

CHAPTER 6

CONCLUSIONS AND IMPLICATIONS

We have now presented the two main results of this thesis. We find that the $6S \rightarrow 7S$ dc Stark shift is $k = 0.7262(8) \text{ Hz(V/cm)}^{-2}$, and we find that the $6S \rightarrow 7S$ tensor transition polarizability is $\beta = 27.024(43)_{\text{expt}}(67)_{\text{theory}}a_0^3$. We will now use the measurement of the dc Stark shift to re-evaluate the uncertainty of the atomic theory calculations, and then use our new measurement of β , along with our previous measurement of $\text{Im}(E_{\text{PNC}})/\beta$, to obtain a value for the weak charge Q_W . This value of Q_W will be compared with the prediction of the standard model. The comparison provides a test of the standard model.

6.1 Re-evaluation of Atomic Theory

The test of the standard model using a measurement of PNC in atomic cesium requires calculations of the cesium matrix elements. If the uncertainty of the experiment is significantly smaller than the uncertainty of the calculations, then the test will be limited by the theory.

There are two groups that have made a considerable effort to develop the theory to determine the structure of cesium: Blundell, *et al.* [43, 14, 45] at the University of Notre Dame and Dzuba, *et al.* [15, 47, 48] now at the University of New South Wales. The most important result of their calculations is the value of

k_{PNC} , which is given by

$$k_{\text{PNC}} = \frac{N}{Q_W} \sum_n \left(\frac{\langle 7S | \vec{D} | nP \rangle \langle nP | H_{\text{PNC}} | 6S \rangle}{E_{6S} - E_{nP}} + \frac{\langle 7S | H_{\text{PNC}} | nP \rangle \langle nP | \vec{D} | 6S \rangle}{E_{7S} - E_{nP}} \right). \quad (6.1)$$

The quantity k_{PNC} is made up of two types of matrix elements: allowed electric dipole matrix elements like $\langle 6P | \vec{D} | 6S \rangle$, and unmeasurable matrix elements like $\langle 6P | H_{\text{PNC}} | 6S \rangle$. Because H_{PNC} is proportional to the Dirac matrix γ^5 , these latter matrix elements are essentially the matrix elements of γ^5 .

Of equal importance to the calculated value of k_{PNC} is the accuracy of the calculation; if the calculation is only good to 2%, then the subsequent test of the standard model is limited to 2%. Therefore, the two groups have spent considerable time evaluating the accuracy of their calculations. Because the behavior of the two sorts of matrix elements is so different—the matrix elements of \vec{D} are sensitive to the value of the wave function at large \vec{r} and the matrix elements of γ^5 are sensitive to values of small \vec{r} —one must be careful to test the predictions of the theory using quantities that behave in a similar fashion to both of these matrix elements. The most objective way to estimate the accuracy of the theory is to compare its predictions with the results of experiments.

In order to provide these comparisons, many groups have performed high-precision measurements of the properties of cesium. Among the most precise of these are measurements of the hyperfine structure constants. These measurements are particularly relevant because of the similarity of their behavior to the behavior of the matrix elements of γ^5 . Other precise measurements are of the lifetimes of excited states such as the $6P_{\frac{1}{2}, \frac{3}{2}}$ states. The lifetime measurements are important because they involve the same radial matrix elements as k_{PNC} , such as $\langle 6P | \vec{D} | 7S \rangle$.

To quote from Ref. [14]: “If all known properties of cesium could be reproduced by some *ab initio* calculation scheme to within 0.1%, we feel it would be rea-

sonable to trust a PNC calculation carried out in the same fashion to this same level, subject to a scatter analysis along the lines presented above.” The “scatter analysis” mentioned is one where the measured values of E_{nS} , E_{nP} , and $\langle nS | \vec{D} | nP \rangle$ are used in different combinations with *ab initio* calculations to estimate how much the value of k_{PNC} would change with small changes in the calculated parameters*.

In Blundell *et al.* [14], the authors quote a variation of 0.78%, but the standard deviation of their calculated numbers is more accurate: $\sigma = 0.40\%$. Thus, if we can show that the theory’s predictions agree with experiments at the 0.4% level, then it should be reasonable to trust the calculation of k_{PNC} to that same level.

All of the measurements made prior to the publication of Ref. [14] show only small differences from the theory (less than 1%) with the exception of the measurement of the dc Stark shift [49], where there was a 2% difference between the theory and experiment (Chapter 4). This difference is now 0.3%. In addition, there are new experiments that reveal errors in earlier lifetime measurements of sodium [60, 61] and lithium [62]. These new experiments eliminate what had appeared to be troubling 1% errors in calculations for these atoms that are equivalent to the calculations in cesium. Finally, the agreement is very good for the newly-measured $7S \rightarrow 9S$ interval in ^{210}Fr [63, 64]. Now is an opportune time to revisit the comparison between the experiments and the predictions and re-evaluate the uncertainty that should be attributed to the theory.

In Table 6.1 we have collected the most precise measurements of several quantities in cesium that provide tests of the theory, along with the “rescaled” predictions of Dzuba, *et al.* and the *ab initio* predictions of Blundell *et al.*

Particularly notable are the top three lines of the table, which show that the agreement has dramatically improved from the 1-2% agreements of the older

*This scatter analysis is performed by Blundell *et al.* to estimate their uncertainty. They do not change the numbers they report, however. When the *ab initio* calculation of Dzuba *et al.* did not “reproduce the energy level, (they) multiplied the correlation correction by some numerical factor to fit the energy.” [47] That is, they use the experimental energies to “correct” their calculations.

Table 6.1: Fractional differences ($\times 10^3$) between measured and calculated values of quantities relevant for testing PNC calculations in atomic cesium. We only list the most precise experiments. The second column lists the most relevant aspects of the wave functions that are being tested. $\langle 1/r^3 \rangle_{nP}$ is the average of $1/r^3$ over the wave function of the electronic state nP . Where the experiment has improved or changed significantly since the publication of Ref. [14], the difference from the old experiment is listed in brackets.

Quantity measured	Calculation tested	Difference ($\times 10^3$)		
		Dzuba ^a	Blundell ^b	σ_{Expt}
6S \rightarrow 7S dc Stark shift ^c	$\langle 7P \parallel \vec{D} \parallel 7S \rangle$	-3.4[19]	-0.7[22]	1.0[4]
6P _{1/2} lifetime ^d	$\langle 6S \parallel \vec{D} \parallel 6P_{1/2} \rangle$	-4.2[-8]	4.3[1]	1.0[43]
6P _{3/2} lifetime ^d	$\langle 6S \parallel \vec{D} \parallel 6P_{3/2} \rangle$	-2.6[-41]	7.9[-31]	2.3[22]
α ^e	$\langle 7S \parallel \vec{D} \parallel 6P_{1/2} \rangle$, and $\langle 7S \parallel \vec{D} \parallel 6P_{3/2} \rangle$	—	-1.4	3.2
β ^f	same as α	—	-0.8	3.0
6S HFS ^g	$\psi_{6S}(r=0)$	1.8	-3.1	—
7S HFS ^h	$\psi_{7S}(r=0)$	-6.0	-3.4	0.2
6P _{1/2} HFS ⁱ	$\langle 1/r^3 \rangle_{6P}$	-6.1	2.6	0.2
7P _{1/2} HFS ^j	$\langle 1/r^3 \rangle_{7P}$	-7.1	-1.5	0.5

experiments* .

The standard deviation of the fractional differences between theory and experiment in Table 6.1 is 4.0×10^{-3} . We believe this to be the most objective number to use to represent the 68% confidence level for the atomic theory, and thus for the uncertainty on k_{PNC} . Using the average of $k_{\text{PNC}} = 0.905 \times 10^{-11} iea_0$ [14] and $k_{\text{PNC}} = 0.908 \times 10^{-11} iea_0$ [15], we find the value

$$k_{\text{PNC}} = 0.9065(36) \times 10^{-11} iea_0. \quad (6.2)$$

6.2 Final Result and Implications

We now have all the numbers—with less than 0.5% uncertainty—to determine the value of the weak charge and to test the standard model. When we

*The footnotes in Table 6.1 are as follows. ^aRefs. [15, 47, 48]; ^b[14, 43, 44, 45]; ^cThis work; ^dRef. [40]; ^eUsing the present work and α/β from Ref. [25]; ^fThis work; ^gDefined; ^hRef. [41]; ⁱRef. [65], ^jRef. [42].

combine the 0.40% value of k_{PNC} from Eq. (6.2) with our new 0.30% value for β and the 0.35% measurement $\text{Im}(E_{\text{PNC}})/\beta$ from Ref. [13], we find

$$Q_W = -72.06(28)_{\text{expt}}(34)_{\text{theory}}. \quad (6.3)$$

This value is virtually the same as our previous result $Q_W = -72.11(27)_{\text{expt}}(89)_{\text{theory}}$ [13] but more precise. It would be extremely satisfying to use this number to say that the standard model is either right or wrong, but a quick look at a recent 105 page review paper on tests of the standard model [66] shows that it is not an easy task to draw such a simple conclusion.

The value of Q_W given by Eq. (2.2) depends on the two coupling constants C_{1u} and C_{1d} . These constants depend on the definition of $\sin^2 \theta_W$, and they depend on which radiative corrections are included in the calculation. Using the analysis of [67, 68], we find that the standard model predicts $Q_W = -73.20(13)$. This value differs from our measured value by 2.5σ . A comparison of our values of Q_W and the standard model prediction is shown in Fig. 6.1.

Assuming that the difference from the standard model is not due to an experimental error or a statistical fluctuation, it suggests several possibilities. The first is that there is some very perverse feature of the calculated electronic wave functions and the γ^5 operator that causes errors in the calculation of this matrix element to be much larger than for directly measurable matrix elements. In this case, our assumption that the standard deviation of the values in Table 6.1 represents the accuracy of k_{PNC} is wrong. The second is that there are contributions or corrections to atomic PNC within the standard model that have been overlooked. We see no justification for either of these possibilities, but they clearly need to be explored further. The first offers a formidable but not overwhelming challenge to both theoretical and experimental atomic physicists.

The final possibility is that the discrepancy is indicative of the presence

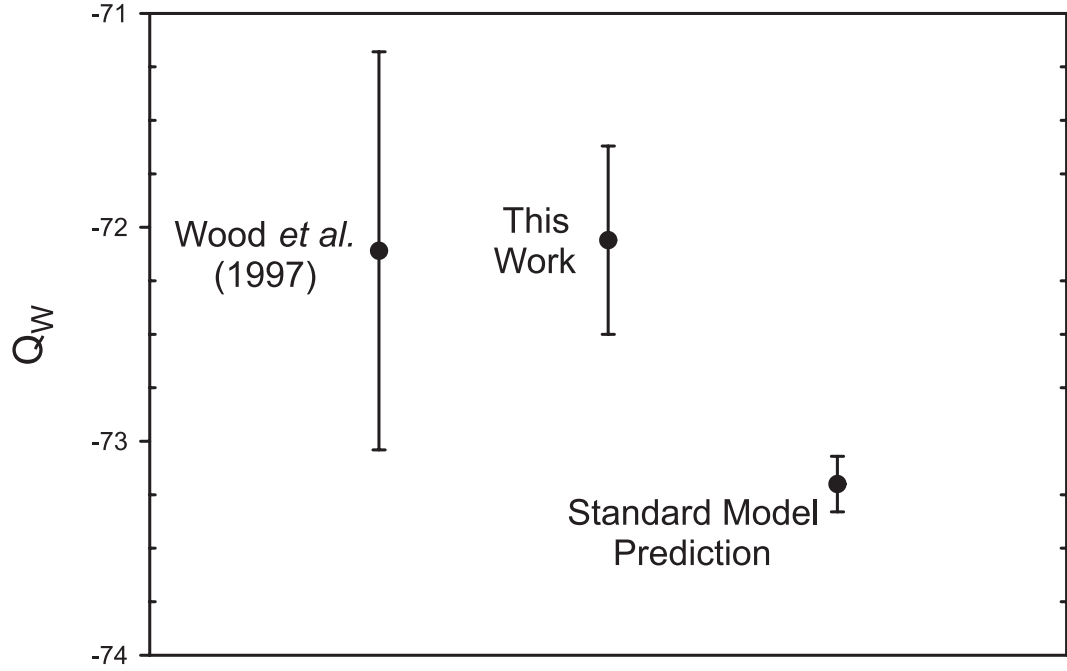


Figure 6.1: A comparison of values from Ref. [13], this work, and the standard model. The difference between this work and the standard model is 2.5σ .

of some new physics not contained in the standard model. Physics that would be characterized by the S parameter is not a likely candidate because the size of the contribution needed ($S = -1.44(35)_{\text{expt}}(46)_{\text{theory}}$) would be in conflict with other data [6]. However, there are other types of new physics, such as an additional Z boson, that would be consistent all other current data. For example, there is a theory that predicts the existence of an extra Z_χ boson [67] with a mass given by

$$\Delta Q_W({}_{55}^{55+N}\text{Cs}) \simeq 0.4(2N + 55) \frac{m_W^2}{m_{Z_\chi}^2}. \quad (6.4)$$

Using our value of Q_W this theory predicts $m_{z_\chi} = 692(29) \text{ GeV}/c^2$. The possibility that this theory—or other theories like it [69, 70, 71, 72, 73]—is correct also needs to be explored further.