Chapter 1

Introduction

Dilute atomic gases present an ideal environment for studying various aspects of many-body physics. These systems can be prepared with precise control and extreme purity. Advancement in the techniques of trapping and cooling based on the detailed understanding of atom-light interactions have allowed experimentalists to cool atomic gases to ultra-low temperatures. This has opened new possibilities for the understanding and investigation of many-body phenomena, for example the prediction of Bose-Einstein condensation (BEC). BEC occurs in bosonic systems where the ground state acquires a macroscopic population when cooled below a certain critical temperature T_c. While first predicted by Bose and Einstein in 1924, it wasn't until 1995 when it was confirmed in the form in which it was originally posed by experiments in a trapped dilute atomic vapor of Rb^{87}, Na and Li^{7} [1, 2, 3]. Recently BEC has been achieved in dilute atomic gases of H, He, K^{41},Cs and Yb, and has also been realized in micro-traps on chips [4] and also in two dimensional surface traps [5].

Since 1995, dilute atomic gases have generated tremendous theoretical and experimental interest and have led to the study of phenomena like atom lasers and vortices. More recently with the advent of techniques based on Feshbach resonances, the interactions between atoms can now be dynamically tuned by changing the external magnetic field. While interactions in condensed matter systems could be modified, for example, by doping a semiconductor material, dynamic control is typically limited. In fact, in some condensed matter systems such as those that give rise to the fractional quantum Hall effect, there is virtually no control over interactions. Feshbach tuning allows the inter-atomic interaction to be dynamically varied from a repulsive to an attractive one. The ability to tune interactions has allowed us to stretch the scope of possibilities to studying BCS superfluidity [6, 7, 8, 9], guiding of matter waves [10] and molecule formation in a Fermi gas [11]. Thus dilute atomic gases have provided us with a new setting to answer some of the questions and test ideas that have previously been inaccessible in the domain of condensed matter physics.

From a theory point of view, dilute atomic gases are extremely challenging since the theoretical framework required to describe them can be very different in different regimes based on the strength of interaction, temperature scale of interest and density of the atomic gas. For example, in a dilute Bose gas with weak repulsive interactions at $T = 0$ (BEC), a description in terms of mean fields given by the Gross-Pitaevskii (GP) equation is sufficient to account for all the observable properties. However for finite temperatures $0 < T < T_c$, a quantum kinetic description that includes the effect of binary collisions may be necessary. In another situation, a dilute Bose gas at $T = 0$ can be configured to be strongly correlated making the mean field description invalid. In this case, as we will see in the later part of this thesis, an effective field theory picture in terms of composite particles is convenient to completely account for the correlations and predict the physical observables.

Given the richness of this area of research, in this thesis we theoretically investigate some particular aspects of the trapped Bose gas system in two different regimes- the weak interaction regime and the strong interaction regime.

(1) Weak interaction regime:

In this regime we focus on the behavior and working of a continuous-wave (cw) atom laser. An atom laser is essentially a device that produces an intense coherent beam of atoms. Since a BEC is a coherent superposition of atoms in the ground state, one could simply think of an atom laser as a beam of Bose condensed atoms. The working of an atom laser can be described in complete analogy to the optical laser. As in the case of an optical laser, if a threshold condition is satisfied, the atom laser will operate far from equilibrium in a dynamical steady state. In this dynamic state, the macroscopically occupied quantum state is continuously depleted by loss to the output field, and continuously replenished by pumping from an active medium. The threshold condition for an atom laser is analogous to the critical point associated with the phase transition from a normal gas to a BEC. A version of a "pulsed" atom laser was demonstrated [27, 28] soon after the first experimental realization of BEC in dilute alkali gases. Therefore one of the objectives of this thesis is to model a possible pumping scheme that would continuously replenish the reservoir allowing the BEC to be generated in a dynamical steady state and hence a cw- atom laser.

In order to study the coherence and phase diffusion properties of any such device, it is very important to understand the collisional regime of the dilute trapped Bose gas. While binary collisions is the necessary mechanism for the evaporative cooling technique employed in the realization of a BEC, it is also responsible for the loss of coherence of the condensed component due to collisions with the non-condensed atoms. In order to better understand the effect of collisions, we discuss the dilute Bose gas in the quantum kinetic regime ($T > T_c$, absence of a condensed component). Here, we derive a non-Markovian generalization to the quantum kinetic theory described by Walser et al. [13] for temperatures above the BEC temperature. We show that within this framework, finite duration effects like quasiparticle damping arise naturally and lead to a systematic Markov approximation from a non-Markovian Born theory. Using numerical simulation we demonstrate the emergence of different time scales for correlation and relaxation as well as predict damping rates and frequencies of collective modes [14]. This formulation can be extended to the temperature regime, $0 < T < T_c$, by including the condensed component. Such a theory will be an important step towards a complete understanding of the cw- atom laser.

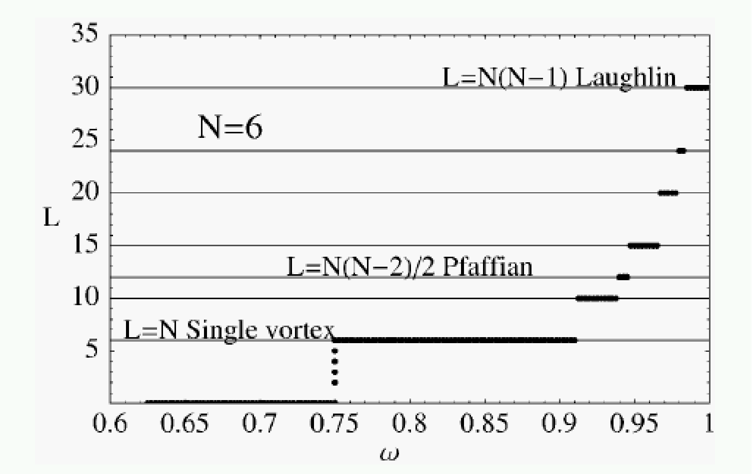

Figure 1.1: The total angular momentum of the stable ground state plotted as a function of the rotational frequency for $N = 6$ atoms with two-body interaction potential in units of harmonic oscillator quanta given by $V = (1/N)\delta(\mathbf{r}_i - \mathbf{r}_j)$. (Figure obtained from N. K. Wilkins and J. M. F. Gunn, Phys. Rev. Lett. **84**, 6 (2000))

(2) Strong interaction regime:

This regime is defined by the situation where the interaction energy is large compared to any other single particle energy scale of the system. A dilute Bose gas can be configured in this regime in various ways. One possibility is to tune the interactions by using Feshbach resonance techniques. Another possibility is to introduce degeneracy in the system, for example by using an optical lattice trapping potential as was used in the experiment involving a superfluid to a Mott transition [89]. In this thesis we consider a two dimensional trapped Bose gas but instead introduce degeneracy by rotating the trap. At a certain critical rotational frequency the system is predicted to enter the strongly correlated regime where the many-body ground state is given by the bosonic variant of the Laughlin wavefunction. This is shown in Fig. 1.1 where the total angular momentum of the stable ground state is plotted as a function of the rotational frequency. The Laughlin state becomes stable only when the rotational frequency is close to the trapping frequency. The Laughlin ground state is precisely the state responsible for the observation of the fractional quantum Hall effect in two dimensional electron systems. Therefore in this thesis we will refer to this regime of the dilute Bose gas as the fractional quantum Hall regime. Using the tunability of interaction made available by Feshbach resonances, which as mentioned earlier is almost impossible to achieve in an electronic fractional quantum Hall system, we study this regime of the dilute Bose gas in the presence of Feshbach interaction. The importance of employing such resonant interactions in the strongly correlated regime will be apparent from the possibility of creating novel states as well as the opening of new possibilities for answering questions related to the many-body physics of fractional quantum Hall effect.

Overview:

The general flow of topics covered in this thesis is shown in the figure below. In Ch. 2, we discuss a model based on an optical pumping scheme to continuously load the reservoir of a continuous wave atom laser. Here we show that despite loss mechanisms, it is possible to generate BEC in a steady state that is crucial to the working of a continuous wave atom laser. In Ch. 3, we develop a non-Markovian quantum kinetic theory and thereby derive a systematic Markov approximation that includes finite duration effects such as quasiparticle damping. The remainder of the thesis is devoted to the strongly correlated regime of the dilute Bose gas. In Ch. 4, we introduce the electronic fractional quantum Hall effect and show how an analogous many-body state can be generated in a trapped dilute Bose gas system. In Ch. 5, we develop a field theory picture required to describe this regime. Within this picture the strongly correlated regime can be thought of as a Bose condensed phase of composite bosons. The idea of composite particles and its connection to the existence of fractional statistics is also discussed. The importance of such a description will be apparent as it will allow us to theoretically study this regime in the presence of Feshbach interactions. The implications of introducing resonant interactions in this regime are discussed in Ch. 6. The main result of this chapter is the prediction of a novel strongly correlated state unique to the trapped Bose system.

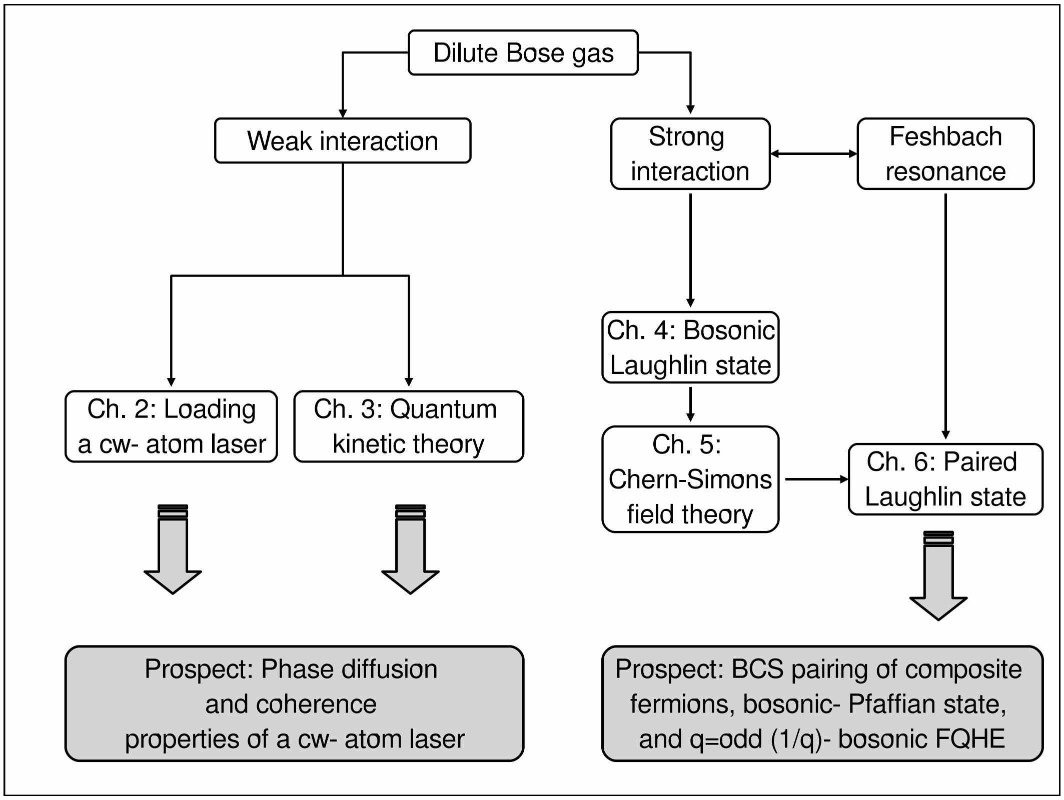

Figure 1.2: Flow chart of the chapters in this thesis. Figure also shows the opening of new possibilities for further study as a result of this work.