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- [128] In the Gross-Pitaevskii equation, the ratio of interaction strength to the harmonic oscillator energy scales as na/ν_r , where n is the density of the condensate wavefunction. For the current experiments, the interactions are weak enough that the density profile is still approximately Gaussian, for which $n \sim N\nu_r^{3/2}$. In contrast, in the Thomas-Fermi limit the interactions are so strong that the density profile is a paraboloid for which $n \sim N^{2/5}\nu_r^{6/5}$.

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- [145] Some smoothing is performed in the wings of the distribution, where signal-to-noise is poor, using the same set of assumptions we use for thermometry.
- [146] As the excluded central region is enlarged, systematic bias in the inferred temperature vanishes, but so does signal-to-noise. We found it necessary to fit to a region which unfortunately samples the outer edge of the degenerate portion of the cloud. From numerical studies of the ideal Bose-Einstein distribution, we derive and apply a modest ($< 10\%$) correction to the measured temperature. The feature we see in the cloud energy occurs regardless of whether we include this correction.
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- [154] We use a 5 cm high breadboard that is more than rigid enough. The excess height of the breadboard reduces the optical access below the cells because our standard height for laser beams is only 15.2 cm. The entire vacuum system may be elevated even higher than the beam heights, but that introduces other problems: the beams are raised closer to eye levels and longer mounts are required, making the entire system more prone to vibrations.
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- [181] We minimize phase uncertainty in the measured frequencies by fitting complete oscillation periods many periods apart. Systematic uncertainties due to anharmonic frequency shifts are below 0.1%.
- [182] The argument in this paragraph is due to C. Monroe and E. Cornell. The results have been numerically validated by B. Verhaar's group.
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- [187] Our noise detection method provides a genuine measure of the noise on the TOP coils and an estimate of the noise near the atoms that is better than a factor of two. Noise power registered on the pick-up coil directly reflects noise on the TOP coil because the pick-up coil is wrapped immediately next to one of the TOP coils. The absolute noise voltage registered on the rf spectrum analyzer is calibrated with a digital FFT. The uncertainty in the measured voltage, determined by statistical noise in the power spectrum, is less than 3 dB. The resonance frequency of the pick-up coil does not influence the measured voltage since the resonance occurs at a much higher frequency than what we measure. We calculate the magnetic field noise at the location of the atoms from an estimate of the distance between the trapped atoms and the TOP coil.
- [188] The details of this approach are discussed in Ref. [185], in the context of noise-induced heating in traps. We similarly reason that the averaging time T is short compared to the time scale over which the level populations change, but is long compared to the correlation time of the magnetic field fluctuations.

- [189] We assume that the loss rate is slow enough that (i) once an atom has made a transition to the $m = 0$ state it is lost from the trap before it can return to the $m = -1$ state; and (ii) the time-averaged magnetic field determines the Zeeman splitting at a point \vec{r} in the potential. The roll-off in the power spectral density at high frequencies implies that atoms further from the trap center, which sample larger magnetic fields, are less likely to be lost from the trap.
- [190] The three-body recombination rate is calculated assuming a per atom rate that is constant with magnetic field, using the initially measured condensate central density. Thus, the rate is an upper limit to the rate of loss we expect from recombination events. The background collision rate is assumed to be constant and equal to the lifetime measured for a classical cloud in a 6.2 Gauss trap.
- [191] At low ω_T , the ferrite-core transformer, which couples the oscillating currents to the TOP coils, has decreased coupling efficiency. The circuit which generates the oscillating currents is also a limitation. At high ω_T , the roll-off in the amplitude gain and phase response in the circuit limits TOP operation above 30 kHz rotation frequencies. As a result of these two constraints, we can only preserve the same bias field (≤ 1 Gauss) for frequencies down to 150 Hz and up to 27 kHz. The limitations to high-frequency operation force us to lower the bias field such that $\omega_L \rightarrow \omega_T$. However, we cannot go to extremely low bias fields without sacrificing lifetime (see Section 6.5). A similar trade-off occurs in studying traps of low ω_T . We can create axial trap frequencies of several hundred Hz, but the increased density of the cloud limits the lifetime through three-body recombination.
- [192] We reduce the number of atoms by almost a factor of 100 by cutting deeply with the rf into a pure condensate. The deeper cuts are less repeatable and cause increased shot-to-shot fluctuations in the number of atoms, which contribute to the larger uncertainty in the lifetimes for these data.
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