Appendix B

Saturation intensity and Rabi oscillation frequency

The saturation intensity, I_{sat}, is defined such that when the laser intensity ($I_0 = \frac{1}{2}c\epsilon_0|E|^2$) equals I_{sat}, one quarter of the atoms will be in the excited state ($f_e = \frac{1}{4}$). An expression for excited population can be written as

$$f_e = \frac{\frac{\Omega}{4}^2}{\frac{\Omega}{2}^2 + \Delta^2 + \frac{\Gamma}{4}^2} \tag{B.1}$$

where Ω is the Rabi frequency, Δ is the laser detuning from resonance, and Γ is the decay rate. For zero detuning, $f_e = \frac{1}{4}$ when $\Omega^2 = \Gamma^2/2$, or

$$\frac{I_0}{I_{sat}} = \frac{2\Omega^2}{\Gamma^2} \tag{B.2}$$

Recall that the Rabi frequency is defined as

$$\Omega = \frac{< ground|d \cdot \hat{e}|excited > E}{\hbar} = \frac{dE}{\hbar} \tag{B.3}$$

With this equation we can write an expression for I_0/I_{sat}

$$\frac{I_0}{I_{sat}} = \frac{\frac{1}{2}c\epsilon_0|E|^2}{I_{sat}} = \frac{d^2E^2}{\hbar^2\Gamma^2} \tag{B.4}$$

The decay rate for a transition can also be related to the the expectation value of the dipole moment in the following way[107]:

$$\Gamma = \frac{\omega_0^3 d^2}{3\pi\epsilon_0\hbar c^3} \tag{B.5}$$

Then we can write the saturation intensity as

$$I_{sat} = \frac{hc\pi\Gamma}{3\lambda^3} \tag{B.6}$$

For our experiments with Ca it was helpful to be able to calculate I_{sat} for the 657 nm and 423 nm transitions. With knowledge of I_0 and a calculated I_{sat}, one can use equation B.2, so solve for the Rabi frequency. The results of these calculations are tabulated in Table B.1.

Table B.1: Saturation intensity and Rabi frequencies

λ	I_{sat}	I_0	Ω
657 nm	0.19 μW/cm^2	6 mW/cm^2	32.6 $\times 10^3$ s^{-1}
423 nm	57.7 mW/cm^2	2 mW/cm^2	27.8 $\times 10^{-6}$ s^{-1}