Appendix B

Single-side-band Generator

A single-side-band generator provides a way to effectively shift the frequency of a signal by a relatively small amount (for example, 10 kHz) [40]. This is useful for measuring the frequency instability of a given signal when it is to be compared against a reference signal of the same frequency [41]. Shifting the reference signal by 10 kHz and mixing the shifted reference and the signal to be measured provides a 10-kHz signal that can be counted with sufficient resolution to detect the frequency fluctuations of the measured signal. The single-side-band generator operates by mixing the original signal, or carrier, with a 10-kHz signal to produce an upper and lower side band on the carrier, and then one of the side bands is interferometrically cancelled.

Figure B.1 illustrates the schematic of a single-side-band generator, with the specific parameters of some of the components given for the single-side-band generator used in the measurements discussed in Chapter 6. The signal to be shifted is first set to a power level of 10 dBm and then split, producing two in-phase 7-dBm signals that each drive the LO port of a frequency mixer. This original signal, at frequency ν, can be considered proportional to $\cos(2\pi\nu t)$. Any phase offset is irrelevant, since it would be equivalent for the two signals produced after splitting the original signal, and is therefore neglected. The two signals driving the mixer LO ports are mixed with two signals at frequency ν' that have a relative phase difference of ϕ_0, where ν' is the frequency difference between the original signal and the side band to be generated. For

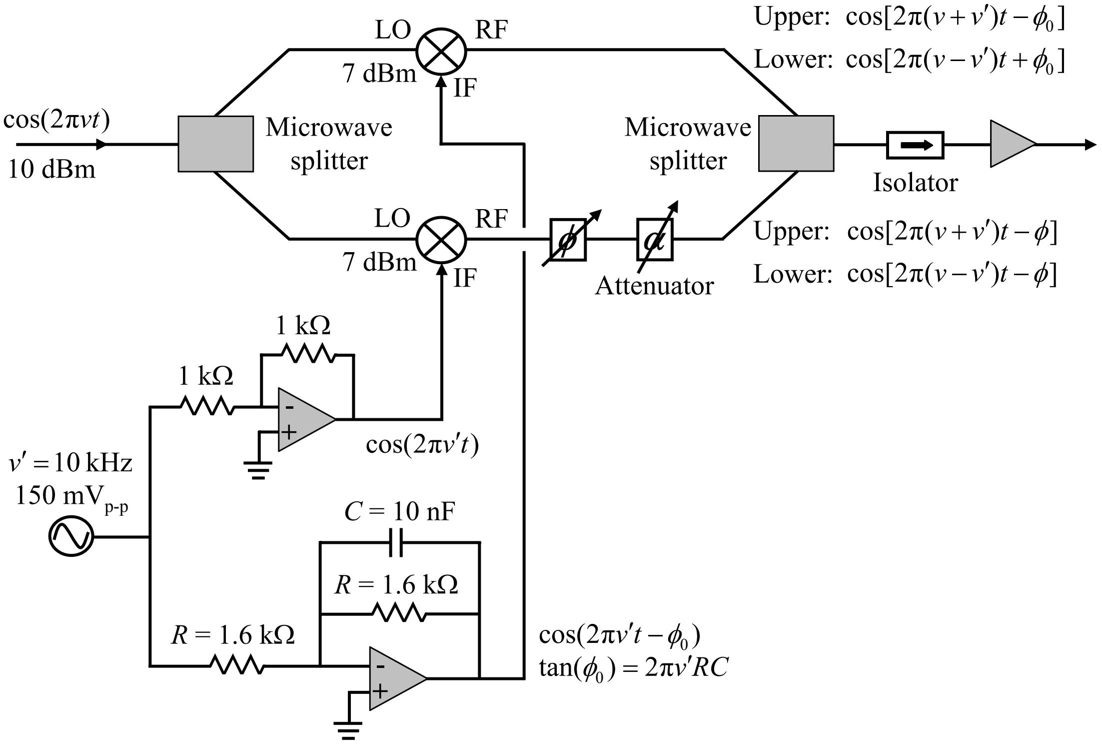

Figure B.1: The single-side-band generator operates by splitting the carrier signal and mixing the two portions with two 10-kHz signals that have a relative phase difference, producing 10-kHz upper and lower side bands on the carrier signals. Combining the mixer outputs and introducing the appropriate relative phase difference between them results in the cancellation of one of the side bands.

the scenario presented here, $\nu' = 10$ kHz. A stable synthesizer operating at 10 kHz and two op-amp circuits produce the two 10-kHz signals with a relative phase difference of ϕ_0. The upper op-amp circuit in Fig. B.1 has unity gain and produces a signal proportional to $\cos(2\pi\nu' t)$. The lower op-amp circuit exhibits a gain of < 1 and produces a signal proportional to $\cos(2\pi\nu' t - \phi_0)$, where ϕ_0 is given by

$$\tan(\phi_0) = 2\pi\nu' RC \tag{B.1}$$

R and C are the resistance and capacitance values of this circuit, as shown in Fig. B.1. Injecting these two 10-kHz signals into the IF ports of the two mixers produces an output from the RF port of each mixer consisting of an upper side band with a phase term that is the sum of the phase terms of the mixer inputs, and a lower side band with a phase term given by the difference of the phase terms of the mixer inputs. An adjustable phase shifter and attenuator after one of the mixers allows the upper side bands of the two mixer outputs to be set to the same amplitude and 180 ° out of phase, so that combining the two outputs results in the complete cancellation of the upper side band.

With the adjustable phase shifter placed after the lower mixer in Fig. B.1 and introducing a phase delay of ϕ, the upper and lower side bands of the lower mixer output are proportional to

$$\text{Upper :} \quad \cos[2\pi(\nu + \nu')t - \phi] \tag{B.2}$$

$$\text{Lower :} \quad \cos[2\pi(\nu - \nu')t - \phi] \tag{B.3}$$

The side bands of the upper mixer output, which is produced using the 10-kHz signal that is lagging in phase by ϕ_0, are proportional to

$$\text{Upper :} \quad \cos[2\pi(\nu + \nu')t - \phi_0] \tag{B.4}$$

$$\text{Lower :} \quad \cos[2\pi(\nu - \nu')t + \phi_0] \tag{B.5}$$

From these expressions, it may at first seem that to cancel the upper side band the optimal choices for ϕ_0 and ϕ are $\phi_0 = 90\,^\circ$ and $\phi = 270\,^\circ$, since the upper side bands will be $180\,^\circ$ out of phase and the lower side bands will be completely in phase. However, in the limit that ϕ_0 approaches $90\,^\circ$, the gain of the lower op-amp circuit approaches 0, eliminating the 10-kHz signal that is delayed by ϕ_0. Instead, a good choice for ϕ_0 is $45\,^\circ$, which corresponds to an attenuation by only a factor of $1/\sqrt{2}$. Then R and C must satisfy the condition $RC = (2\pi\nu')^{-1}$, which they do for the values given in Fig. B.1 when $\nu' = 10$ kHz. With $\phi_0 = 45\,^\circ$, the upper side band is cancelled with $\phi = 225\,^\circ$, which corresponds to a phase difference between the lower side bands of $90\,^\circ$. Note that selecting $\phi = 135\,^\circ$ would instead cancel the lower side band while preserving the upper side band. After the cancellation of one side band, the remaining side band is amplified to a level suitable for use in subsequent measurements. A microwave isolator is placed before the amplifier to prevent reflections from disturbing the destructive interference of the cancelled side band.

Figure B.2 shows an example spectrum of the output from the single-side-band generator, obtained with an RF spectrum analyzer. The original signal has a frequency of 760 MHz, and from Fig. B.2 it is evident that this carrier frequency is still present in the output of the single-side-band generator. This is because of leakage of the carrier through the mixers. The largest peak, 10 kHz below the carrier, is the uncancelled lower side band. The upper side band has been nearly completely cancelled and is only slightly above the noise floor of the spectrum analyzer. After mixing the output from the single-side-band generator with a signal at the carrier frequency for which the frequency fluctuations are to be measured, any low-frequency noise on the 10-kHz mixing product caused by the carrier mixing with the signal to be measured is easily eliminated with a 1-kHz high-pass filter. The peaks in the spectrum that are 20 and 30 kHz from the carrier are due to additional frequency mixings in the two mixers. The ratio of the lower-side-band power to the power of these peaks is maximized by varying

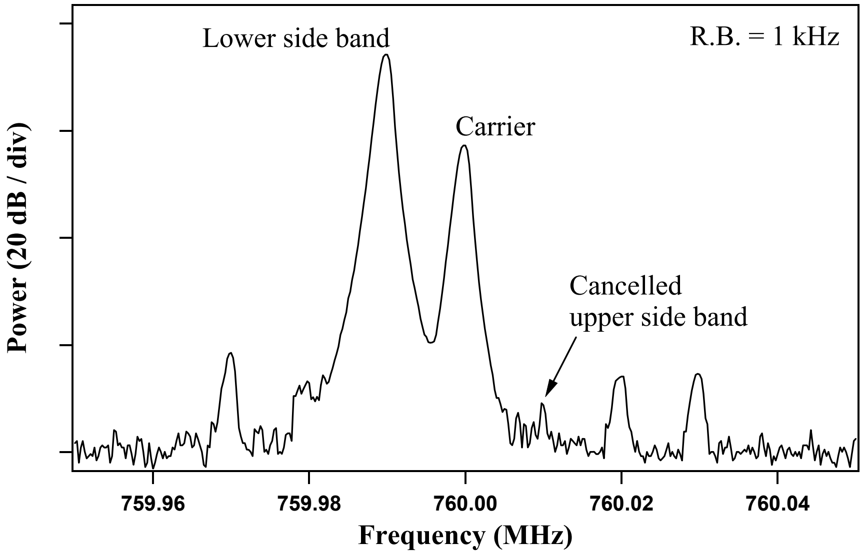

Figure B.2: The spectrum of the output from the single-side-band generator shows that the largest spectral peak is the desired lower side band. Because of leakage through the mixers, the carrier frequency is also present in the output. The upper side band has been nearly completely cancelled. R.B., resolution bandwidth.

the amplitude of the 10-kHz synthesizer used to produce the 10-kHz signals that are mixed with the original signal in the single-side-band generator. The optimal amplitude for the synthesizer is 150 mV peak-to-peak, as indicated in Fig. B.1.

When the single-side-band generator is used for determining the frequency instability of a signal as described earlier, it is important to consider what limits the stability of the single-side-band generator itself. Suppose the frequency of a 1-GHz signal is to be measured with respect to a 1-GHz reference with a sensitivity of a part in 10^{15} for an averaging time of 1 s. This corresponds to a 1-μHz deviation of the reference. Therefore, the 10-kHz frequency, which shifts the 1-GHz reference for comparison against the signal to be measured, must have fluctuations less than 1 μHz over a 1-s averaging time. Though this corresponds to an easily attainable fractional frequency deviation of the 10-kHz source of 1×10^{-10} for a 1-s averaging time, few commercially available high-stability 10-kHz sources provide an absolute frequency precision better than 1 μHz. Therefore, the fractional frequency instability of a single-side-band generator designed for a 1-GHz reference is typically $> 1 \times 10^{-15}$ over a 1-s averaging time, limited by the precision of the 10-kHz source.