Chapter 1

Introduction

1.1 Bose-Einstein condensation and the “super” systems

The phenomenon of Bose-Einstein condensation(BEC) has a rich history over the past century [155], beginning with the recognition of two classes of particles in nature, bosons and fermions, obeying Bose-Einstein and Fermi-Dirac statistics respectively: In a system of indistiguishable particles, an arbitrary number of bosons may occupy a single quantum state whereas fermions are restricted by the Pauli exclusion principle to a single particle per state. Remarkably, according to the “spin statistics theorem,” particles in nature with integer spin are bosons while those with half-integer spin are fermions. The different particle statistics have fundamental and far-reaching physical consequences; however, effects become evident only at low[1] temperatures where particles gather in the lowest energy states of a system and the average occupation of a state approaches unity. Early in the 1900s, theory by Bose [56] and Einstein [76] predicted that if a gas of ideal(non-interacting) bosons is cooled down, a quantum phase transition will occur at a critical temperature $T = T_C$, below which a macroscopic population “condenses” into the ground state of the system. At $T = 0$, the condensate fraction reaches one. It certainly is not surprising that all the particles end up in the ground state at $T = 0$ since it is the state with the lowest energy. Significantly, however, the macroscopic

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[1] A “low” temperature is defined relative to the characteristic spacing of a system’s energy levels and need not be low in any particular absolute terms.

occupation persists at finite temperatures up to the critical temperature.

Following the theory of Bose and Einstein, London [115] was the first to suggest that BEC could account for the experimentally observed superfluid transition in liquid ^{4}He, even though the strong interatomic interactions in the liquid clearly put the system far from the concept of an ideal Bose gas. Implicit in this discussion is the fact that ^{4}He, although composed of fermions (electrons, protons, neutrons), is nevertheless a composite boson, the constituent spins of which add up to give a total integer spin. The original motivations of London are not delved into here; instead, some of the relevant consequences are pointed out. In a liquid helium sample, the condensate is associated with the superfluid component. As a macroscopically occupied quantum state, it has an associated wavefunction or order parameter $\Psi = \sqrt{\rho(\vec{r})}e^{i\mathcal{S}(\vec{r})}$. As Tilley points out [155], this fact in itself made London's proposal a bold suggestion. After all, wavefunctions were associated at the time with the microscopic quantum world of atoms but London was suggesting that a continuous wavefunction exists over the scale of a bucket of liquid helium!

Given the existence of an order parameter, currents of the superfluid or "superflows" are associated quantum mechanically with gradients in the phase $\mathcal{S}$. As originally emphasized by Onsager [131] and Feynman [85], this has a direct bearing on the meaning of a rotating quantum fluid. Since the order parameter needs to be single valued, the phase around a closed loop must return on itself or, if there is a hole in the superfluid, it can increment by an integer multiple of 2π. The latter case describes the multiply-connected topology of a superfluid pierced by one or many quantized vortex cores. The connection between quantum mechanical phase and velocity is a rather strong statement permitting rotation to enter a superfluid *only* in the form of quantized vortices. In the "rotating bucket" experiments with liquid helium, a rotating normal fluid is cooled below the superfluid transition. Above a critical rotation rate Ω_C, singly-quantized vortices are nucleated at the edge of the superfluid and migrate inwards. Higher rotation rates lead to the nucleation of more vortices, which come to form stationary arrays in the rotating frame. In 1979, Yarmchuk and Packard [160] reported the direct observation of individual vortex cores in arrays of up to eleven vortices. The visual observation of vortex lattices, together with the quantization of vortex circulation as first demonstrated by Vinen [156], represents one of the hallmarks of superfluidity.

Following superfluid ^{4}He, the list of physical systems exhibiting a macroscopically occupied quantum state grew to include superfluid ^{3}He, superconductors and lasers. The latter two systems have resulted in countless technological advances. In the case of superconductors, conduction electrons pair up to form composite bosons which produce persistent supercurrents, directly analogous to the superflows in liquid Helium II. Magnetic fields play the role of rotation in a superconductor. Thus, above a critical applied magnetic field, Abrikosov lattices of magnetic flux quanta are observed [1], having the same triangular structure predicted for vortex lattices in superfluid ^{4}He.

In 1995, the newest member of the macroscopic quantum states was demonstrated experimentally - BEC in a trapped dilute alkali gas. There are currently in excess of 30 gaseous BEC experiments around the world using several different atoms (Cs, H, He*, Li, K, Na, Rb ...). With a density 10^{-6} less than that of air at STP, a dilute-gas condensate involves interatomic interactions that are relatively weak compared those in helium liquid (although this is an evolving frontier [62]). The result is a theoretical description that is more tractable. The dilute nature of the gas also means that the ground state fraction is large, essentially 100% compared with ∼10% in Helium II, where the interactions deplete the ground state [155]. A novel feature in the atomic vapour experiments is the ability to directly see the density profile of the condensate's wavefunction with optical imaging, central in fact to the results of this thesis. For this reason in particular, dilute gas condensation as a visually accessible phenomenon has become synonymous with the concept of ”BEC”.

One of the most interesting (and fun) aspects of atomic BEC is the interplay it has with other areas of physics, be it quantum optics or condensed matter. For example, a significant amount of evidence has been amassed to connect superfluidity in ^{4}He to the phenomenon of Bose-Einstein condensation. One may ask though: is dilute-gas BEC a superfluid? The question is more than just semantics. To answer it, one arguably should demonstrate the canonical experimental properties that *define* superfluidity [114], such as the rotating bucket experiments mentioned above, although the choice of experiments ends up being somewhat subjective. It is useful to try to distill the basic features of a condensate that contribute to its superfluid character. Consider rotation properties specifically. The existence of a macroscopic wavefunction alone leads to the quantization of circulation characteristic of a superfluid; however, equally crucial is the role of repulsive interactions in the condensate since they are responsible for the "persistence" of quantized vortices as stable topological objects in the fluid. Garcia-Ripoll *et al.* [88] refer to this as the "structural stability" of a vortex, which may be understood as follows: A quantized vortex cannot disappear from the condensate wavefunction by "spinning down." The multiply-connected topology of a wavefunction containing a vortex core, like a rubber sheet with a hole in it, can be destroyed only by cutting the density of the condensate along a radial line from its edge to the vortex core. Since the repulsive interatomic interactions give the condensate a finite compressibility, the suppression of the condensate density along a radial line entails an energy cost. It is this energy cost which makes a quantized vortex a structurally stable topological feature. In the absence of sufficiently strong repulsive interactions, an asymmetry introduced in the condensate's confining potential would presumably destabilize the vortex [88]. An analogous situation has been experimentally demonstrated for an optical vortex contained in a laser beam propagating in free space [125]. Under an asymmetric perturbation from a cylindrical lens, the optical vortex dynamically inverts its topological charge, which is thus clearly not a stable quantity.

1.2 Rotating Bose-Einstein condensates

This thesis addresses the experimental creation of quantized vortices, from one to many, in a dilute-gas condensate of ^{87}Rb. Concurrent with the results presented here, a number of other experimental groups have also addressed the rotational behaviour of a condensate, including studies of vortices [118, 48, 104, 103, 75] and the measurement of a condensate's non-classical moment of inertia [119, 100]. The overall body of work, both theoretical and experimental, is already sizeable, making rotating condensates its own little sub-field of gaseous BEC.

A general BEC workshop took place at JILA in 1999 near the outset of this thesis work. At the time, there was considerable discussion and uncertainty how a vortex could be nucleated into a condensate. The discussion centered around the analog of the "rotating bucket" experiment for trapped gases. The prime question related to the nature of the energy barrier that a vortex would need to overcome to be nucleated into the condensate. This whole issue was initially circumvented when Williams and Holland proposed a novel way to *directly* create a condensate containing a vortex with wavefunction engineering [159]. Based on earlier experimental work in this group with coupled two-component condensates, the wavefunction engineering technique led to the first creation of a (single) vortex in a dilute-gas condensate [122]. At about the same time, the ENS group was successful at nucleating vortex lattices by stirring the condensate in a deformed confining potential [118]. In very rapid succession, there were results from several groups providing different methods of nucleation and elucidating vortex behaviour. Vortices, it seemed, were "everywhere." In retrospect, it seems strange that there was any uncertainty at all in 1999 about vortex nucleation in a condensate! The field was and continues to be extremely active, which has made it a fascinating area to work in.

The wavefunction engineering technique, with its ability to introduce a vortex into a condensate "artificially" with reproducible initial conditions, has proven to be a useful method to study a wide variety of vortex dynamics in a $T = 0$ condensate [Chapter 3]. Additionally, the technique allows the creation of two-component vortices in which the vortex state is composed of one internal spin state of ^{87}Rb while the core is filled with fluid composed of another simultaneously trapped spin state. The mutually repulsive interaction between the two components opens up the possibility to study vortex dynamics in mixtures of interacting superfluids, where the vortex angular momentum need not be confined to one component.

As a general technique, wavefunction engineering has even been extended to make dark solitons in a condensate. The dark soliton, which is the matter wave equivalent of the optical phenomenon, corresponds to a nodal plane cutting across the condensate and an associated π phase discontinuity. Unlike vortices, solitons are not topological features in the condensate wavefunction. Indeed, after their creation, the solitons are observed to rapidly decay to vortex rings in our spherical trapping geometry. Although interesting in its own right, this decay process can be considered an example of wavefunction engineering whereby an unstable precursor state decays to a desired target state, the vortex ring. This may prove useful in the future for making more complicated topological structures in the condensate.

The vortex work based on wavefunction engineering focused on the "quantum limit" of single vortices in a condensate essentially at zero temperature. The variety of experiments not only provided some interesting vortex physics but, perhaps most importantly, led to the acquisition of a number of experimental techniques for the manipulation and detection of vortices. These techniques have continued to prove useful as the experimental direction has moved from the creation and study of condensates with single vortices to the opposite limit of highly rotating condensates containing large lattices of a hundred or more vortices.

The route to large vortex lattices in a condensate ultimately involves the mechanical addition of angular momentum to the trapped gas, just as for the original liquid helium experiments. Coming full circle with the 1999 workshop, a point of controversy has been the exact nature of the vortex nucleation mechanism, this time in the context of experimental results. The group at ENS and later those at MIT and Oxford have nucleated vortex lattices by directly addressing the condensate with a stirring potential. In this “extrinsic” nucleation technique, the stirring action of the potential has been found to drive surface waves on the condensate to instability. The crests of the waves break, nucleating vortices in the resulting turbulence of the “surf”. The approach to vortex nucleation taken in this thesis [Chapter 4] has followed a somewhat different route, which at the time offered the possibility of a different physical mechanism for nucleation. In this new method, an ultracold gas above T_C is first brought into rotation and then evaporatively cooled to quantum degeneracy. Above a critical rotation rate, vortices are nucleated into the condensate. This “intrinsic” nucleation is due only to the interaction of the normal cloud with the condensate since the trapping potential is left static (and rotationally symmetric to conserve angular momentum) throughout the evaporation. After a flurry of discussion in the literature, the nucleation mechanism appears to be due again to surface-wave instabilities, driven not by a rotating trap roughness but instead by thermal fluctuations from the rotating normal cloud.

Nevertheless, a significant benefit of the intrinsic nucleation method is the generation of large amounts of vorticity in a condensate. A key feature of the evaporative technique is that it is designed to simultaneously cool and spin up a trapped gas to higher rotation rates. As a result, highly rotating condensates of a million or more atoms can be formed containing 130 or more singly quantized vortices. They are crystallized in remarkably ordered lattices, spanning ten or more lattice constants across the condensate. In a particular sense, these vortex lattices are very much the opposite of the “quantum” limit of a single vortex. For the large numbers of vortices in the condensate, the correspondence principle would suggest that the rotation field, coarse grained over the vortex structure, should go over to the classical limit of rigid-body rotation. In fact, the condensate does centrifugally distend like a rotating classical gas. From the distortion to the condensate shape, an effective rigid-body rotation rate as high as 95% of the centrifugal limit has be inferred (and this value is rising even higher in current experiments). The large rotation rates and condensate number have been brought to the point of reproducibility that the BEC machine is now truly a *vortex machine*.

The reproducible production of highly rotating condensates containing large, equilibrium vortex lattices is an ideal spring board for a number of different experiments. In the first instance, a study has been made of the non-equilibrium lattice dynamics induced when large amplitude shape deformations are applied to the condensate. The anisotropic stress of the shape deformations induces a variety of strain responses in the vortex lattice including shifting of lattice planes, changes of lattice structure, and the formation of sheet-like structures in which individual vortices appear to have merged.

Finally, it was mentioned above how in presence of large amounts of vorticity a condensate attains the classical limit of rigid-body rotation. However, in the limit of truly extreme rotation rates one approaches what may be called the "true quantum limit" where the amount of vorticity approaches the number of particles within the condensate. Under these circumstances, recent theory [63, 133, 101, 146] predicts the transformation of a condensate from a vortex lattice state to a strongly correlated quantum system, analogous to the quantum Hall regime in condensed matter physics. Given the versatile set of experimental tools that can be brought to bear on the condensate, it is a possibility more than a hope that quantum Hall physics will be accessed in dilute-gas BEC in the near future.

1.3 JILA Mark III: keeps on going, and going . . .

Underlying all the physics explored in this thesis is a BEC apparatus, JILA Mark III, originally constructed by previous members of the group as a "next generation" machine [78, 120]. Diode laser technology and a vacuum system without moving parts have made this experiment extraordinarily reliable as far as experiments go. Consider that the glass cells have been under continuous high vacuum since their construction in 1996. The original ^{87}Rb dispenser is still being used with several more available in-chamber. Having a cycle time of ∼1min, the machine has produced about a half-million condensates during the time of this thesis and averaged two successful experiments per year. Admittedly, the apparatus has undergone a gradual overhaul with time. Lasers have been changed and several more added. There are now at least as many lasers devoted to manipulating and probing the condensate as there are ones devoted to its production. Although practically every part of the setup has undergone modification, the same basic apparatus underlies it all (if you can still see it under the optics and coils!) Only now is the experiment beginning to suffer from some limitations, particularly in optical access, leaving room for plenty of ingenuity.