

**Theoretical Investigation of Cold Alkali Atom
Collisions**

by

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The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

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Theoretical Investigation of Cold Alkali Atom Collisions

Thesis directed by Prof. Chris H. Greene

Theoretical techniques are developed to solve the coupled Schrödinger equations that describe cold magnetically trapped alkali atom dynamics. In particular, a multichannel quantum defect formulation is presented to separate cleanly the atom-dependent from the atom-independent contributions to the final scattering matrices. The resulting approach provides a more comprehensive understanding of cold alkali collisions and improves substantially on the efficiency of calculations. Improved Rb two-body interaction potentials are extracted from measured ^{87}Rb inelastic collision rates and from the measured position and width of a ^{85}Rb Feshbach resonance. Using these state-of-the-art potentials, specific atomic hyperfine states have been identified whose collisional properties suggest intriguing possibilities for interesting and novel degenerate gas studies. In addition, a detailed analysis of ^{39}K photoassociation lineshapes has permitted the determination of accurate two-body interaction potentials for this atom as well. Finally, three-body recombination of doubly polarized trapped atoms is investigated. Two qualitatively different mechanisms which control the recombination rate are identified for positive and negative two-body scattering lengths a . Surprisingly, the recombination rate is found to scale approximately as a^4 in both cases.

To Angela

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