

**The First Experiments with Bose-Einstein
Condensation of ^{87}Rb**

by

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The First Experiments with Bose-Einstein Condensation of ^{87}Rb

Thesis directed by Prof. Eric Cornell

Bose-Einstein Condensation (BEC) is the macroscopic occupation of the ground-state of a system of bosons that occurs when the extent of the wavefunctions of the particles is comparable to the interparticle spacing. Although predicted by Albert Einstein in 1924, BEC in a dilute system was observed only recently in an atomic vapor of ^{87}Rb by our group in 1995. This thesis describes the first experiments to explore the properties of this new state of matter. In early experiments, we studied how interparticle interactions modify the ground-state wavefunction and mean energy. We observed phonon-like collective excitations of the condensate. We studied modes of different angular momenta and energies. Our observations of how the characteristics of the modes depend on interactions quantitatively supported the mean-field picture of the dilute BEC. Shortly thereafter, we developed thermometry and calorimetry to study the ground-state fraction and mean energy of the Bose gas as a function of temperature. The BEC transition temperature and the temperature dependence of the ground-state fraction are in good agreement with predictions for an ideal Bose gas. However, the measured mean energy is larger than that of the ideal gas below the transition. We observe a distinct change in the energy-temperature curve near the transition, which indicates a sharp feature in the specific heat.

In an effort to produce larger condensates we constructed a double-MOT apparatus that became the third-generation machine at JILA to observe

and study BEC. The new apparatus produces condensates five times more quickly than the original experiment, increasing the number of atoms in the condensate from several thousand to 1-2 million atoms. Using the improved apparatus, we studied the TOP (time-averaged orbiting potential) magnetic trap. An important, new observation is that the trap symmetry is affected by the sag due to gravity, an effect which can be exploited to create very harmonic, spherical potentials. We also measured a sharp decrease in trap lifetime for bias fields below 1 G. The improved understanding of the TOP trap should enable future interesting experiments with BEC.

I dedicate this thesis to my family, and especially to Mom.

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