopy to minimize the contribution of Doppler effects to the measured linewidth [14]. The cell pressure is controlled by stabilizing its cold-finger temperature, which is set at -5 °C. The absolute frequency of the 515-nm transition is determined using the Cs-stabilized frequency comb to measure the funda-

mental frequency of the I_2 -stabilized cw laser. Figure 4.1(a) shows an example record over ~ 3300 s of the deviations from the mean of the heterodyne beat between the cw laser and the nearest comb component. These measurements are obtained by counting the heterodyne beat using a 1-s gate time for the frequency counter. The Allan deviation is used to express how the fractional fluctuations of the measured optical frequency depend on averaging time, as discussed in Section 2.5. It is calculated for the data in Fig. 4.1(a) using Eqn. (2.16) and normalizing by the optical frequency. Shown in Fig. 4.1(b), the Allan deviation for the optical frequency measurement is identical to that of the Cs standard, which is provided by a commercial Cs beam clock. This indicates that the measurement is limited by the stability of the Cs standard to which the frequency comb is referenced.

From the measurement of the heterodyne beat frequency between the cw laser and the frequency comb, the absolute frequency of the cw laser (and therefore the 515-nm I_2 transition to which it is stabilized) is determined using Eqn. (2.10). The mode number, m , of the comb component against which the cw laser is beating is determined by independently measuring the cw laser frequency with a resolution better than half the mode spacing of the comb. This corresponds to a resolution better than 375 MHz, which is achieved using a medium-resolution wavelength meter. The frequency of the 515-nm transition is measured over a period of two weeks for three different I_2 cells, all of which were prepared by the Bureau International des Poids et Mesures (BIPM). Two were made recently before the measurements were performed, while the third was made more than eight years prior to the measurements. The results are shown in Fig. 4.2 with respect to the value adopted for this transition by the Comité International des Poids et Mesures (CIPM) in 1997 from the previous measurements using wavelength interferometry. They reveal that there is no statistically significant difference among the three I_2 cells. The weighted mean of the results indicates the frequency of the transition measured with the Cs-referenced comb is 71.8 kHz higher than the CIPM recommended

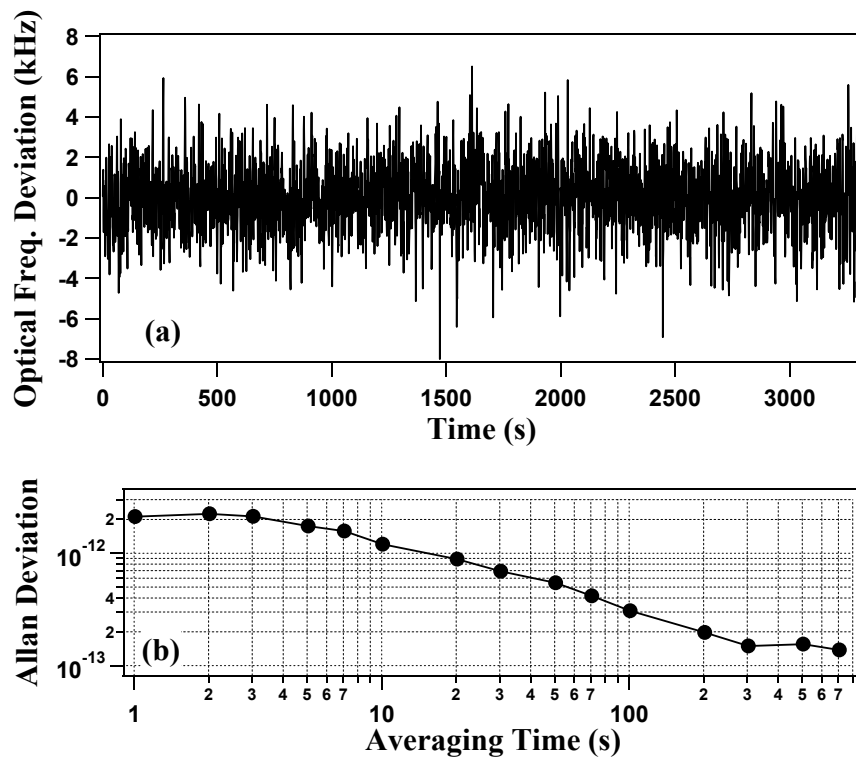


Figure 4.1: (a) The deviations from the mean of the heterodyne beat between the cw laser stabilized to the 515-nm I_2 transition and a mode of the Cs-referenced frequency comb. A gate time of 1 s is used for counting the beat frequency. (b) The Allan deviation of the I_2 -stabilized cw laser frequency, calculated from the data in (a). It is limited by the stability of the Cs standard referencing the frequency comb for the averaging times shown here.

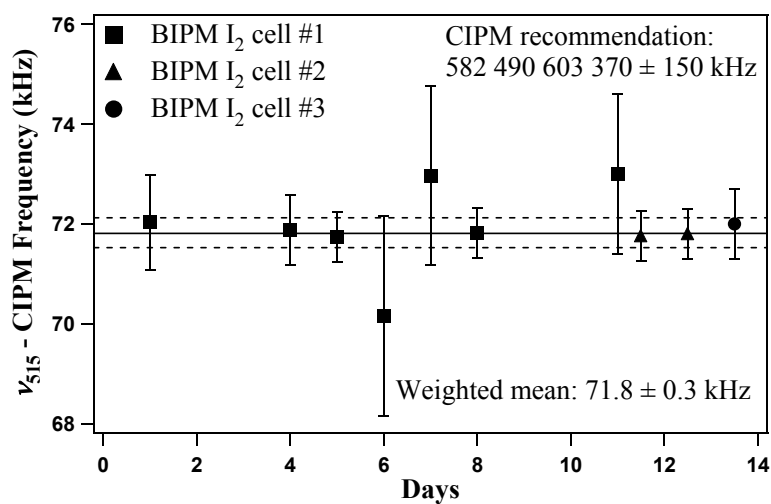


Figure 4.2: The absolute frequency of the 515-nm I₂ transition is measured with the Cs-referenced frequency comb over a period of two weeks for three different I₂ cells. A mean value is found that is 71.8 kHz above that recommended by the CIPM, which is within the uncertainty of the CIPM recommendation.

value of 582 490 603.37 MHz [62], with a 0.3 kHz statistical uncertainty. This offset is well within the 1-sigma uncertainty (150 kHz) of the recommended value. Allowing for various systematic effects on the transition frequency, the total uncertainty in the comb-based measurement is estimated to be 1.5 kHz, which represents an improvement by 2 orders of magnitude from the uncertainty in the CIPM recommendation.

Although Fig. 4.2 demonstrates that the frequency comb can be used to make consistent frequency measurements over a period of a few weeks, its reliability for providing accurate frequency measurements can be further verified by using it to measure the frequency of a Nd:YAG laser stabilized to the narrow I_2 transition near 532 nm and comparing the results with those obtained previously using a different comb system. The measurements of the I_2 -stabilized fundamental laser frequency are shown in Fig. 4.3, with respect to the CIPM recommended frequency. Five different periods of

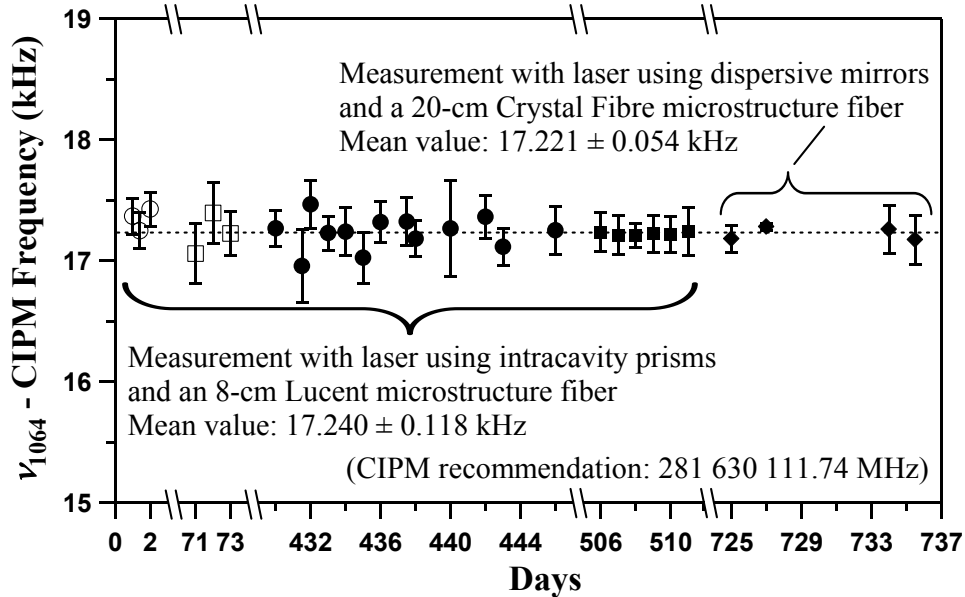


Figure 4.3: The results for measuring the 532-nm I_2 transition frequency with the current comb system agree very well with those obtained over a period up to two years prior using a significantly different comb system.

measurement of this I_2 transition are shown, spanning more than two years. The last period contains the results using the current frequency comb produced by the mode-

locked laser utilizing intracavity dispersive mirrors for dispersion compensation. The previous four sets of measurements were performed with a different comb system consisting of a mode-locked laser that uses intracavity prisms for dispersion compensation and a different microstructure fiber for broadening the spectrum to stabilize f_{ceo} . The agreement of these measurements made with two different comb systems over two years is excellent, with a difference between the mean value of the previous measurements and that for the results obtained with the current comb system that is less than 20 Hz. This is well within the standard deviations of the previous measurements and the current ones, which are 118 Hz and 54 Hz, respectively. As for the 515-nm measurement, the short-term frequency-measurement noise of the 532-nm system is limited by the instability of the Cs reference. The reproducibility of I₂ systems, however, is limited by the baseline fluctuations in the I₂ spectrometer.

Since the stability of the measurements of the I₂ transitions is being limited by the stability of the Cs standard, as evidenced by the Allan deviation of Fig. 4.1(b), it is useful to use a more stable microwave frequency reference for the comb. One such reference that is available is a hydrogen maser, which exhibits an instability of $\sim 2 \times 10^{-13} \tau^{-1/2}$ for an averaging time τ — roughly an order of magnitude better than the Cs standard. Stabilizing the repetition frequency of the comb to a microwave source referenced to the maser allows the stability of the I₂-stabilized laser to be explored at a level unattainable by the Cs-referenced comb. For an absolute frequency measurement, which must be referenced to the Cs primary standard that defines the second, the maser is calibrated relative to the NIST primary Cs frequency standard. Figure 4.4(a) shows the deviations from the mean of the heterodyne beat between the maser-referenced comb and a cw laser stabilized to the 532-nm I₂ transition, using a 10-s gate time for the frequency counter. The comb remains stabilized to the microwave reference during the entire measurement period shown here of ~ 5 h. This level of reliability is critical for using the comb in the implementation of an optical clock. The Allan deviation for

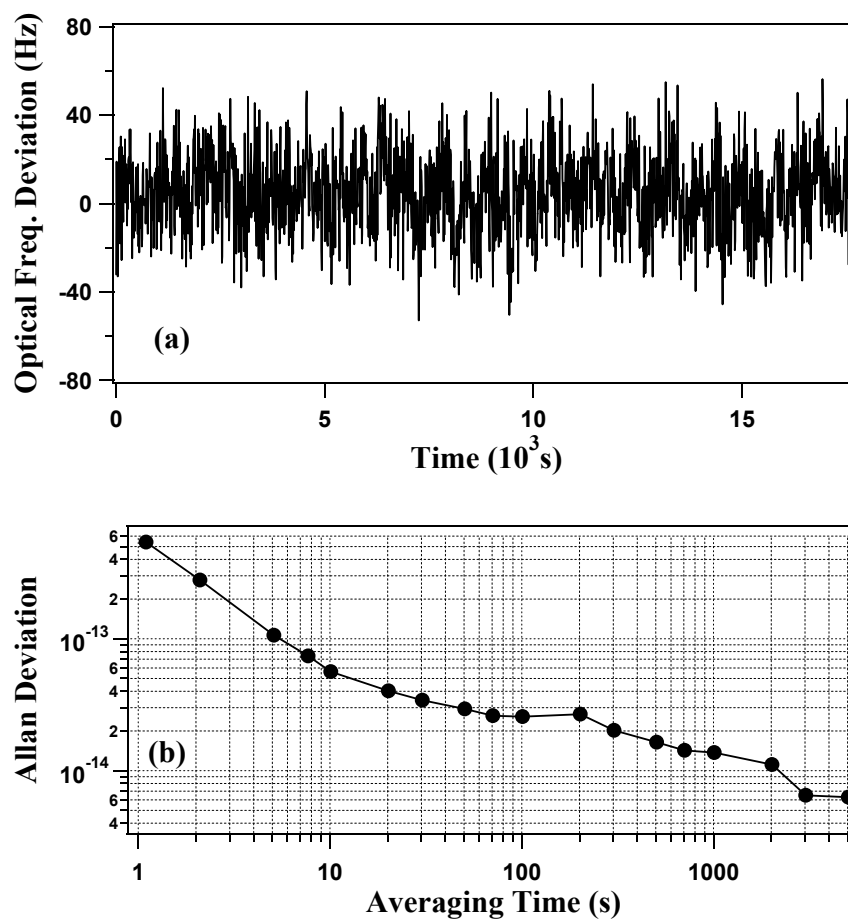


Figure 4.4: (a) The deviations from the mean of the heterodyne beat between the cw laser stabilized to the 532-nm I_2 transition and the maser-referenced comb, using a 10-s gate time for frequency counting. (b) The Allan deviation of the I_2 -stabilized cw laser frequency, measured with the maser-referenced comb. It is limited by the stability of the microwave reference.

this optical frequency measurement [shown in Fig. 4.4(b)] at averaging times of 10 s and below is obtained from data taken with corresponding gate times to eliminate the effects of dead time between gate intervals, but at longer averaging times it is calculated by combining the 10-s data as described in Section 2.5. The maser-referenced microwave frequency to which the comb repetition frequency is stabilized must be transmitted from its source within the NIST laboratories in Boulder to our laboratory in JILA on the university campus [86]. Issues involved with this transfer will be discussed in detail in Chapter 6. The Allan deviation of the measured optical frequency corresponds to that of the transmitted maser-referenced microwave signal, revealing that the stability of this frequency measurement is still limited by the microwave reference.

The second method of using the comb to connect optical and microwave frequencies — establishing a phase-coherent link between f_{rep} and an optical frequency by stabilizing the heterodyne beat between the comb and the optical frequency — can also be demonstrated with the I_2 system. It is in this manner that the comb would be incorporated into an optical clock. After stabilizing the beat between the comb and the cw laser locked to the 532-nm I_2 transition, the fluctuations of the comb repetition frequency are measured by mixing f_{rep} with the microwave signal transmitted from NIST that is referenced to the maser. This generates a difference frequency that can be counted with sufficient resolution to determine the instability of f_{rep} with respect to the microwave reference. Of course, from the previous measurements, it is expected that the instability of the transmitted microwave reference will limit this measurement. Figure 4.5(a) shows a record over ~ 3 h of the fractional fluctuations of f_{rep} obtained in this way, using a gate time of 20 s. The Allan deviation of f_{rep} at averaging times of 20 s and below is obtained using data taken with corresponding gate times, and at longer averaging times it is determined by combining the 10-s data. It is shown in Fig. 4.5(b) and confirms that the measurement is limited by the stability of the microwave reference. To evaluate the true stability of this I_2 -based optical clock, it is necessary to compare

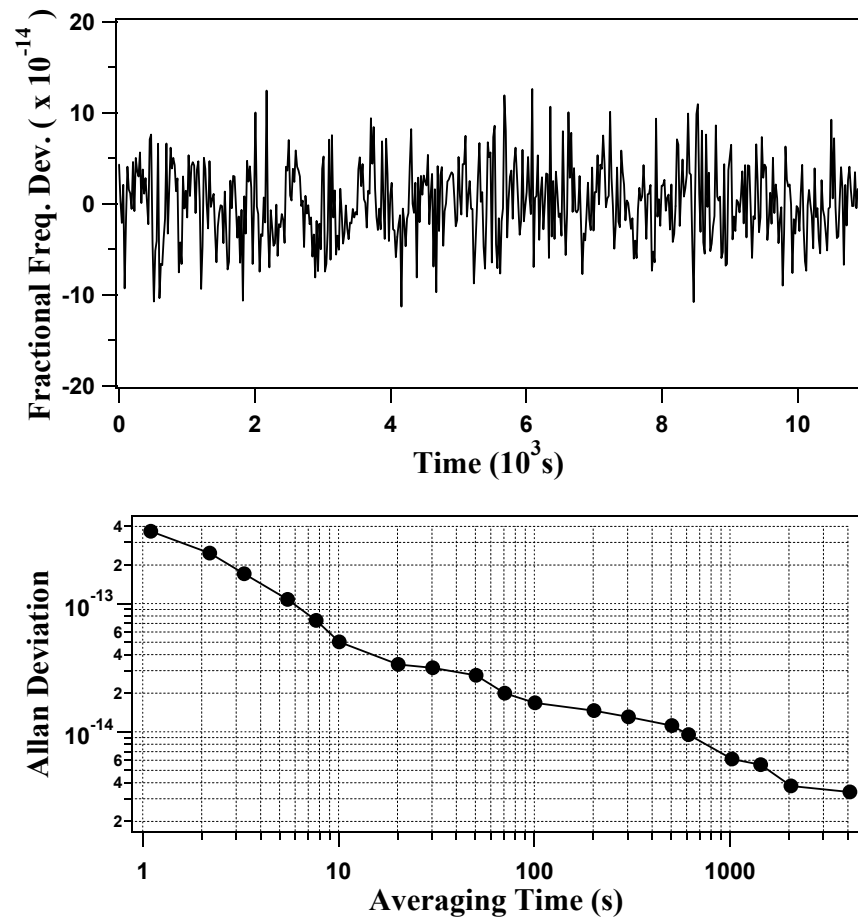


Figure 4.5: (a) The fractional frequency fluctuations of the repetition frequency of the comb when stabilizing the comb to the 532-nm I_2 transition. This represents the manner in which the comb would be used in an optical clock. The fluctuations are measured with respect to a maser-referenced microwave source, using a 20-s gate time for frequency counting. (b) The Allan deviation of f_{rep} for the I_2 -stabilized comb. It is limited by the transmitted maser-referenced microwave signal against which the fluctuations of f_{rep} are measured.

its performance against another system which has a stability equal to or better than that of the I_2 clock.

The stability of the cw laser locked to the 532-nm I_2 transition is determined by comparing it to the highly cavity-stabilized laser at 563 nm that serves as the local oscillator for the single-mercury-ion-based optical standard at NIST [89]. To span the 250-THz frequency gap between the two lasers, a frequency comb at NIST is stabilized to the mercury-ion-standard local oscillator to transfer its stability across the visible spectrum. This enables the measurement of any visible frequency at an instability of 4×10^{-15} for a 1-s averaging time [86]. The stability of the I_2 -stabilized laser is measured by transferring it over an optical fiber to NIST and counting the heterodyne beat between this laser and the stabilized frequency comb. Figure 4.6 shows the Allan deviation of the I_2 -stabilized laser computed from data taken with a 1-s counter gate time, taken from [86]. It is well above the stability limit of the mercury-ion-standard

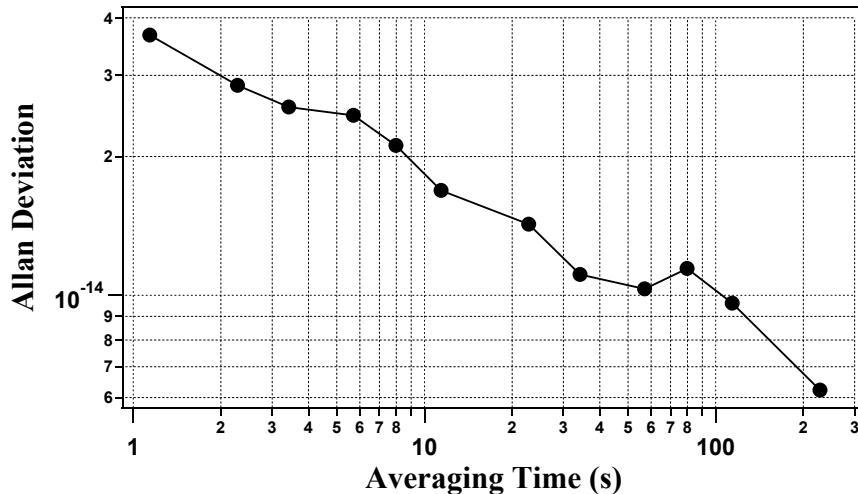


Figure 4.6: The Allan deviation of the I_2 -stabilized cw laser is determined by comparing it with the local oscillator for the single-mercury-ion-based optical standard at NIST. This data is obtained from [86]

local oscillator. Also, by comparing the outgoing frequency of the I_2 -stabilized laser with that transmitted on a round trip to NIST and back, it is verified that the measured

instability is greater than that introduced by the fiber transmission. Therefore, this represents the true stability of the I_2 system. The instability for a 1-s averaging time is $< 4 \times 10^{-14}$ and reaches 1×10^{-14} at 100 s.

4.2 Cold-atom-based optical standard

Although cell-based standards are compact and reliable, the motion of the atoms or molecules limits the performance of these systems. Laser-cooling the atoms [56] significantly reduces their thermal motion, reducing the Doppler-related linewidth of the transitions by a factor of $\sim 10^{-4}$ – 10^{-5} . The most stable optical frequency standards being developed are based on transitions in laser-cooled atoms and ions, as discussed in Section 1.4. Strontium is one example of an atom that can be laser-cooled [49, 52, 57] and offers a few extremely narrow transitions that could potentially be used in an optical clock [50, 37, 67]. A simplified energy level diagram of Sr is shown in Fig. 4.7.

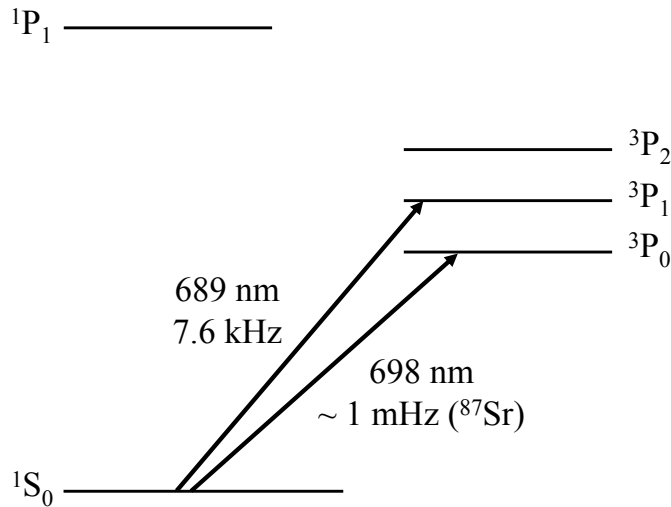


Figure 4.7: The weakly allowed $^1S_0 - ^3P_0$ transition of ^{87}Sr could potentially be used in an optical clock. The broader $^1S_0 - ^3P_1$ transition of ^{88}Sr is useful for demonstrating how a frequency comb can be used to link optical and microwave frequencies.

The $^1S_0 - ^3P_0$ transition is weakly allowed in the fermionic Sr isotope, ^{87}Sr , and this narrow transition is currently being studied for use in an optical clock [74]. The $^1S_0 -$

$^3\text{P}_1$ transition of ^{88}Sr is not one of the ultra-narrow transitions, but it is suitable for demonstrating the use of the frequency comb to link optical and microwave frequency standards. Stabilizing the repetition frequency of the comb to a microwave source referenced to a hydrogen maser that is itself referenced to a Cs fountain clock allows the comb to be used for measuring the absolute frequency of this ^{88}Sr transition.

The $^1\text{S}_0 - ^3\text{P}_1$ transition of ^{88}Sr is centered at 689 nm, and has a natural linewidth of 7.6 kHz. The cw laser used to probe this transition is pre-stabilized to a high-finesse, stable optical cavity, and is then locked to the $^1\text{S}_0 - ^3\text{P}_1$ transition of ^{88}Sr atoms contained in a vapor cell. This produces a probe laser linewidth that is <100 Hz. An AOM is used to shift the frequency of the stabilized probe laser for scanning through the transition of laser-cooled ^{88}Sr atoms. At each position of the scan, the atoms are cooled to ~ 1.3 μK in a magneto-optical trap, and then the trapping laser beams and the quadrupole magnetic field are turned off before probing the atoms and measuring the fluorescence. Measuring the absolute frequency of the probe laser with the maser-referenced comb during the entire scanning period and fitting a Voigt profile to the data of the measured lineshape provides the absolute frequency of the center of the transition [39]. Figure 4.8 shows the measurements of the $^1\text{S}_0 - ^3\text{P}_1$ transition frequency obtained over a period of a little more than a month. These data have a statistical uncertainty of 20 Hz, which corresponds to a fractional uncertainty of $\sim 5 \times 10^{-14}$. This is an order of magnitude lower than the fractional uncertainty for the measurement of the 515-nm I_2 transition, which is $300/(582 \times 10^{12}) \sim 5 \times 10^{-13}$. Taking into account systematic uncertainties as well, this measurement is more than 200 times more accurate than previous measurements of this ^{88}Sr transition using a thermal atomic beam [19].

Since the trapping laser beams shift the transition frequency, they must be turned off before probing the atoms, which results in Doppler-related systematic shifts of the transition and limits the interrogation time. In future work, the Sr atoms will be confined in an optical lattice. As discussed in Section 1.4, carefully choosing the frequency

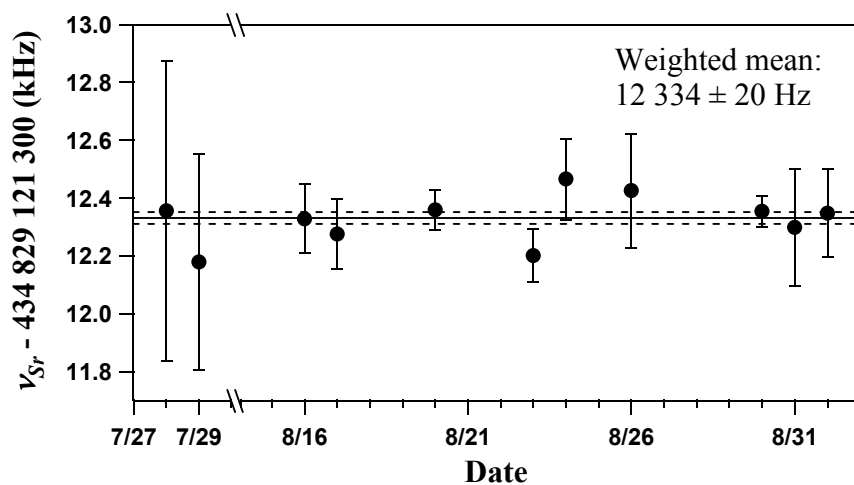


Figure 4.8: The absolute frequency of the $^1S_0 - ^3P_1$ transition in ultracold ^{88}Sr atoms is measured with the maser-referenced frequency comb over a period of ~ 1 month. The maser is itself referenced to a Cs fountain clock. The fractional statistical uncertainty of these data is an order of magnitude lower than that for the measurement of the 515-nm I_2 transition.

of the trapping laser for the lattice minimizes the shift of the transition, allowing the atoms to be probed while trapped. This will allow the ultra-narrow $^1S_0 - ^3P_0$ transition of ^{87}Sr to be studied.

Although the I_2 -based optical clock offers portability and a level of simplicity not possible with a clock based on ultracold atoms, laser-cooled atoms obviously provide much higher stability and accuracy. Even the $^1S_0 - ^3P_1$ transition of ^{88}Sr exhibits an order of magnitude better performance than the I_2 system, and this is not the narrowest clock transition available in Sr. Since systems of laser-cooled atoms or ions are too complex to be easily transported, making use of the superior performance provided only by these systems for the various applications mentioned in Section 1.6 requires the development of reliable transfer of a frequency reference linked to the optical standard. This is also necessary for studying the stability and accuracy of these systems, which can be accomplished only by comparing against similar systems that are often located in separate laboratories.