Appendix A

ADK ionization rate calculation

The numerical calculations of the ionization rate use the expression in Ref. [71] for the Ammosov-Delone-Krainov (ADK) theory including the Keldysh theory after Coulomb correction. The tunnel ionization rate is given by

$$\omega(t) = \omega_p |C_{n*}|^2 (\frac{4\omega_p}{\omega_t})^{2n*-1} \exp(-\frac{4\omega_p}{3\omega_t}) \tag{A.1}$$

with

$$\omega_p = \frac{I_P}{\hbar}$$

$$\omega_t = \frac{eE_l(t)}{(2mI_p)^{1/2}}$$

$$n* = Z(\frac{I_{ph}}{I_p})^{1/2}$$

$$|C_{n*}|^2 = 2^{2n*}[n*\Gamma(n*+1)\Gamma(n*)]^{-1}$$

where I_p is the ionization potential of the gas species, I_{ph} is the ionization potential of the hydrogen atom, $E_l(t)$ is the electric field of the laser, and m is the mass of the electron.

From this rate, it is straightforward to calculate the ionization fraction as a function of time during the laser pulse given by:

$$\eta(t) = 1 - \exp[-\int_{-\infty}^{t} dt' \omega(t')]. \tag{A.2}$$

Matlab Code

```
Eion = 0.58065; % Ionization Potential for Argon in a.u.
Eion2=27.6/27.21; % 2nd Ionization Potential for Argon in a.u.
%Eion = 21.5646/27.21; % Ionization Potential for Neon in a.u.
%Eion2=40.1/27.21; % 2nd Ionization Potential for Neon in a.u.

QN = 1./sqrt(2*Eion);
QN2 = 2./sqrt(2*Eion2);
Ain = 5.0E14; % Peak intensity in W/cm2
Ain = sqrt(Ain/3.51E16); % Electric field amplitude

Tpul = 30.; % Pulse FWHM in fs
Tpul = Tpul*1E-15*4.1322E16;
Tf = Tpul*0.84932;
wo = 0.05695;
TT = 2.*pi/wo;
Nx = 10000; % Number of time steps
HT = 4.*Tf/Nx;
for i=1:Nx
    T = i*HT;
    E(i)=Ain*exp(-(T-2.0*Tf)^2/Tf^2)*cos(wo*(T-2*Tf));
    Int(i)=(Ain*exp(-(T-2.0*Tf)^2/Tf^2))^2;
end

N(1)=1.;
N1(i)=0.;
N2(1)=0.;
T(1)=0.;
Rint=0.;
for i=1:Nx-1
    T(i+1) = i*HT;
    F = abs(E(i))+1.E-30;
    C2=2^(2*QN)/(QN*gamma(QN+1)*gamma(QN));
    C22=2^(2*QN2)/(QN2*gamma(QN2+1)*gamma(QN2));
    Rate1(i)=Eion*C2*(4.0*Eion*sqrt(2.0*Eion)/F)^(2.0*QN-1.0)*...
        exp(-4.0*Eion*sqrt(2.0*Eion)/(3.0*F));
    Rate2(i)=Eion2*C22*(4.0*Eion2*sqrt(2.0*Eion2)/F)^(2.0*QN2-1.0)*...
        exp(-4.0*Eion2*sqrt(2.0*Eion2)/(3.0*F));
    Rint=Rint+Rate1(i)*HT;
    N(i+1)=exp(-Rint);  % Number of Neutrals
    N1(i+1)=1.-N(i)-N2(i);  % Number of Single Ions
    N2(i+1)= N2(i)+Rate2(i)*N1(i)*HT;   % Number of Double Ions
    Rate1(i)=Rate1(i)*N(i);
    Rate2(i)=Rate2(i)*N1(i);
end

Neutral=1-N;
Ion1=N1;
Ion2=N2;
T=(T-2*Tf)/TT*2.67;  % Convert to fs
```