Chapter 3

Collective Excitations of a Bose-Einstein Condensate in a Dilute Gas

3.1 Introduction

Early experiments with BEC showed that interatomic interactions influence the energy and wavefunction of a zero-temperature condensate [121, 6]. Using simple Gaussian or paraboloid models of the density distribution of the condensate, we observed good qualitative agreement between our energy data and the predictions of the Gross-Pitaevskii mean-field theory [124, 125, 126]. Unfortunately, the model profiles fit the subtle features of the condensate profile too crudely, and thereby compromised a quantitative and detailed test of theory. Gross-Pitaevskii theory also makes predictions for the existence of elementary condensate excitations [9, 10, 11, 12, 134, 135]. Such time-dependent behaviour was more amenable to a precise measurement since it was easier to measure a frequency accurately than to determine the absolute features of the density profile. We report herein on studies of low-energy, collective excitations of a dilute condensate of ^{87}Rb. Characterizing these low-lying excitations was a first step towards understanding the dynamics of this novel quantum fluid [136].

3.2 Experimental Procedure

The apparatus and procedures for creating and observing BEC were the same as used in Section 2.3. For the experiments described here, the final evaporation took place in a trap with a radial frequency of 132 Hz (373 Hz axial). To approach the zero-temperature limit, we evaporated well below the BEC phase transition [137] so that the expanded clouds showed no sign of having a thermal component. At higher temperatures, this component appeared as a broad, symmetric Gaussian background [22]. Typically we had 4500 ± 300 atoms in the condensate.

3.3 Theory of Excitations

The standard theory for BEC in a dilute atomic vapor uses a nonlinear Schrödinger equation, the Gross-Pitaevskii equation, to describe the condensate wavefunction in the limit of zero temperature [138]. This equation comes from a second-quantized mean-field theory with the interatomic interactions modeled by an s-wave scattering length. The elementary excitations of BEC in a finite, harmonically-confined dilute gas were studied theoretically using this model [9, 10, 11, 12, 134, 135]. The lowest energy normal modes of the condensate, corresponding to rigid-body center-of-mass motion ("sloshing"), were predicted to occur at the trap frequencies. The frequencies of the next lowest condensate excitation modes, however, were expected to deviate from the spectrum of a cloud of ideal gas, for which the excitation frequencies are simply multiples of trap frequencies. Not surprisingly, the amount of deviation depends on the strength of the interatomic interactions.

3.4 Exciting collective modes

We excited these collective modes of the condensate by applying a small time-dependent perturbation to the transverse trap potential. We generated the perturbations by applying a sinusoidal current to the coils responsible for the rotating field of our TOP trap (in addition to the normal TOP currents). The response of the condensate depends on the symmetry of the driving force as well as the driving frequency, ν_d. By appropriately setting the phases of the currents through the coils, we generated perturbations with either of two different symmetries. We label these two driving symmetries $m = 0$ and $m = 2$, where m, the angular momentum projection onto $\hat{z}$, is a good quantum number because of the axial symmetry of our unperturbed magnetic trap. Equipotential contours for the two trap perturbations are shown in Fig. 3.1. The $m = 0$ drive preserved axial symmetry and corresponds to an oscillation in radial size. For the $m = 2$ drive symmetry, the trap spring constants along $\hat{x}$ and $\hat{y}$ were modulated 90° out of phase. This corresponded to a normal mode resembling a transverse ellipse whose major axis rotates in the $x - y$ plane.

The basic spectroscopic approach was as follows: we distorted the cloud by applying the perturbative drive for a short time, then allowed the cloud to evolve freely in the unperturbed trap for a variable length of time. Finally, we turned off the confining potential suddenly and imaged the resulting cloud shape after 7 ms of free expansion.

3.5 Observation of Excitations

Initial studies were made in a 132 Hz trap. The perturbative drive pulse duration was 50 ms, the center frequency was set to match the frequency

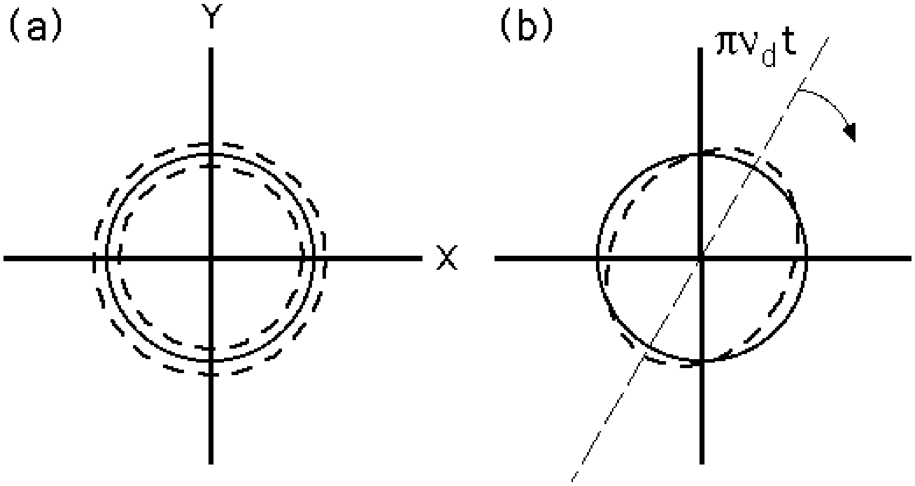

Figure 3.1: In the unperturbed trap, contours of equipotential in the transverse plane are symmetric (solid line). To drive the $m = 0$ excitation (a) we applied a weak harmonic modulation with frequency ν_d to the trap radial spring constant. The $m = 2$ drive (b) broke axial symmetry with elliptical contours which rotate at $\nu_d/2$. The amplitude of perturbation is shown exaggerated for clarity. Figure taken from Ref.[8].

of the excitation being studied, and the amplitude was 1.5% of the radial spring constant of our trap. We observed two different collective excitations of the condensate. The observables in both cases were the widths of the expanded clouds as a function of the free evolution time. In one case we observed a sinusoidal oscillation of the radial width at a frequency of $(1.84 \pm 0.01)\nu_r$, where ν_r is the radial trap frequency and the error quoted reflects only statistical uncertainties. This mode was driven by the axially symmetric $m = 0$ trap perturbation and was not observed for the drive with $m = 2$ symmetry (with the same drive amplitude and frequency). The cloud widths oscillated in both axial and radial directions, with approximately opposite phase. This response is shown in Fig. 3.2. The observed phase difference between the oscillations of axial and radial widths is not exactly π; however, the free expansion of the condensate prior to imaging complicates analysis of this phase shift [139]. The second excitation oscillated freely at $(1.43 \pm 0.01)\nu_r$, and appeared in response to a $m = 2$ drive, and not to a $m = 0$ drive. In this case, the radial width oscillated, with no observable response in the axial width. The two-dimensional projection of an ellipsoidal cloud whose major axis rotates in the transverse plane would exhibit this behavior.

We calibrated the observed excitation frequencies in units of the trap frequencies by making similar measurements on non-condensate clouds. The temperature of these clouds, in units of the BEC transition temperature, is $T/T_c \approx 1.3$; consequently the density and thus the interactions were very small. Here, we saw a response that oscillated at 264 Hz and was driven with either symmetry. A harmonically confined, noninteracting gas pulses at twice the radial trap frequency, so this gave $\nu_r = 132 \pm 1$ Hz. We also checked that the thermal cloud did not respond when driven at $1.43\nu_r$.

3.6 Effect of Interactions

In the s-wave scattering approximation, the interactions for ground state ^{87}Rb atoms in our trap are repulsive (positive scattering length) [108, 131], providing an effective potential energy which favors a lower central density of the condensate compared to the non-interacting case. This interaction energy determines the excitation spectrum of the condensate [140]. We were able to examine this effect because BEC in trapped neutral atoms offers the advantage of an adjustable interaction energy. In the standard mean-field picture, the strength of the non-linear interaction term in the Gross-Pitaevskii equation, relative to the harmonic trap's energy-level spacing, scales with $Na\sqrt{\nu_r}$ for the regime of interactions in which the ground-state density is well-approximated by a Gaussian [128, 129, 130, 5]. Thus, by varying the trap frequency or the number of atoms, N, we changed the relative importance of interactions in the condensates.

We measured the excitation frequencies of the $m = 0$ and $m = 2$ modes of the condensate as a function of both N and ν_r. In the 132 Hz trap, we changed the relative interaction strength by reducing the number of atoms. To change ν_r, we evaporated to BEC in the 132 Hz trap, then adiabatically ramped the trap fields until the condensate was held in a trap with an axial frequency of 43.2 Hz. In this lower frequency trap, we excite the condensate with a 100 ms pulse and a drive amplitude equal to 3% of the radial spring constant. The observed fractional amplitude of the oscillations in the cloud width was typically 11% of the mean width for this drive. We measured the free oscillation frequency of the $m = 0$ and $m = 2$ modes in this trap to be $(1.90 \pm 0.01)\nu_r$ and $(1.51 \pm 0.01)\nu_r$, respectively, for $N \approx 3000$.

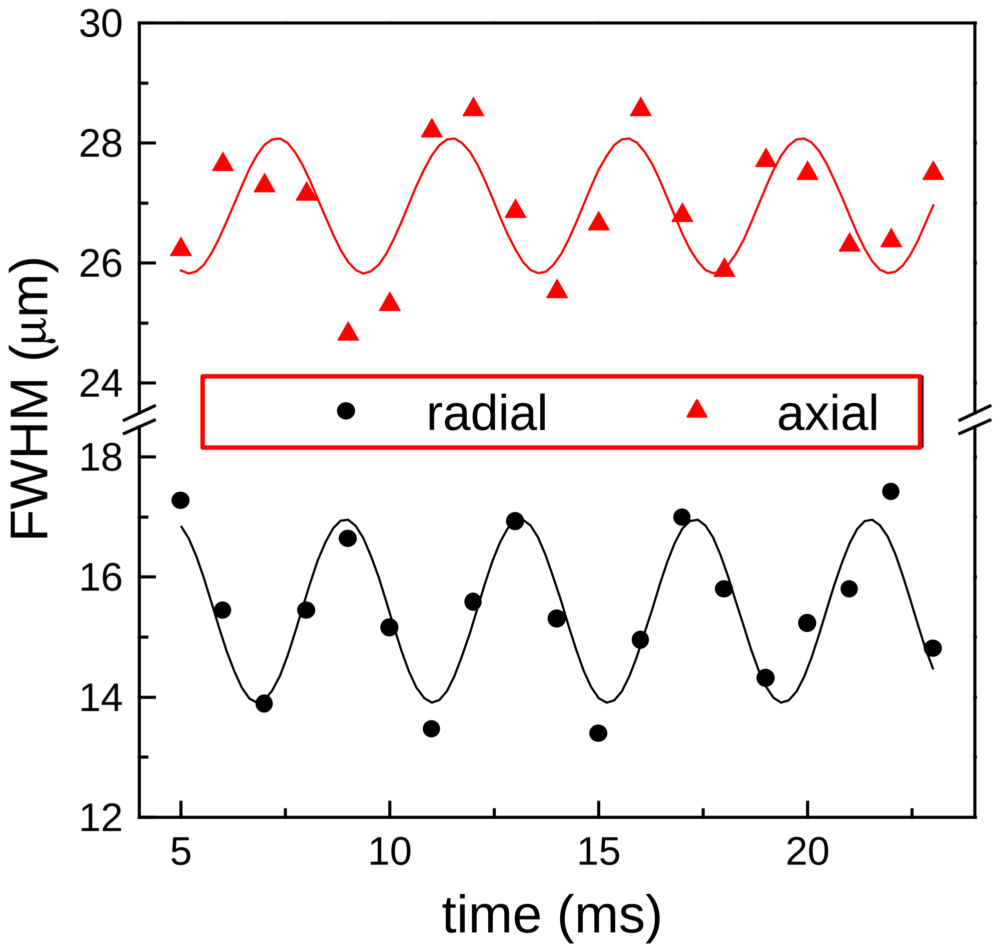

Figure 3.2: We applied a weak $m = 0$ drive to a $N \approx 4500$ condensate in a 132 Hz (radial) trap. Afterward, the freely evolving response of the condensate showed radial oscillations. Also observed is a sympathetic response of the axial width, approximately 180° out of phase. The frequency of the excitation was determined from a sine wave fit to the freely oscillating cloud widths. Each data point represents a single destructive condensate measurement. Figure taken from Ref.[8].

The measured excitation frequencies as a function of interaction strength are shown in Fig. 3.3. By using the product $N\sqrt{\nu_r}$ for the dependent variable we combine our different number and trap frequency data into one graph. The solid lines in Fig. 3.3 show the mean-field theory calculation by Edwards *et al.* [9, 10], using the best value of the scattering length for ground state ^{87}Rb atoms available at the time, $a = 110a_o$ [108], where a_o is the Bohr radius. An extension of this calculation by Esry and Greene is shown with dotted lines [11]. Finally, dashed lines indicate the prediction by Stringari for the "strongly interacting" limit [12], in which the kinetic energy of the ground state is ignored. Our data agreed reasonably well with these mean-field theory results; the measured energies of the low-lying collective excitations of the condensate deviate from the simple harmonic trap spectrum as predicted, with larger deviation for larger interaction strength. Error bars in Fig.3.3 indicate statistical error in the determination of the frequencies, but do not include possible systematic errors such as day-to-day variations in the trap magnetic fields, and therefore frequencies (estimated to be less than 0.5%). A significant systematic also arose from our method of using the $m = 0$ non-condensate mode to calibrate the trap frequency: Measurements of finite temperature excitations [141] indicated this caused our measured excitation frequencies to be low by 1%. Also, the theoretical curves are strictly valid only in the limit of zero temperature and zero amplitude.

In the limit of low excitation amplitude, the spectrum of low-lying collective excitations corresponds exactly to the Bogoliubov quasi-particle spectrum [9, 10]. The collective condensate response to our trap perturbation, in the limit of low amplitude, is simply a coherent state of these elementary excitations. To explore the question of whether or not our experiments were

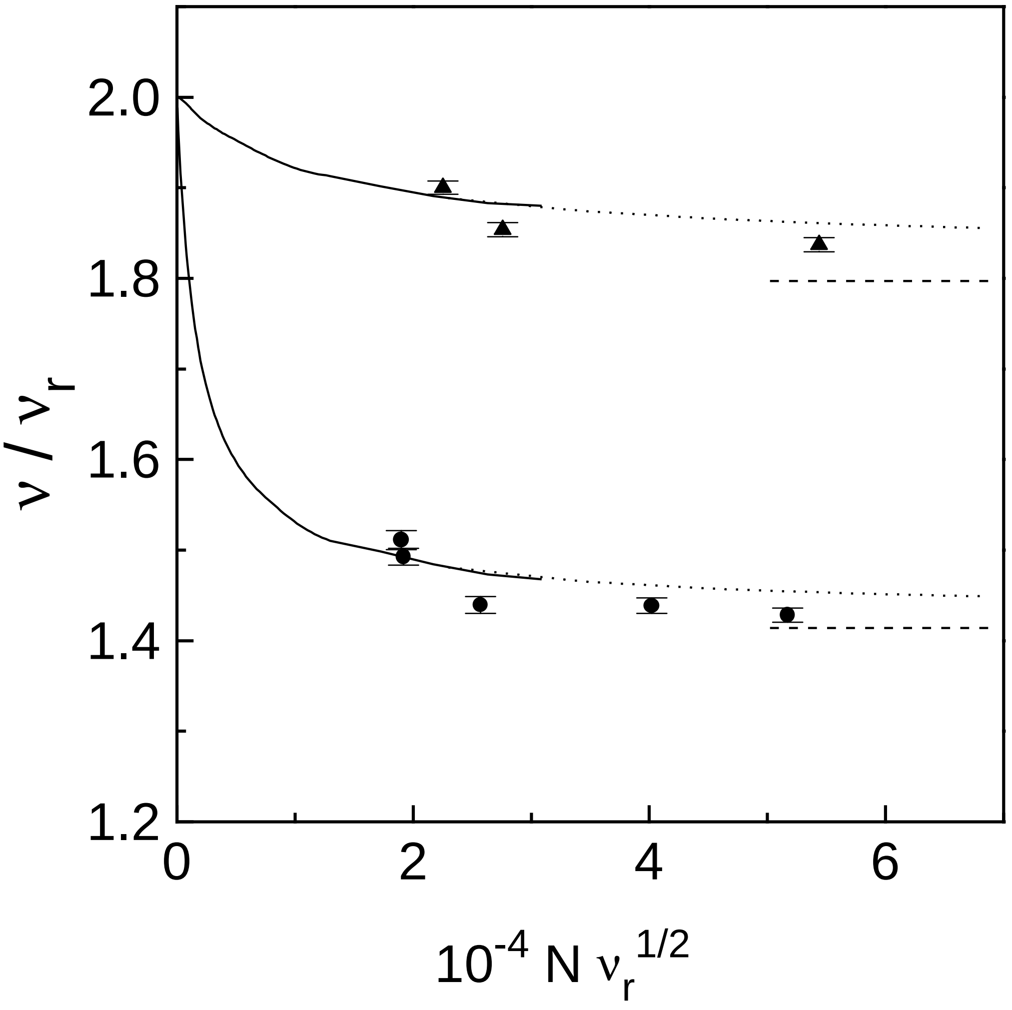

Figure 3.3: We measured the frequency of the $m = 0$ (triangles) and $m = 2$ (circles) condensate modes as a function of interaction strength. The relative interaction strength in the condensate varied as the product of number of atoms, N, and the square root of the radial trap frequency, ν_r. Solid lines show the mean-field calculation by Edwards and co-workers [9, 10], dotted lines show the results of similar calculations by B. Esry and C. Greene [11] and dashed lines show the prediction by Stringari for the strongly-interacting limit [12]. Figure taken from Ref.[8].

performed in this limit, we measured the condensate response for different driving force amplitudes. In this test, we drove the $m = 2$ mode in the 132 Hz trap, with $N \approx 4500$. The results are shown in Fig. 3.4 where we plot the frequency of the oscillating radial width as a function of the amplitude of that response. The solid line shows a fit to a parabola, a form which describes an oscillator with anharmonic terms. As our measurements of excitation frequency were performed for a response amplitude between 9 and 14%, which caused a shift of only 1% in the frequency, this data suggests that we are in the regime where the measured spectrum corresponds quite closely to the elementary excitations of BEC in a dilute gas.

3.7 Damping

Finally, we examined the damping of a condensate excitation. For comparison purposes we first studied the damping in a noncondensed thermal cloud ($T/T_c \approx 1.3$) in the 132 Hz trap. We excited the 264 Hz $m = 0$ mode, because damping in this mode is not influenced by angular momentum conservation. We fit a sine wave with an exponentially decaying amplitude to the observed oscillations in the radial cloud width. This gave an excitation lifetime of 49 ± 13 ms. Since the mean free path in these clouds is long compared to the excitation wavelength and the effect of the trap anharmonicity is small, the excitation lifetime should scale inversely as the atom-atom collision rate. This rate in turn scales with the product of the density times the velocity of the atoms. For a given harmonic oscillator confining potential, this collision rate is proportional to the optical depth of the cloud. Using this scaling principle we predicted that the damping lifetime in a classical cloud with the same optical

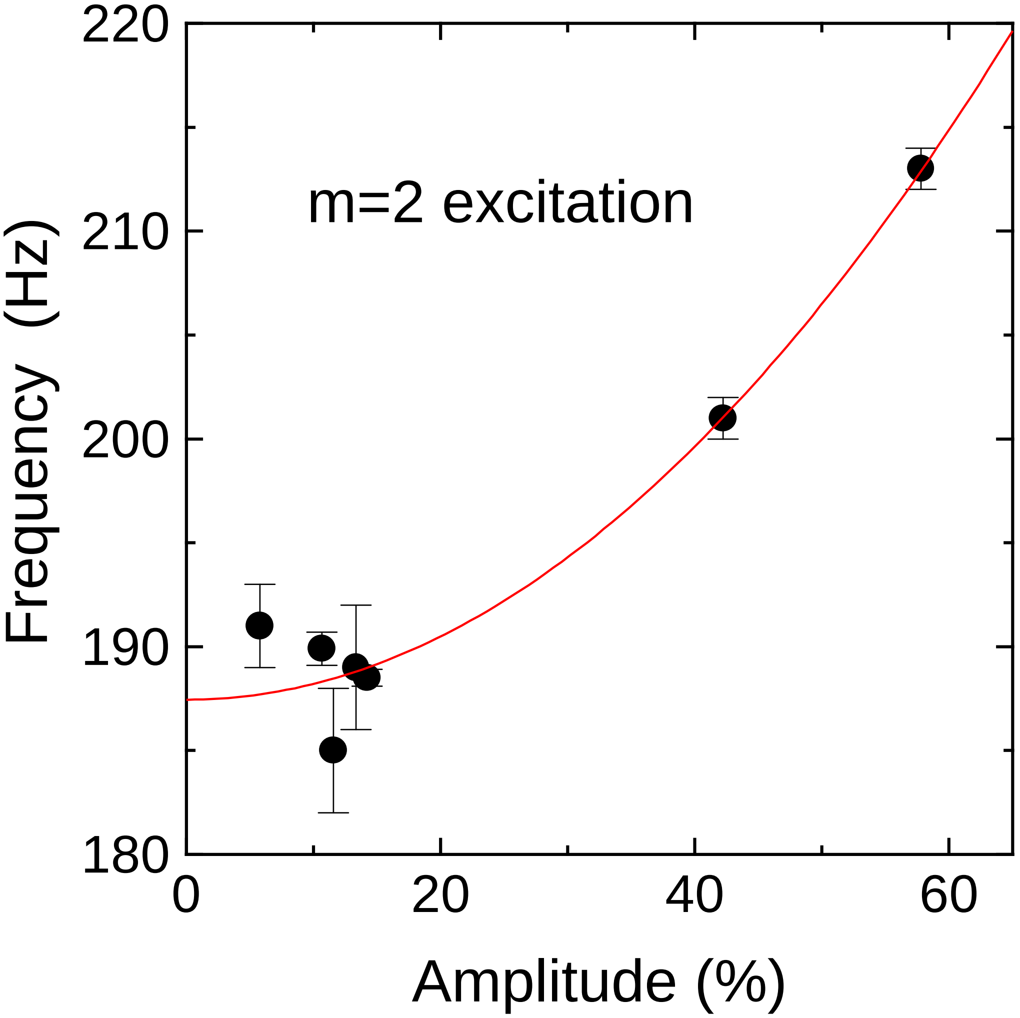

Figure 3.4: The freely oscillating frequency of the condensate is shown as a function of response amplitude. The condensates, consisting of 4500 atoms, were held in a 132 Hz radial frequency trap and driven with $m = 2$ symmetry. The solid line shows a parabolic fit to the data. Figure taken from Ref.[8].

depth as the condensate would be 28 ± 8 ms.

But when we performed the same experiment on the 4500 atom condensate we obtained an excitation lifetime of 110 ± 25 ms. Thus, the condensate excitation persisted nearly four times longer than can be explained in a classical picture.

In summary, we observed low-lying collective excitations of BEC in a dilute atomic vapor. Both $m = 0$ and $m = 2$ modes were identified, and their frequencies measured as a function of relative interaction strength. The data were taken in a linear regime where the collective modes should correspond to the elementary excitations of BEC in this system, and reasonable agreement was found between the experiment and mean-field theory results. The damping lifetime of the $m = 0$ excitation was measured and found to be significantly longer than the prediction of a classical model. We believe further study of these elementary excitations, particularly at different temperatures, will help deepen the understanding of the quantum phenomena of Bose-Einstein condensation of a gas.