Appendix B

Matrix elements

The matrix element of Eq. (3.41) can be evaluated easily by expanding the sin functions into co- and counter-propagating complex exponents and by an additional partial integration. This results in eight separate terms, i.e.,

$$\begin{aligned}
\phi^{ijkl}/a_{\mathrm{S}} &= \frac{4}{\pi}\int_0^{\pi} \sin(i\,x)\sin(j\,x)\sin(k\,x)\sin(l\,x)\,\frac{dx}{x^2}, \\
&= F(i+j+k-l)+F(i+j-k+l) \\
&+ F(i-j+k+l)+F(i-j-k-l) \\
&- F(i+j-k-l)-F(i-j+k-l) \\
&- F(i-j-k+l)-F(i+j+k+l), \qquad \text{(B.1)}
\end{aligned}$$

where

$$F(n) = \frac{1}{2\pi^2}[\cos(n\pi)+n\pi\,\mathrm{Si(n\pi)}], \qquad \text{(B.2)}$$

$$\mathrm{Si}(z) = \int_0^z \frac{\sin(t)}{t}\,dt. \qquad \text{(B.3)}$$

An asymptotic expansion of the sine integral leads to the following approximation that is correct at the 1% level, i.e.,

$$\begin{aligned}
F(0) &= \frac{1}{2\pi^2}, \qquad \text{(B.4)}\\
F(n>0) &\approx \frac{1}{2\pi^2}[\frac{\pi}{2}|n\pi| - \frac{\sin(n\pi)}{n\pi} + \frac{2\cos(n\pi)}{(n\pi)^2}].
\end{aligned}$$