

# **Vortices in a Bose-Einstein Condensate**

by

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Vortices in a Bose-Einstein Condensate

Thesis directed by Prof. Eric A. Cornell

Since the advent of Bose-Einstein condensation in the dilute alkalis, there has been considerable interest in observing effects in atomic condensates akin to the hallmark effects associated with superfluidity and superconductivity. In particular, the study of quantized vortices and vortex lattices represents an important connection between the traditional "super" systems such as liquid Helium and this new atomic system.

This thesis explores some of the first vortex experiments in a condensate of magnetically trapped Rubidium-87. Single vortex lines and rings are created using a wavefunction engineering technique, which is an ideal starting point to study the dynamical behavior of vortices within the condensate. An entirely different approach of "intrinsic nucleation" has been developed to create rapidly rotating condensates with large amounts of vorticity. A novel variation of forced evaporation is used to simultaneously cool and spin up an ultracold gas. In this way, condensates can be formed that contain large, extraordinarily regular lattices of well over 100 vortices. Direct detection of the vortex cores makes it possible to study the microscopic structure of the vortex arrangements both at equilibrium and under dynamical conditions where severe applied stresses distort the lattice far from its equilibrium configuration.

In conclusion, the techniques developed in this thesis have opened up a new area of rotating condensate physics and, in the future, may lead to regimes of extreme rotation and quantum Hall physics.

## Dedication

To my parents and my two brothers, for a level of support that continues to baffle and amaze me.

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