

**Ultracold Bose gases - from the Gross-Pitaevskii
to the fractional quantum Hall regime**

by

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Ultracold Bose gases - from the Gross-Pitaevskii to the fractional quantum Hall regime

Thesis directed by Prof. Murray Holland

Ultra-cold Bose gases present an ideal environment for the study of many-body physics. These systems can be prepared under various experimental conditions with precise control. Techniques like Feshbach resonances allow us to dynamically tune the inter atomic interaction, from strongly attractive to a strongly repulsive one.

In the first part of the thesis, we study the weakly interacting Bose gas in connection with the dynamics of an atom laser. Here we propose a possible optical pumping model for loading the reservoir of a continuous wave atom laser. The finite temperature effects like phase diffusion require a thorough understanding of the kinetic regime of the dilute Bose gas. In this respect, we develop a non-Markovian quantum kinetic theory and thereby show the emergence of different time scales for correlation and subsequent relaxation to an equilibrium states. Using numerical simulations, we also predict the damping rates and frequencies of collective modes.

In the second part, we study the strongly correlated regime where the interaction energy is greater than any other (single particle) energy scale of the problem. Here, in the presence of a Feshbach interaction we predict the generation of novel strongly correlated paired states. Such states while similar to the one observed in a $5/2$ fractional quantum hall effect, are unique in symmetry to the Bose gas system.

Dedication

To my parents

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