Chapter 1

Introduction

1.1 Superfluids great and small

Traditionally in physics one learns that the world is divided into two. There is the microscopic world of quantum mechanics and there is the macroscopic world where classical physics still reigns. Superfluids are astonishing in that they shatter this barrier between worlds. These highly quantum systems can exist on length scale of a μm, a cm, or even 10^4 km. With their dissipationless flow and exotic topologies such fluids have puzzled researchers in diverse fields of physics for almost a century.

An understanding of superfluids starts with the early 1920's work of Satyendra Nath Bose [5] and Albert Einstein [6]. In these papers they argue that at a small but finite temperature a gas of bosons will quickly collect in the ground state. This transition to the ground state far exceeds the expectation for an equally cold ideal gas of atoms and can result in a nearly complete occupation of a single quantum state. This condensation, although an impressive first step, is however not sufficient to explain superfluidity. It was Fritz London [7, 8] who, while trying to explain the phenomena observed in superfluid Helium in the 1930's, would suggest that this macroscopically occupied state is governed by a single wave function, or order parameter, of the form

$$\Psi = \sqrt{\rho(\vec{r})}e^{iS(\vec{r})} \ . \tag{1.1}$$

In the 1930's this was a radical idea but even today it is a little unsettling to think a thousand atoms, or a million atoms, or even Avagadro's number of atoms could be described by a single wavefunction commonly associated with a single particle. Ultimately what makes superfluids so interesting is that this macroscopic wavefunction extends quantum mechanics into the macroscopic world.

The experimental side of the story of superfluids arguably begins with Kamerlingh Onnes [9] who was, in many ways, the Christopher Columbus of superfluids. Onnes was first to liquify ^{4}He and in 1908 he was attempting to cool this new liquid further in order to produce solid ^{4}He. While unsuccessful in this attempt, Onnes did cool the liquid well into the superfluid regime that would later be called He II. Unfortunately Onnes did not recognize the superfluid for what it was. It would take 30 more years before a study of He II would find it to have no viscosity and for the term "superfluid" to be coined.

In the mean time Kamerlingh Onnes was at it again. It was 1911 and Onnes was attempting to understand the change in resistivity of a metal at very low temperatures. Mercury was chosen for the experiment because the metal could be cooled effectively. From here the story is well known. As Onnes cooled his mercury sample, the resistivity decreased roughly as expected until reaching 4.2 K, at which point a most unexpected thing happened, the resistivity of the sample dropped abruptly to zero. Onnes had made the first observation of superconductivity. A theoretical description of superconductivity would take much more time but ultimately Bardeen, Cooper and Schrieffer would demonstrate that the pairing of electrons into Cooper pairs would allow fermions to condense like bosons. Thus superconductors, like He II, become governed by a single coherent wavefunction.

Over the years many other similar systems have been discovered. In 1971 Lee, Osheroff and Richardson discovered two superfluid phases in the fermionic liquid ^{3}He. Like superconductors ^{3}He condenses through the pairing of fermionic atoms. Much as electrons in a superconductor sense each other through a Coulomb repulsion, the ^{3}He atoms interact via their strong magnetic moments. This super system has generated a great deal of interest due to its relative accessibility [9]. The lack of background lattice structure or impurities make this system clean to work with and the magnetic susceptibility makes NMR a convenient diagnostic tool.

Superfluids are not just exotic laboratory creations, but can also occur in nature. Neutron stars, some of which predate Onnes' work by 10^7 years, are predicted to have a superfluid interior. Such neutron stars are a superfluid system that truly stretches the definition of "low" temperature. Having a density of roughly $10^{14}\, g/cm^3$ [10] these stars are thought to contain paired neutrons with a critical temperature, Tc, of roughly 10^9 K. Meanwhile core temperature of these stars is predicted to be a fairly cool couple-million K suggesting a highly degenerate state for the paired neutrons (and to a lesser degree paired-protons and paired-electrons [11]).

Contrasting starkly with the neutron star is a recent addition to the family of superfluids and the subject of this thesis, the dilute gas Bose-Einstein Condensate (BEC). Weighing in at densities of $10^{-11} - 10^{-13}\, g/cm^3$ these condensates exist at temperatures ranging from μK to pK. These systems are clearly more experimentally accessible than their astrological counterpart, and their dilute nature considerably simplifies the theory. The low density and weak interactions also lead to a much higher condensate fraction (nearly 100%) than the other systems discussed here (He II for example is only 10% condensate) [9].

Finally one should mention the newest addition to the superfluid family, the Fermionic condensates [12, 13, 14]. First observed in 2003, it is probably too soon to know exactly where this dilute gas system stands with respect to the other superfluid systems. However, it is clear that this promising new system will be able to probe the gap between fermion and bosonic supersystems. With the addition of this new system it is also clear that the superfluid field is still very much alive and developing.

1.2 Vortices

One of the most distinct features of superfluidity is its response to rotation or, equivalently, to an applied magnetic field in the case of superconductors. In any wavefunction, rotation manifests itself as a phase gradient along the direction of flow. More specifically, a loop integral around the center of rotation of the superfluid must yield a net change in phase regardless of loop size. At the same time the wavefunction of a superfluid must be, in all places, single valued. In the 1950's, Onsager [15] and Feynman [16] observed that quantized vortices, or tiny tornados in the fluid, were needed to fill such requirements and allow for rotation. Each vortex can be thought of as a quantum of angular momentum and contains, at its center, a phase singularity around which an integer 2π phase winding occurs. This integer 2π winding, and fluid depletion at the vortex center allow for the wavefunction to be single valued while providing the necessary phase gradient for rotation. Without these singularities the fluid velocity field is curl free and cannot support circulation. This is why superfluids without vortices are considered irrotational. It is interesting to think that a vortex-free superfluid is rotationally at rest, not in the lab frame or Earth frame, but with respect to some greater axis in the Universe.

When thinking of rotation one tends to visualize the typically classical, rigid-body rotation of a record on a player or a tire. It should be noted that the presence of a single vortex in a superfluid leads to an extremely non-rigid body flow. In this case, flow is rapid near the center of the vortex, where the phase gradient is steep and falls off as $1/r$ where r is the distance from the vortex center. However with increased rotation more vortices can be added to a superfluid. The repulsive vortex-vortex interaction causes vortices to distribute about the fluid and form a triangular lattice, or Abrikosov lattice seen in figure 1.1 a). As one would expect, with increasing quanta of angular momentum the system begins to behave classically, and this distribution of vortices across the superfluid causes the system to rotate in an increasingly rigid-body like manner.

Not surprisingly, vortices can have profound effects on the system. For superconductors in sufficiently high magnetic fields, vortices can consume the sample and force it into a normal state [19]. In superfluid He II systems the generation of "vortex tangles" leads to finite viscosity even for flows below the critical velocity [9]. In neutron stars it is predicted that vortices, pinned to the crust of the star, can break free in an avalanche of vortex motion that causes the rotation of the entire core of the star to jump irregularly.

Vortices in a rotating dilute gas Bose-Einstein condensate [20, 21, 22, 23, 24, 25] are no exception and many of the profound effects they have on the BEC system will be detailed in the rest of this thesis. The study of vortices, in a dilute gas BEC is in no small way motivated by the connections shared with these other supersystems. We are blessed in that there has clearly been decades of work in these other systems that can guide our research. We are also blessed in that the BEC's offer a very clean system in which to study vortex effects. The perfectly smooth magnetic potential used to confine a BEC makes us immune to the effects of vortex pinning that complicates studies in other systems. Additionally, impurity isotopes are simply not an issue. Also interactions in the dilute BEC system are well understood making comparison with theory much easier. Thus, while BECs can mimik superfluid effects seen in other systems, they also promise novel effects of their own.

Tkachenko oscillations, or sound waves in a vortex lattice, represent a striking example of the interconnected nature of superfluid vortex systems. Tkachenko proposed that the triangular, Abrikosov, lattice structure seen in superconductors would also be supported in superfluid Helium. In a 1966 mathematical tour de force [26], Tkachenko demonstrated that this lattice structure should support sound waves. These elliptically polarized transverse waves in the lattice would later be dubbed Tkachenko modes. Tkachenko was so certain of this prediction that he spent the rest of his life looking for

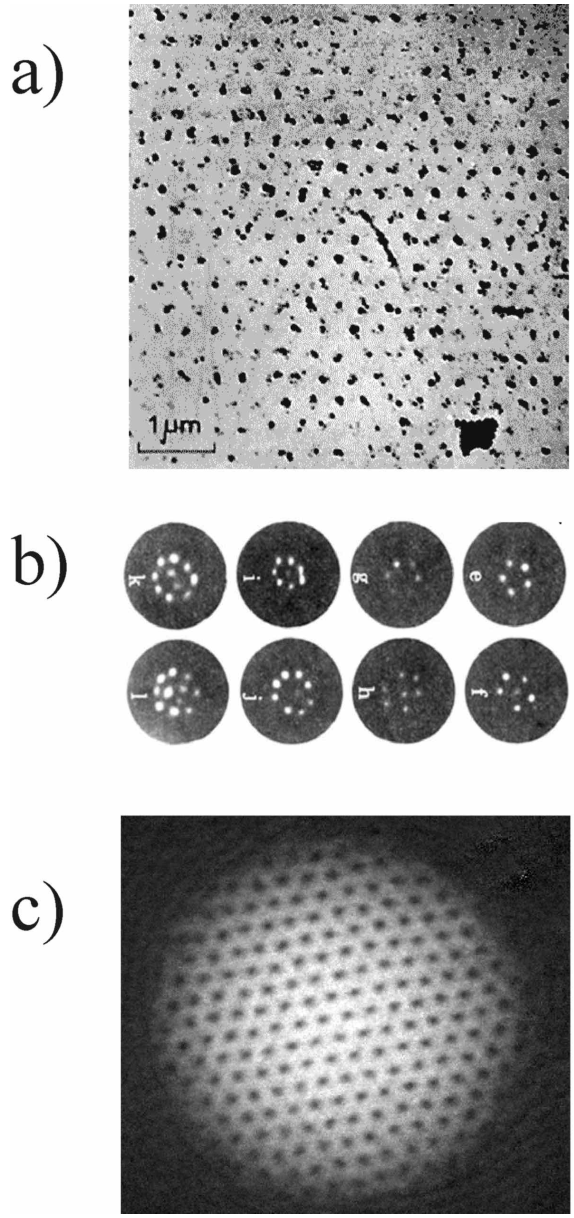

Figure 1.1: Vortices in supersystems. Image a) the first image of an Abrikosov lattice taken in 1967 [17]. Black dots are cobalt particles that are attracted to the magnetic field of the vortices in this superconducting lead rod. Image b) pictures of vortices in He II taken in 1979 by Yarmchuk **et al.** [18] In a heroic experiment, impurity ions where added to the superfluid and became trapped in the vortex cores. Images were taken by adding a strong electric field to project the ions onto a phosphorous screen that could be imaged onto film. Image c) vortices in a rotating Bose-Einstein condensate taken at JILA in 2003.

these excitations in He II, but ultimately met with frustration. It was Glaberson [27] who would first report a signature of these modes in He II. Performing a rotating disk experiment, Glaberson measured the response of the system at different excitation frequencies. While these experiments ultimately demonstrated the existence of Tkachenko modes they were a little unsatisfactory in that they necessitated the simultaneous excitation of many Tkachenko modes. Worse still, vortex pinning effects coupled the Tkachenko modes to other modes in the system, and ultimately it was impossible to study a Tkachenko mode in isolation. This would remain the state of the field for some time...

1.3 Rotating Bose-Einstein condensates

Recently Anglin and Crescimanno have predicted that these modes should exist in a dilute gas BEC [28]. In this initial work they develop a hydrodynamic theory of vortices that accounts for the finite and inhomogeneous density profile of the condensate. This paper yields several interesting results, the most relevant of which is that there exists a radially symmetric mode in condensates with a wavelength on the order of the condensate radius. This mode, dubbed the (1,0) mode, is predicted to have a frequency far smaller than any previously observed bulk fluid mode of the condensate. Two interesting observations about the vortex lattice can be made from the existence of this mode. First, the radially symmetric “s-bend” that it induces in the lattice could only be possible in the presence of a finite shear modulus. A finite shear modulas is not normally associated with a superfluid and could only exist because the superfluid is permeated with the vortex lattice. Secondly, the extremely low frequency of this mode suggests that this shear modulus is quite small and that the lattice is potentially very delicate under shear.

In Anglin and Crescimanno’s notation the (1,0) mode corresponds to one radial node and zero azimuthal nodes. While this thesis primarily studies this one mode it should be noted that there are others that are accessible or potentially accessible. As mentioned in §4.4 the (2,0) mode has been observed as well. Non-radially symmetric modes are also predicted and it is a little surprising that they have never been detected. The (0,2) mode in particular is predicted to be an extremely low energy mode (sub 1 Hz frequency for our system) and one might expect it to exist in nearly any vortex lattice.

By mid 2002 we began the study of Tkachenko modes in our dilute gas BEC. We are able to observe these modes quite easily, in contrast to He II where such modes are not directly observable. By gently perturbing the central density of the condensate we couple to a pure (1,0) Tkachenko mode. Because our initial conditions are so reproducible its becomes possible to map the vortex oscillation patterns in time, obtaining both oscillation frequency and an approximate wavelength. Surprisingly, as low as Anglin and Crescimanno's predicted Tkachenko frequencies are, our measured frequency is much lower, particulary at high rotation.

The explanation for this discrepancy would come in a slew of theory responses [29, 30, 31, 32, 33]. A particularly relevant point, made in nearly all of these works, is that Anglin and Crescimanno's theory is based on assumptions of incompressibility that are true in He II but not completely true for a highly rotating BEC. As the BEC rotation rate becomes comparable to the speed of sound the fluid becomes compressible. Interestingly this compressible regime is also expected to occur in neutron stars [34]. Accounting for compressibility effects lowers the predicted Tkachenko frequencies to a point where they are consistent with experiment.

A second effect, entering the lowest Landau level (LLL) regime, is also expected to decrease the lattice shear modulus and further lower the predicted Tkachenko frequencies. In the rotating frame and as the condensate rotation rate approaches the trapping frequency, the single-particle harmonic oscillator energy states organize into Landau levels or band like structures of nearly degenerate states. If condensate interactions are weaker than the energy splitting between Landau levels, then the condensate primarily occupies the near-degenerate states of the lowest Landau level. This places significant constraints on the condensate wavefunction and one would expect to see a number of interesting effects including a weaker shear modulus in the vortex lattice [29].

Tkachenko modes aside, BECs in the lowest Landau level are intriguing on their own, as this system is somewhat analogous to type-II superconductors in a strong magnetic field [19]. In the extreme case exotic quantum Hall like effects are predicted to occur. For rotating BECs in the LLL, three regimes have been identified, distinguished by the filling factor (the ratio of atoms in the fluid to vortices). For high filling factors, the condensate is in the mean-field lowest Landau level regime [35, 19, 36] and forms an ordered vortex lattice ground state. Filling factors around 10, although currently out of reach, are expected to lead to melting of the vortex lattice. At such filling factors, the shear strength of the lattice is predicted to drop to a point where quantum fluctuations begin to melt the vortex lattice [37, 38], and a variety of strongly correlated vortex liquid states similar to those in the Fermionic fractional quantum Hall systems are predicted to appear [37]. For filling factors around 1, exotic quasiparticle excitations obeying fractional statistics [39] are predicted.

By monitoring the frequency of the (1,0) Tkachenko mode and the frequency of the axial breathing mode [40] we observe a transition to the 2D, high-filling factor, LLL regime. Two theory predictions in particular are addressed for this regime. First there is the Ho prediction [35] that the radial condensate profile should become gaussian as one enters the lowest Landau level. This prediction arises from the fact the condensate wavefunction in the lowest Landau level is so constrained that the position of the vortices dictates the entire condensate profile. If one assumes a perfect Abrikosov lattice then the resulting condensate profile is Gaussian. However our experimental observation shows a decidedly parabolic profile that implies that the lattice, far from the condensate center, distorts slightly as predicted by several other groups [41, 42, 43]. It is interesting to note that this predicted distortion closely resembles the lattice distortion predicted by

Sheehy and Radzihovsky [44] for the Thomas Fermi regime.

Also in the lowest Landau level the vortex core size becomes comparable to the lattice spacing. This is highly suggestive of type-II superconductors in a strong magnetic field where vortices become increasing tightly packed. When the applied magnetic field in these systems reaches a critical value (generally dubbed Hc_2) the vortices effectively swallow the entire system, forcing it into a normal state. In the dilute-gas BEC, early extrapolations from Thomas-Fermi theory [45, 46] suggest a similar effect. Alternatively it is argued by argued by Fischer and Baym [19] and Baym and Pethick [36] that kinetic energy considerations prevent this overlap and that the ratio of core size to spacing will ultimately saturate. A challenging set of experiments discussed in §5.5 demonstrates that the latter prediction is correct.

Multiply quantized vortices also offer intriguing comparisons across supersystems. Vortices discussed so far have all been singly quantized but, in principle, a vortex can have any integer 2π phase winding. Such multiply quantized vortices are just one of the exotic vortex phases seen in ^{3}He-A systems [47] and can also be observed at pinning sites in superconductors [48, 49, 50]. To date, doubly quantized vortices have been observed as the result of topological vortex formation [51], but higher quanta have not been observed. In this thesis, I discuss a somewhat less elegant means of forming a "giant" vortex. Here we suppress the fluid density in the center of the condensate by employing a focused, resonate laser to burn a hole in the cloud. This is not a true multiply-quantized vortex, since density suppression does not actually lead to vortex core overlap. Nonetheless, these experiments yield interesting information about the dynamical stability of such giant vortex features. The Giant vortex itself drastically demonstrates the effects of Coriolis force and the changed dynamics in the rapidly rotating system

1.4 Outlook

Predicting the future course of the TOP trap apparatus has traditionally been very hazardous. However, one obvious direction is the study of the exotic, low-filling-factor states discussed earlier. It should be noted however that this has also been our stated goal for much of the last three years. While this goal has yielded a great deal of interesting science, it has also become clear that low signal and fragile condensates will make low filling factors hard to achieve. There is some hope that the installation of an optical lattice could make this regime accessible and a serious effort to do so is currently underway.

On a separate note, vortex experiments could truly benefit from a Feshbach resonance. It is exciting to think that with such a resonance one could tune a rotating BEC from the He II regime to a compressible neutron star regime to the lowest Landau level regime, instantly and without adjusting atom number or rotation. Theory predictions have even suggested that with a Feshbach resonance, a rotating BEC could serve as nothing less then a model system for the expanding universe [52]. Unfortunately, this is complicated by the fact that our current atom, ^{87}Rb, does not offer any easily usable resonances [53]. On the plus side experience has taught me that while it is hazardous to predict the future of the TOP trap it is equally hazardous to rule anything out.

1.5 Thesis overview

The structure of this thesis is follows. Chapter 2 discusses experimental techniques for generating and imaging vortices. Chapter 3 is a discussion of giant vortices and dynamically stable density deformations in a rotating condensate. Chapter 4 details the excitation of the (1,0) and (2,0) Tkachenko modes in a vortex lattice. These modes are compared to theory and to similar bulk fluid modes of the condensate. Chapter 5 discusses the crossover to the lowest Landau level regime. Breathing modes and

Tkachenko modes are used to demonstrate two-dimensionality and passage into the lowest Landau level regime respectively. Additionally, the condensate and vortex density profiles in this regime are discussed, with particular attention paid to fractional core area effects. Finally, Chapter 6 characterizes several equilibrium vortex lattice effects. Specifically, we attempt precision measurements of vortex lattice spacing and the vortex core size over a range of condensate densities and rotation rates. Lastly, the effects of finite temperature on vortex contrast are studied.