Macroscopic Quantum Tunneling and Entangled States in Bose-Einstein Condensates

Lincoln D. Carr

Physics Department

Colorado School of Mines

in collaboration with

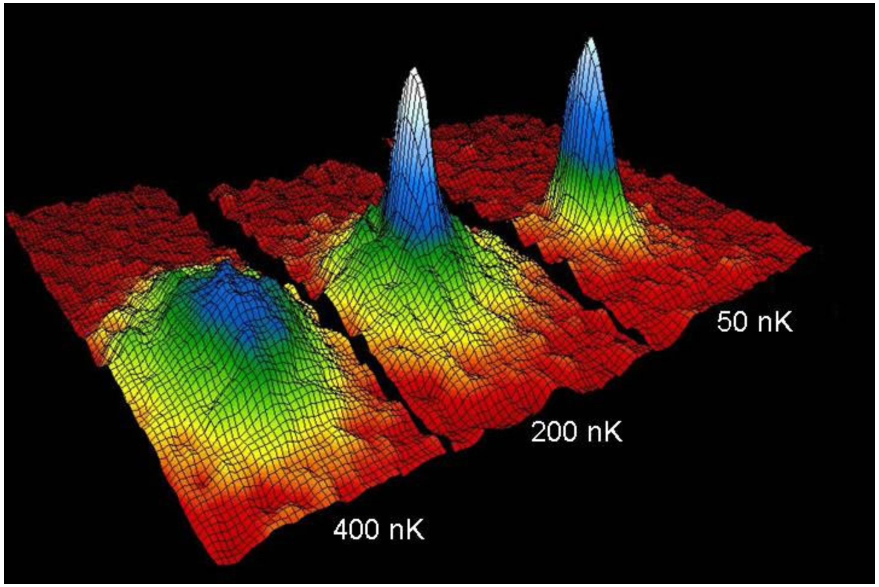
Dimitri Dounas-Frazer and Ann Hermundstad

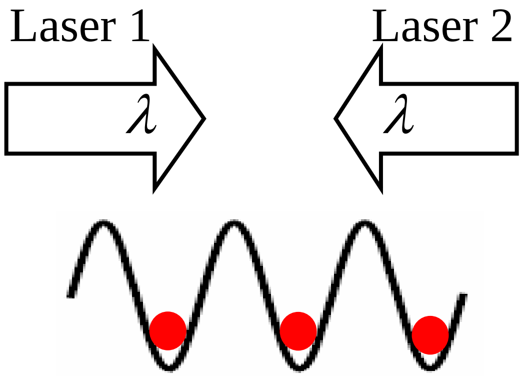
Overview: Macroscopic Quantum Tunneling and Entangled States in Bose-Einstein Condensates

- Introduction
- BEC in a double well
  - Two mode approximation
    - Potential decoherence
    - Tunneling resonances
  - Four mode approximation
    - Role of higher levels in each well
- Macroscopic Quantum Tunneling
  - Quantum sloshing in a tilted double well
- Conclusions and Outlook

Bose-Einstein Condensation in Optical Lattices

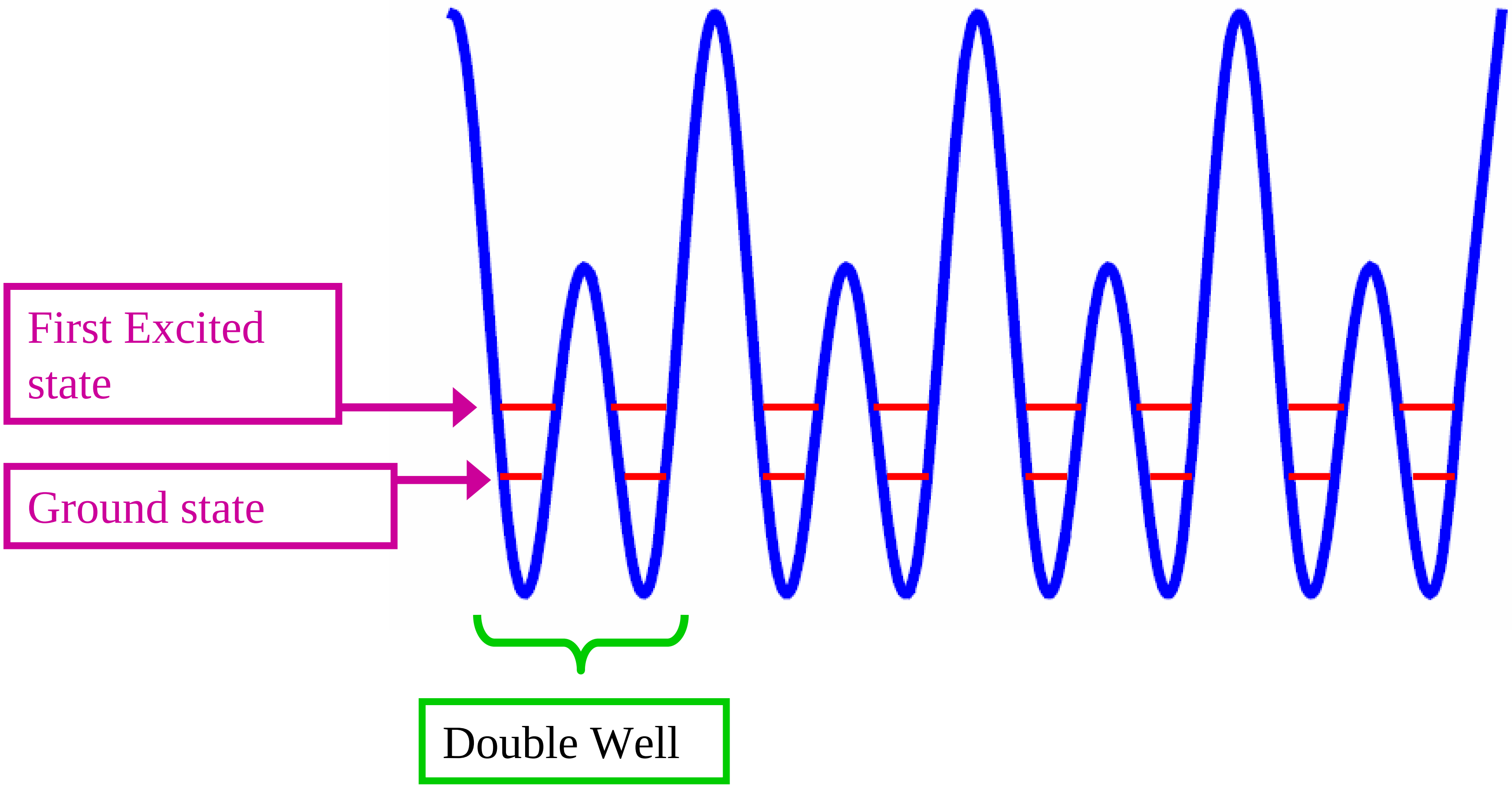
- Combine Bose-Einstein condensate (BEC)
  - JILA: 10^6 gaseous ^{87}Rb atoms in a harmonic trap
    - Nobel prize, 2001
- With a light crystal formed by laser standing waves in 2D/3D





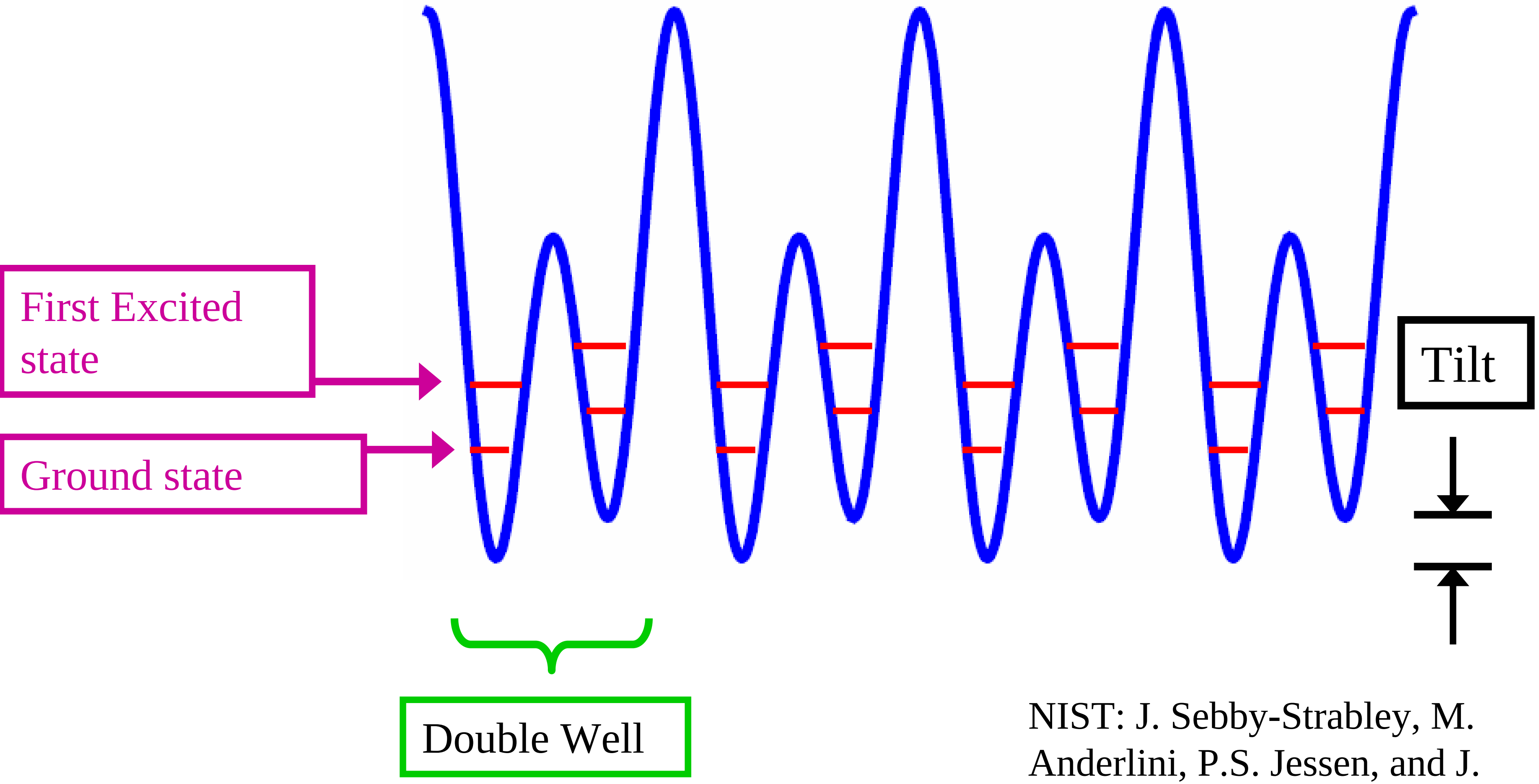
Lattice of Double Wells

- With two frequencies, we obtain



Lattice of *Tilted* Double Wells

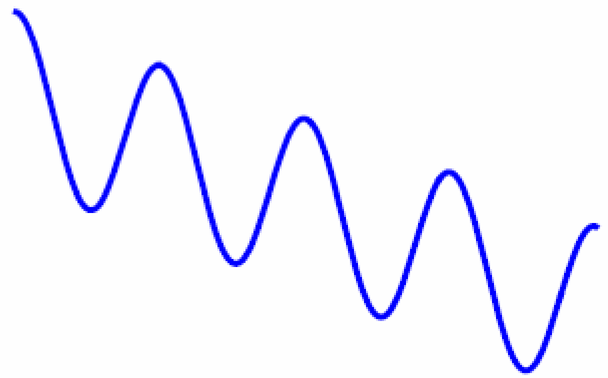
- With more experimental cleverness, one obtains



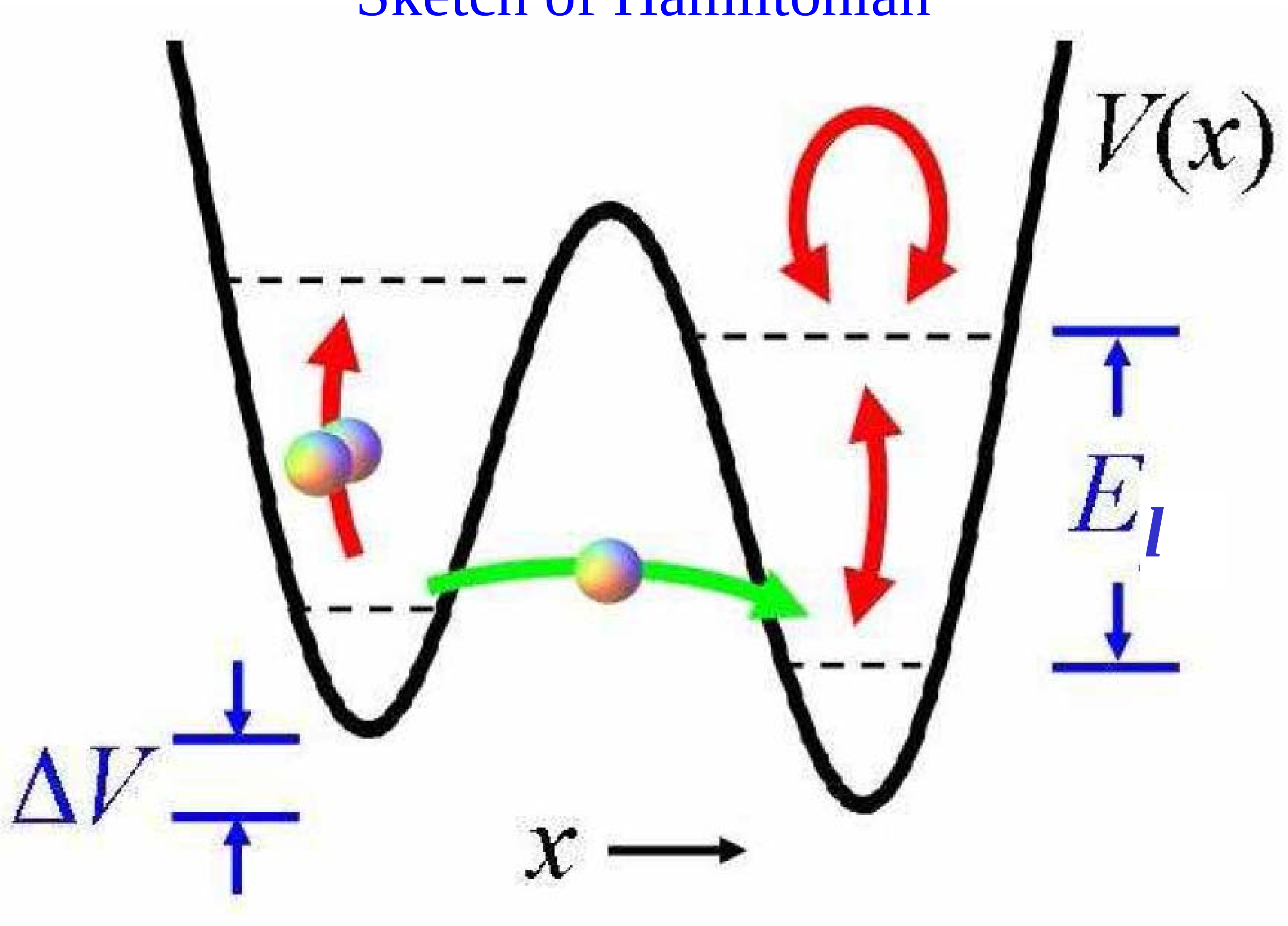
NIST: J. Sebby-Strabley, M. Anderlini, P.S. Jessen, and J. V. Porto, PRA in press (2006)

Motivation

- Quantum many body theory of the tilted double well
  - Controlled 2-qubit gates for quantum computing
  - Gravitometry, atom laser
  - BEC in a double well
    - Many-body entangled states
  - Macrosopic quantum tunneling
    - Push the limits of quantum mechanics



Sketch of Hamiltonian



Fundamental Equations I: Two mode Hamiltonian

- 1. Second quantized continuum field theory for weakly interacting gas in s-wave limit
- 2. Construct localized states and discretize
- 3. Obtain Bose-Hubbard-like Hamiltonian

Hopping

Level index: 0,1

Site index: L,R

$$\hat{H}^m = -J^m \sum_j \hat{b}_j^{m\dagger} \hat{b}_j^m + U^{mm} \sum_j \hat{n}_j^m \left(\hat{n}_j^m - 1\right) + \Delta V \hat{n}_L^m ,$$

Interaction

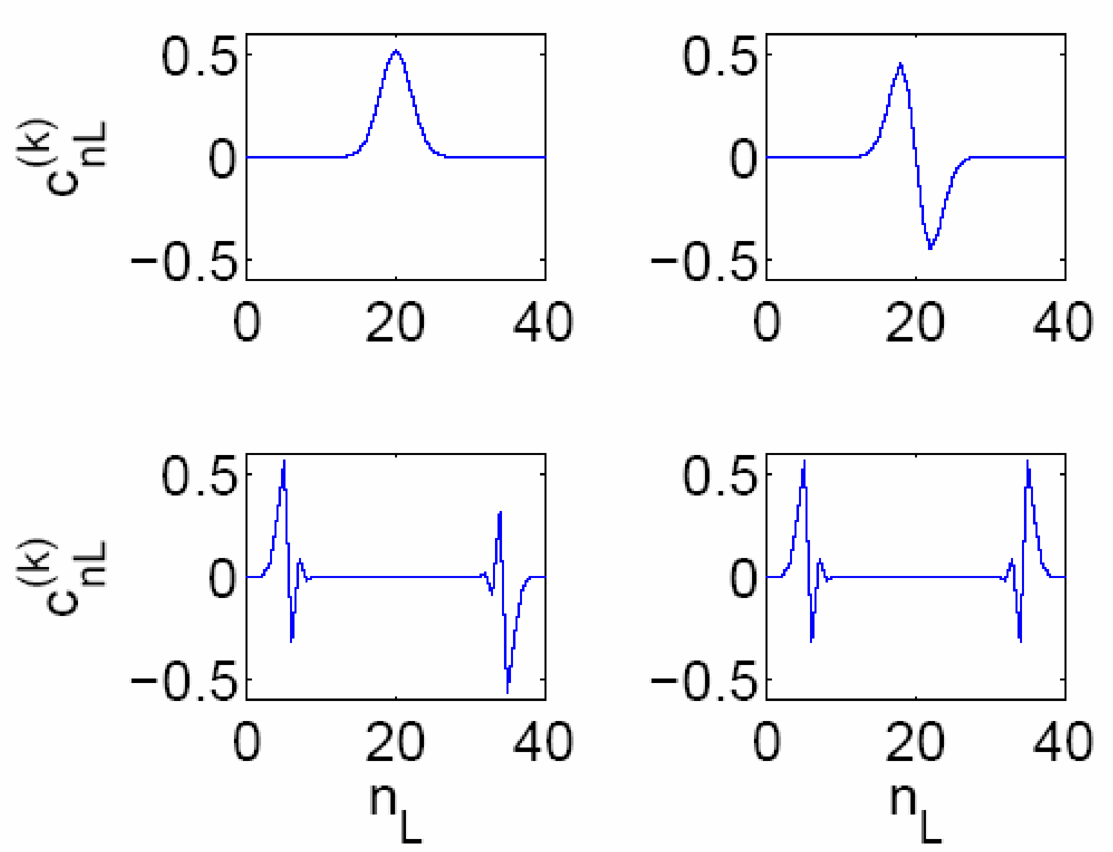
Tilt

See, e.g., M. Jaaskelainen and P. Meystre, Phys. Rev. A 71, 043603 (2005)

Typical solutions

- 2 sites, 1 band, no external potential, Fock states

$$\left|\psi^{(k)}\right\rangle = \sum_{n_L} c_{n_L}^{(k)} \left|n_L, N-n_L\right\rangle$$

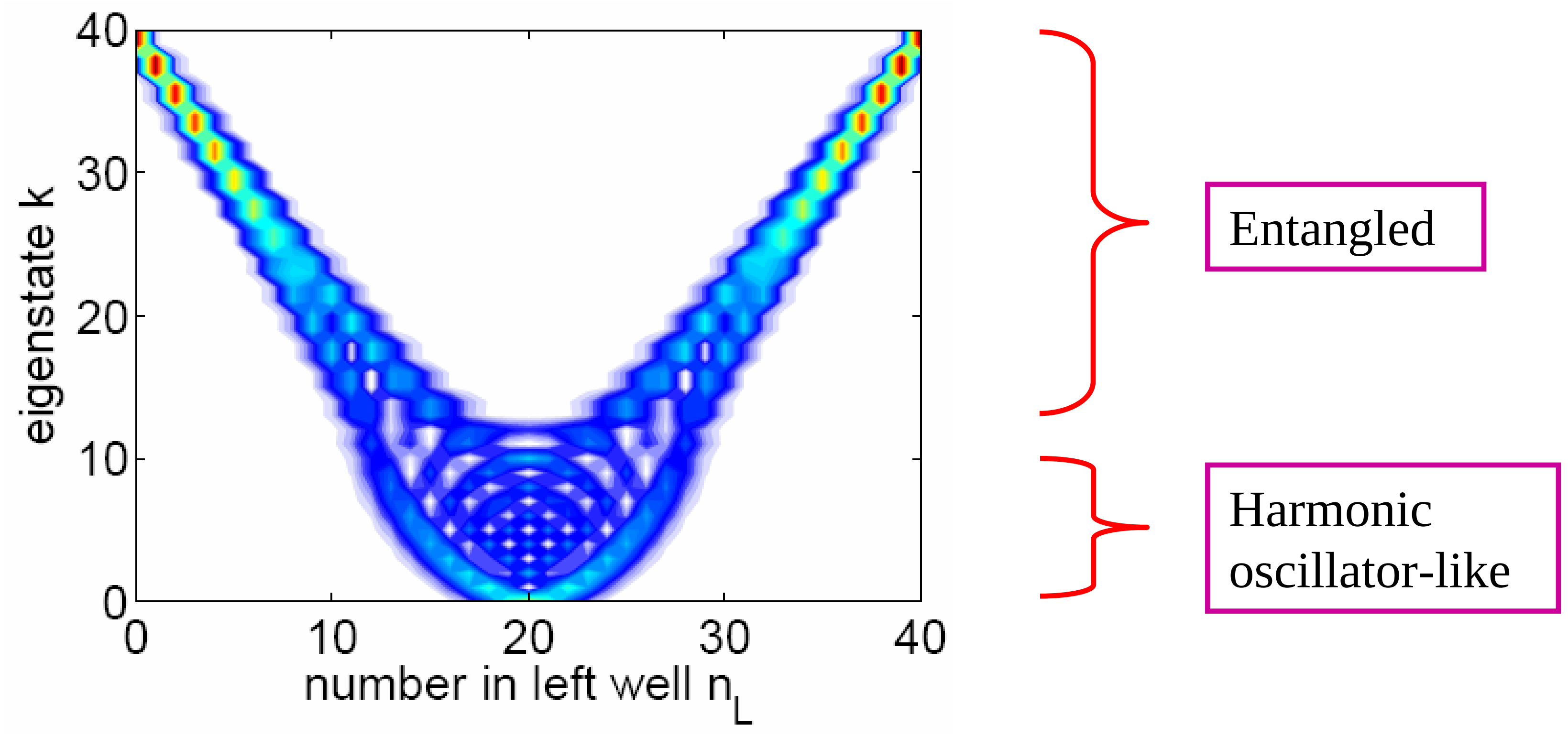


(1) Harmonic Oscilllator-like: Superfluid to Mott

(2) Entangled: Antisymmetric and Symmetric

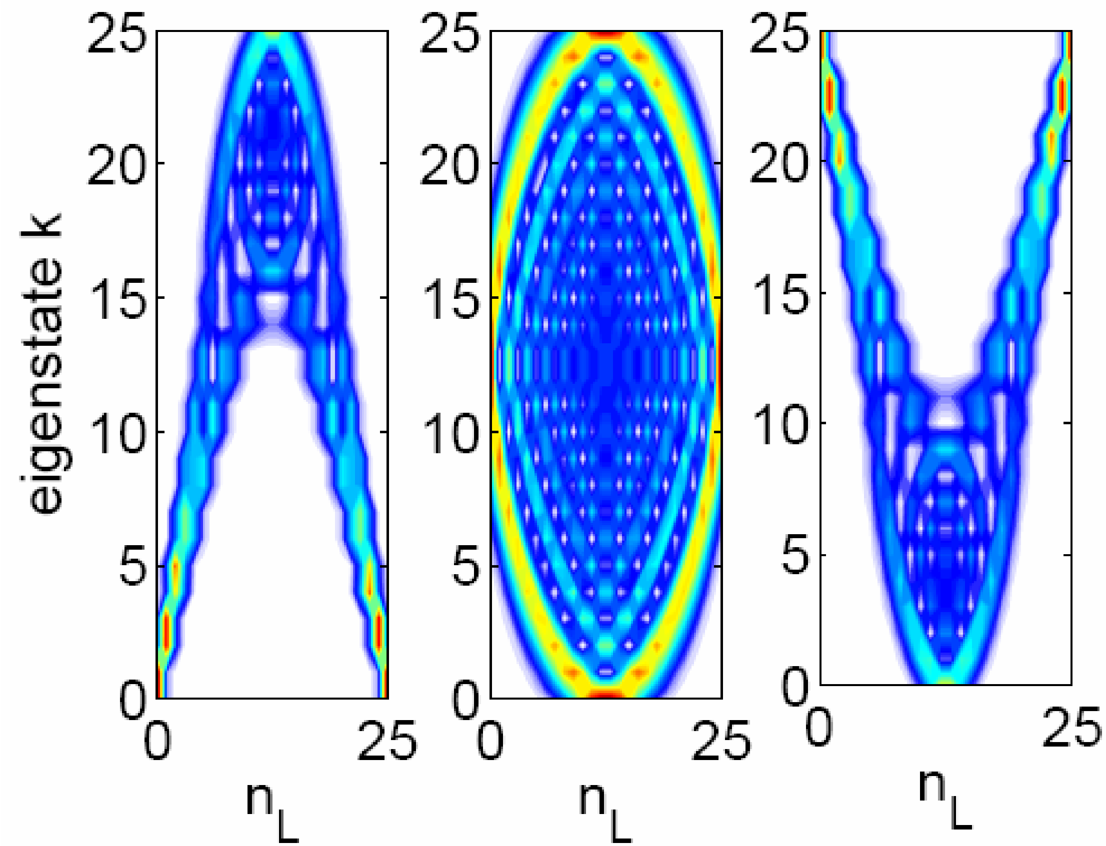
All eigenstates of two-mode Hamiltonian

- Probability density



Ratio of Hopping/Interaction

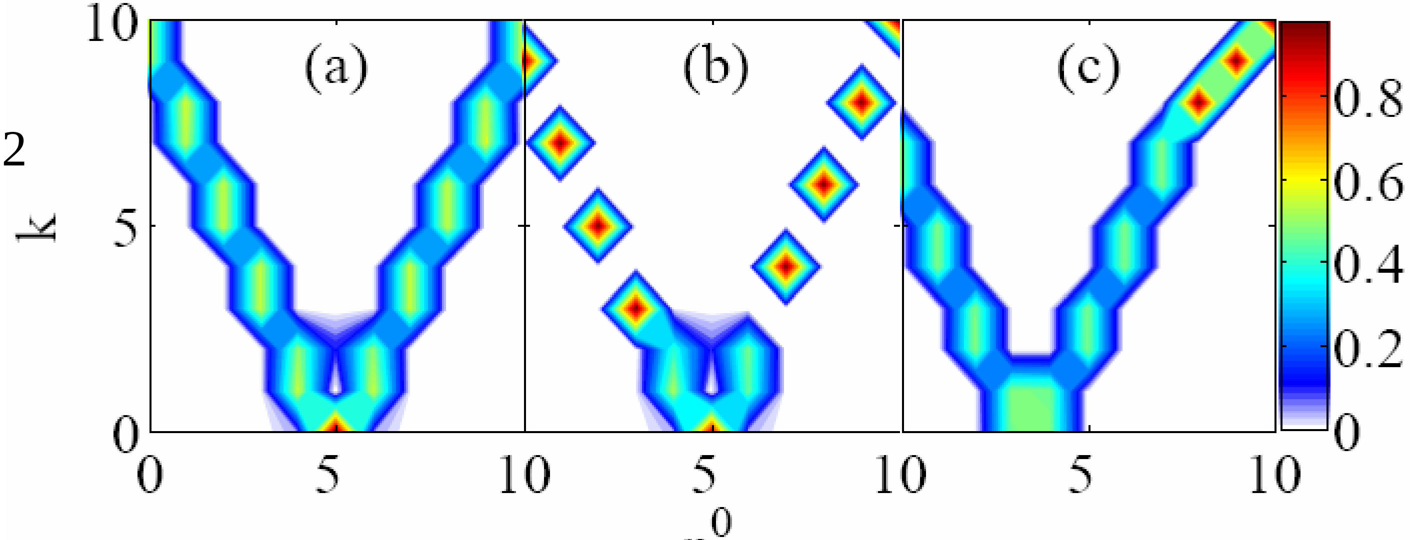
- Probability Density

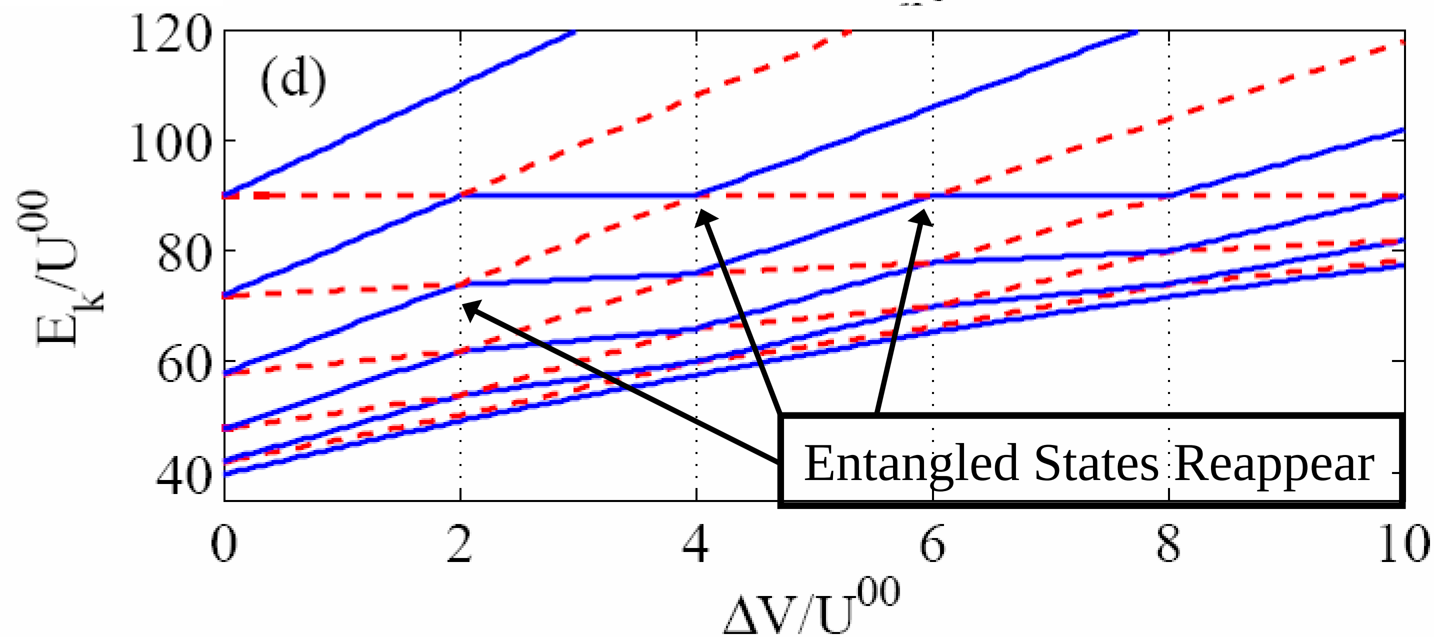


Strong Attractive Interactions	Weak Interactions	Strong Repulsive Interactions

Response to Tilt: N=10

- (a) No tilt
- (b) $\Delta V/U=10^{-2}$
- (c) $\Delta V/U=6$





Tunneling resonances

- Entangled states are fragile with respect to *potential decoherence*
- Reappear when

$$\Delta V = \Delta V_n^m \equiv 2nU^{mm}, \;\; n = 1,\, 2,\, \ldots,\, N^m - 1$$

- Width of reappearance
  = width of avoided crossing
  = energy difference ΔE between symmetric/anti-symmetric pairs of states

$$\Delta V_{\mathrm{crit}}^m \equiv \frac{2\Delta E_{n_L}^m}{N^m - 2n_L^m}, \quad 0 \leq n_L^m < N^m/2$$

Fundamental Equations II: Four mode Hamiltonian

- Two-mode Hamiltonian for ground and first excited levels plus coupling terms

$$\hat{H} = \sum_m \hat{H}^m + E_\ell \hat{N}^1$$

Level energy difference

$$+ U^{01} \sum_{j,m\neq m'} \left[2\hat{n}_j^m \hat{n}_j^{m'} + \hat{b}_j^{m\dagger} \hat{b}_j^{m\dagger} \hat{b}_j^{m'} \hat{b}_j^{m'} \right]$$

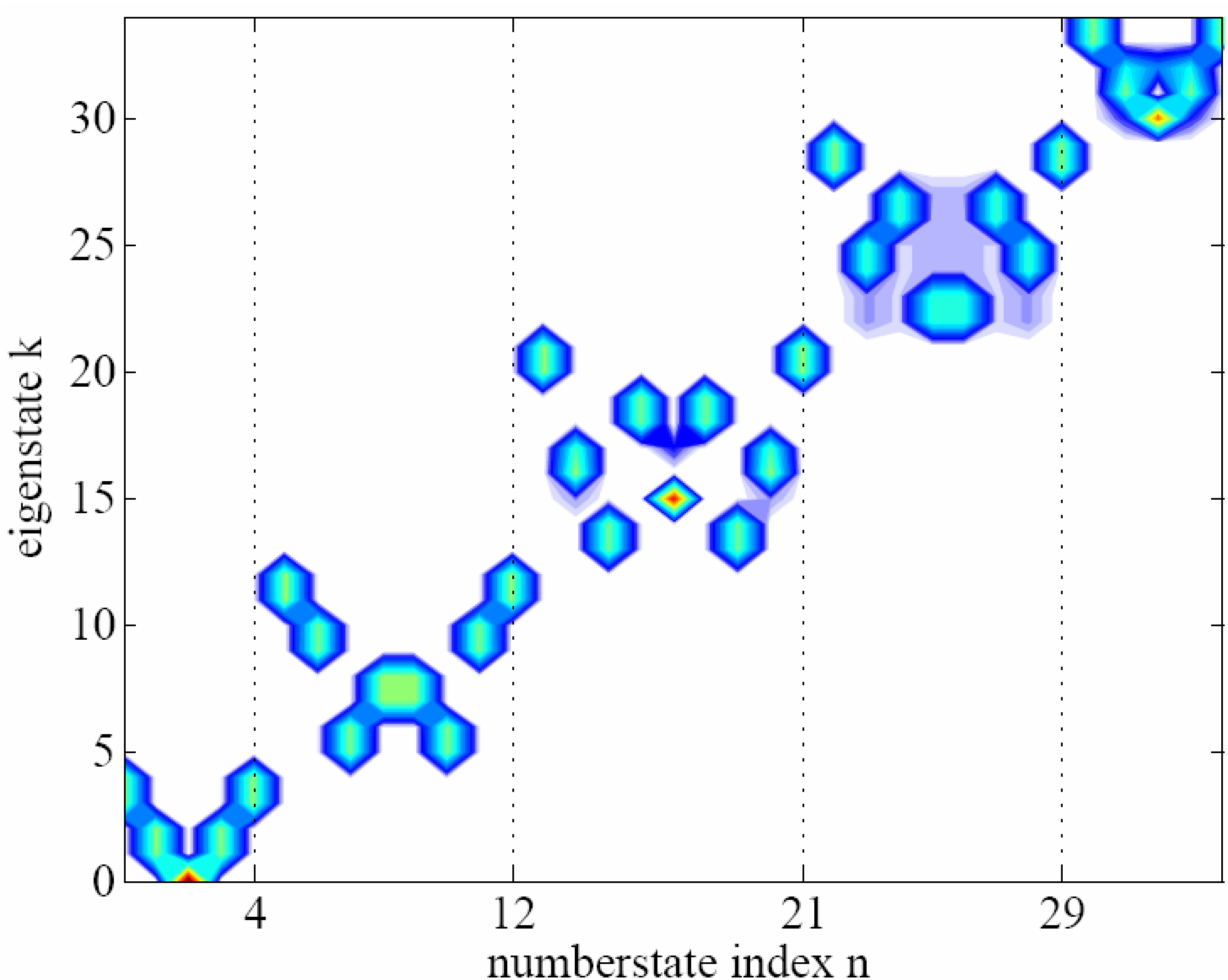
Inter-level interaction

Inter-level pair hopping

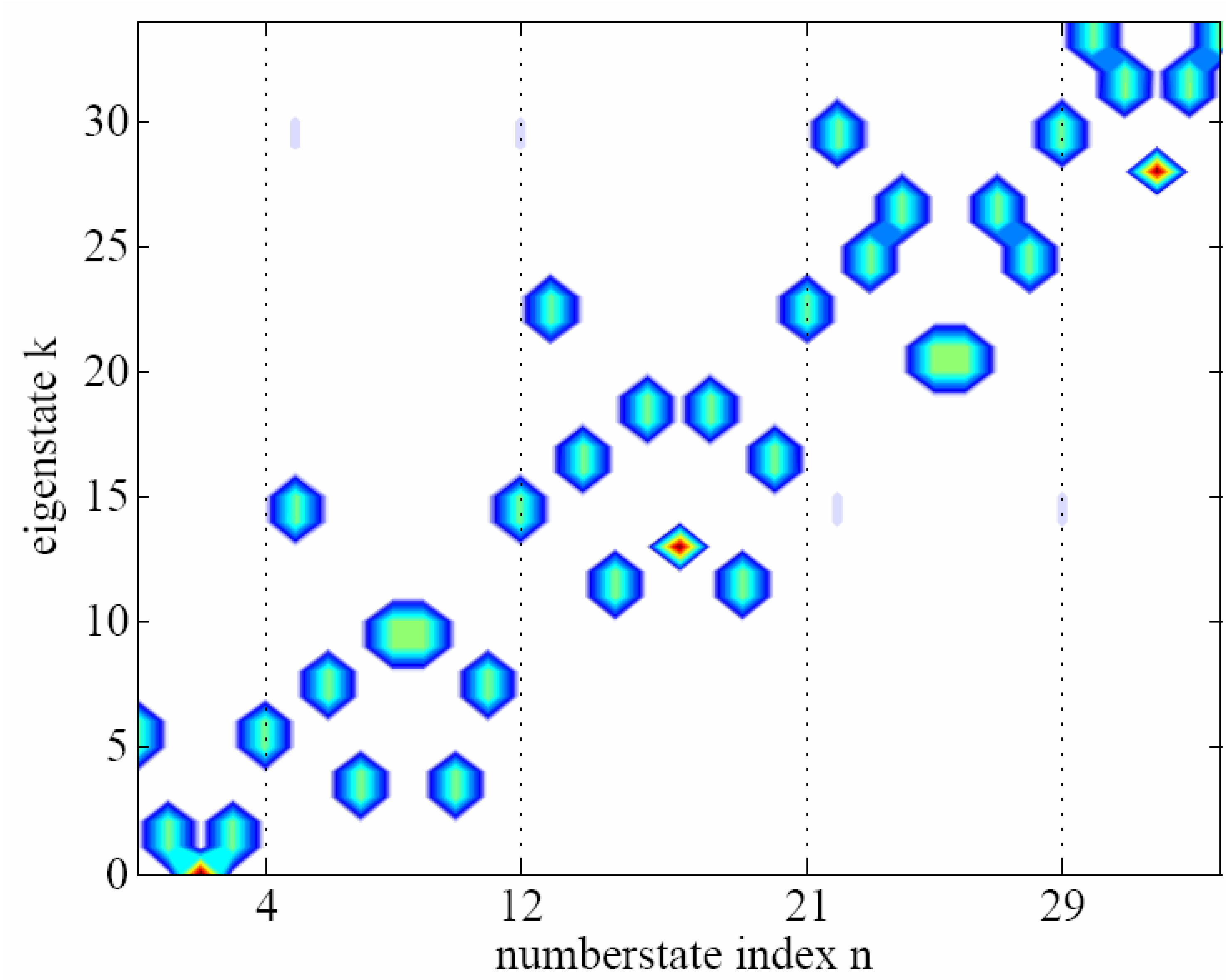
- Fock state Hilbert space

$$|\Psi\rangle = \sum_n c_n |n\rangle, \;\; |n\rangle = |n_L^0, n_R^0\rangle \otimes |n_L^1, n_R^1\rangle$$

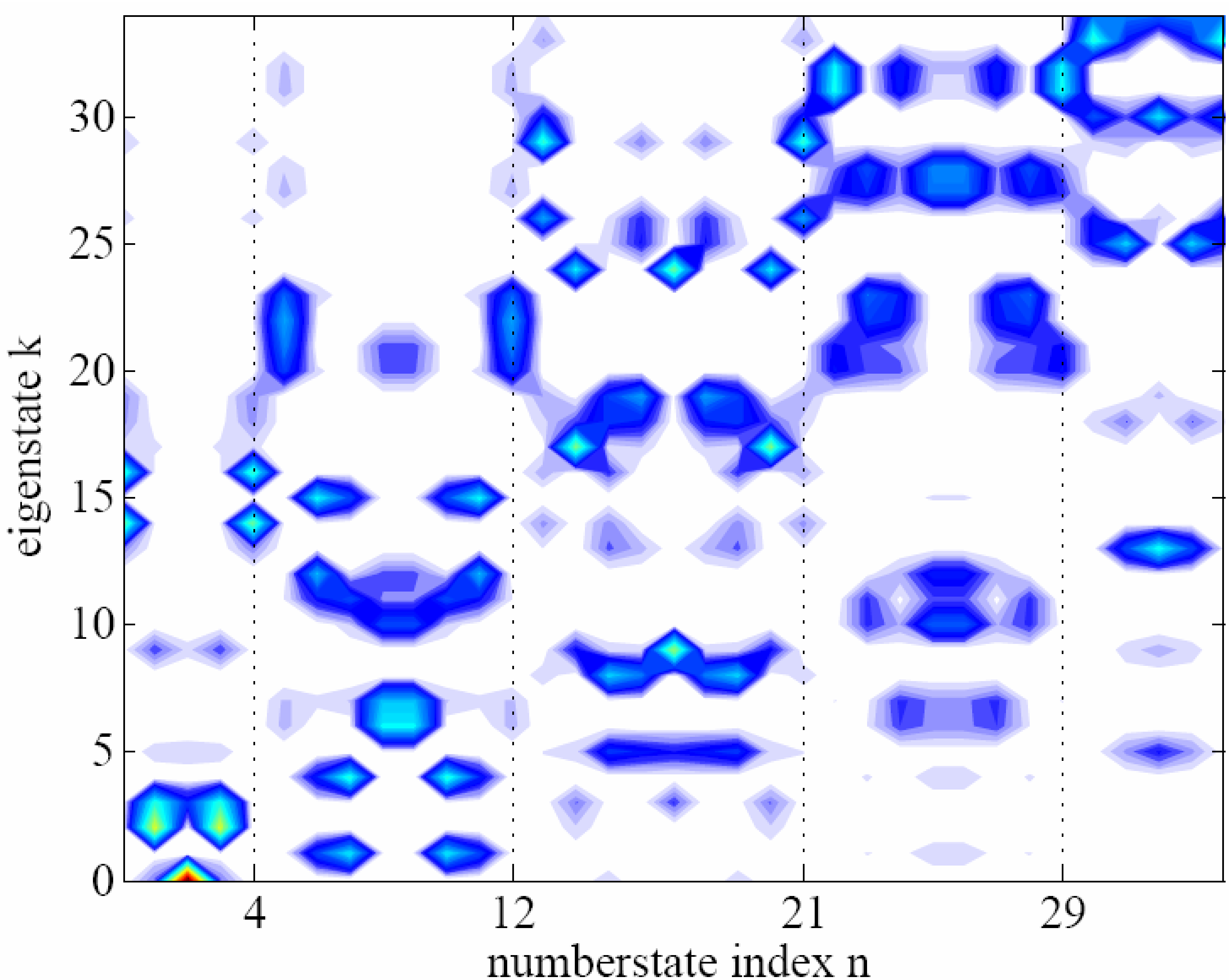
Four-mode Model Stationary States I



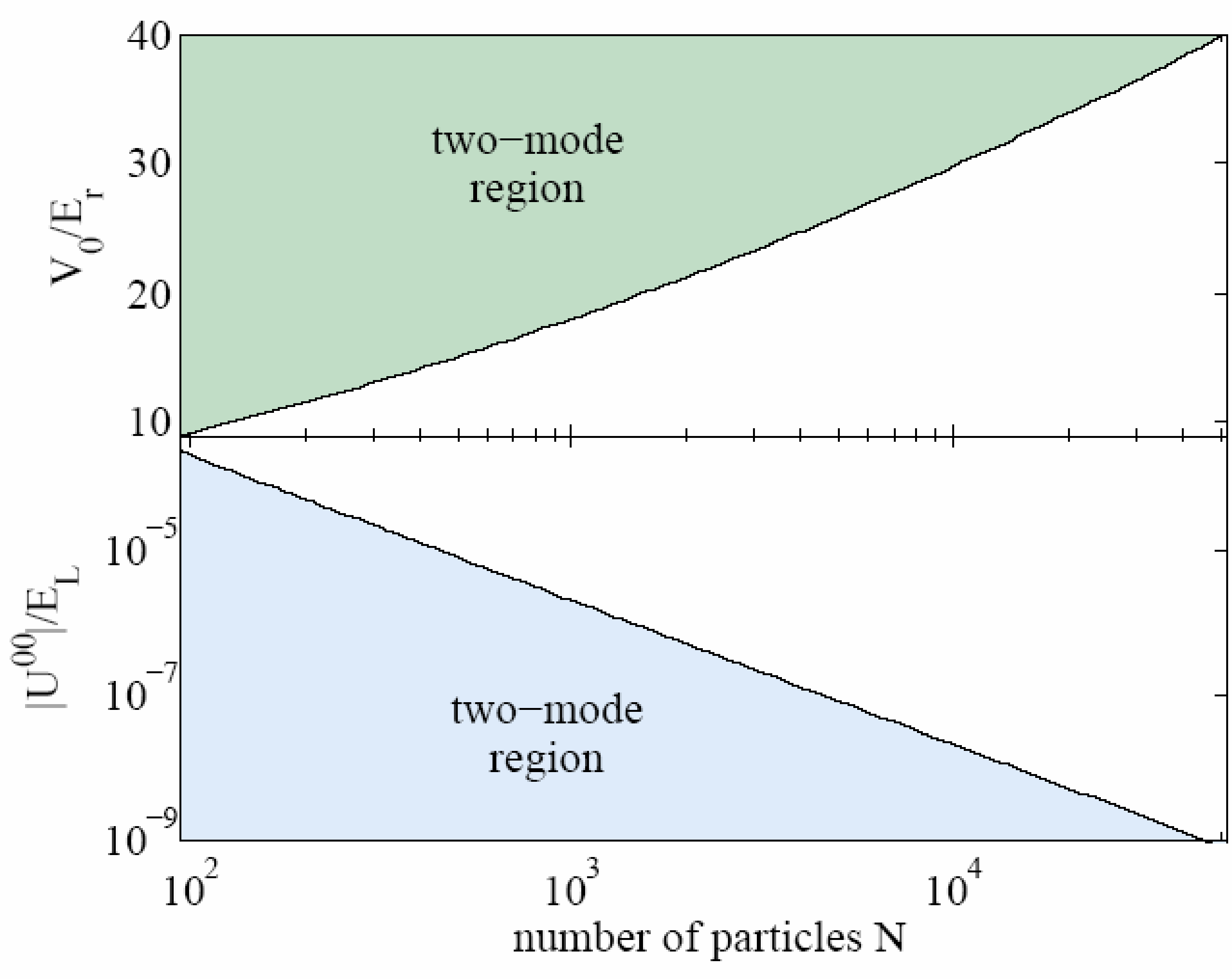
Four-mode Model Stationary States II



Four-mode Model Stationary States III



Bounds on Use of Two-mode Approximation

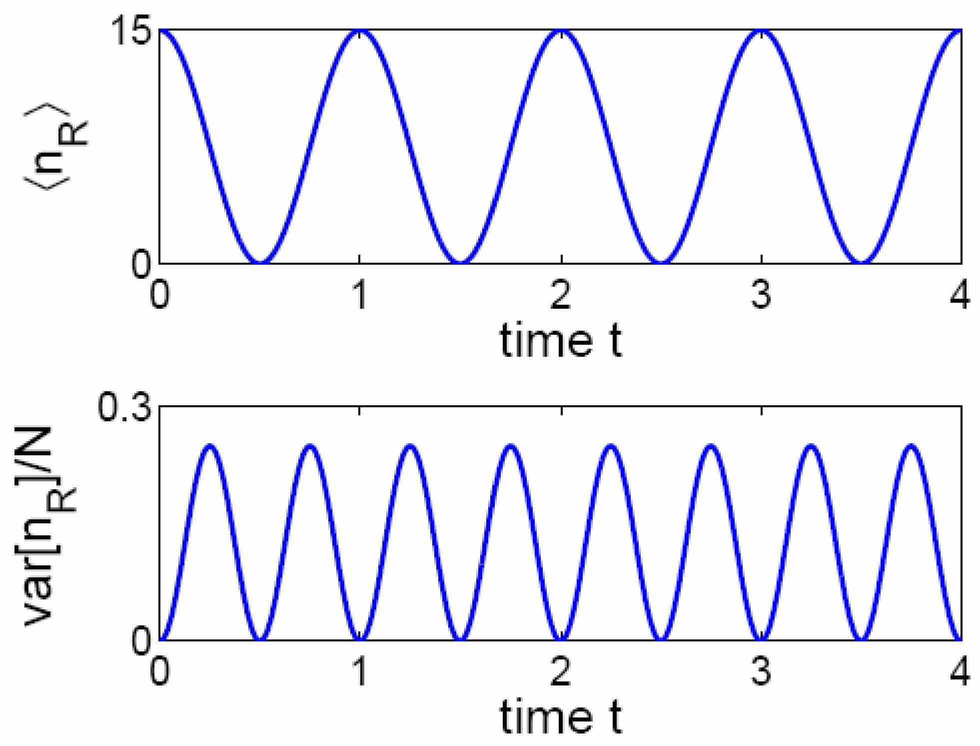


Sloshing in a Tilted Double Well: The Basic Idea

- Build up a picture of MQT particle by particle
- Tune towards Mott border (raise lattice barrier height)
- Observable in experiments

Dynamics: Non-interacting (U=0)

- Average number and average normalized variance

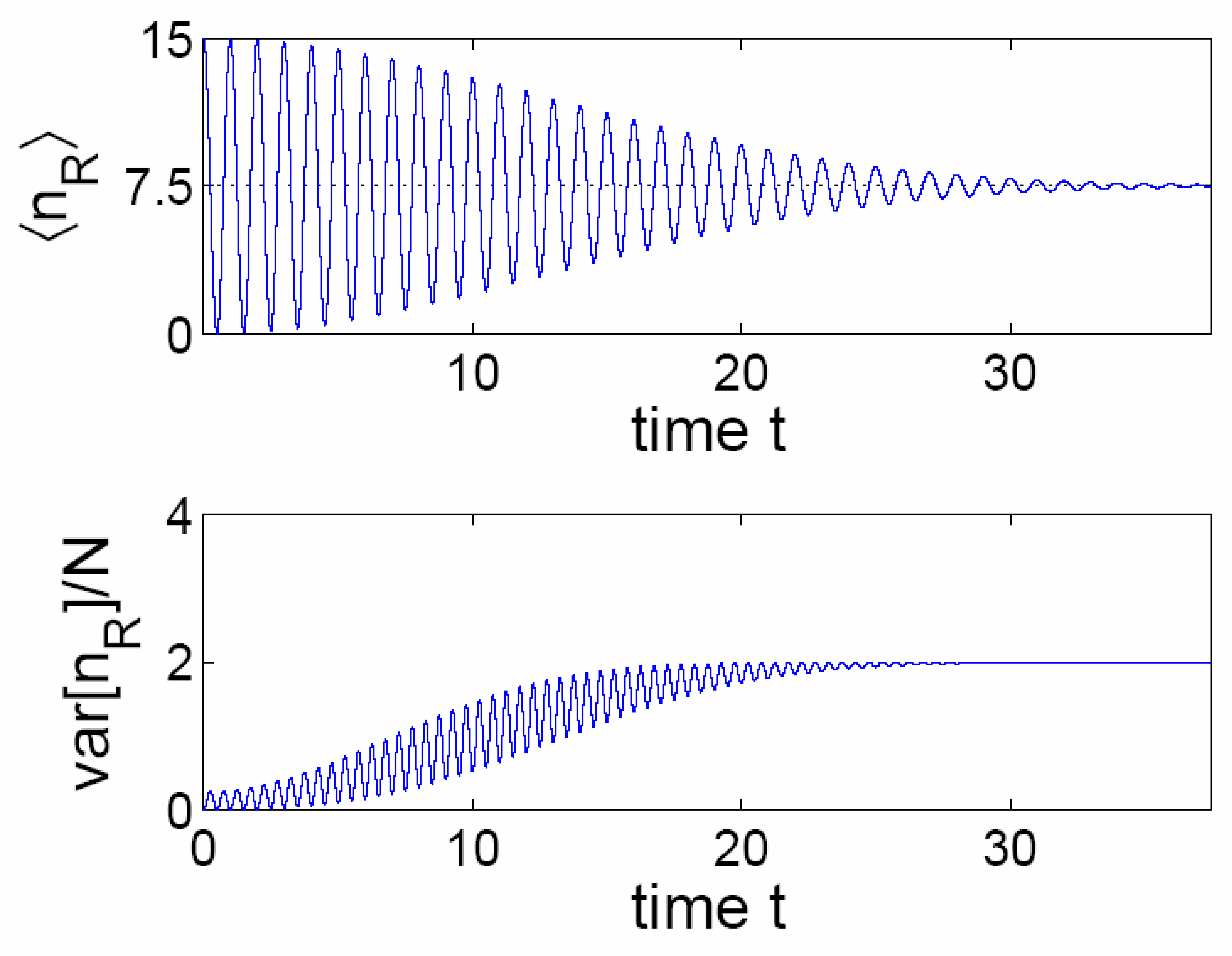


Time in units of

$\pi\hbar/J$

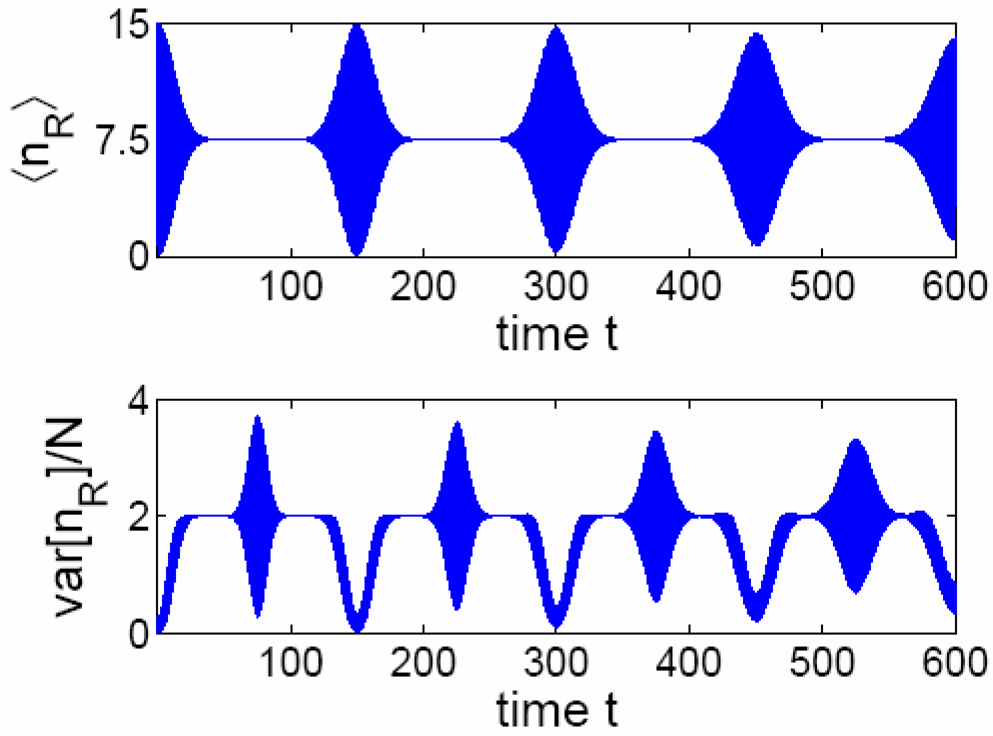
$$\langle \hat{n}_R \rangle = N(1 + \cos 2Jt/\hbar)$$

Weak Interactions (J/NU >> 1), Short Time Oscillations



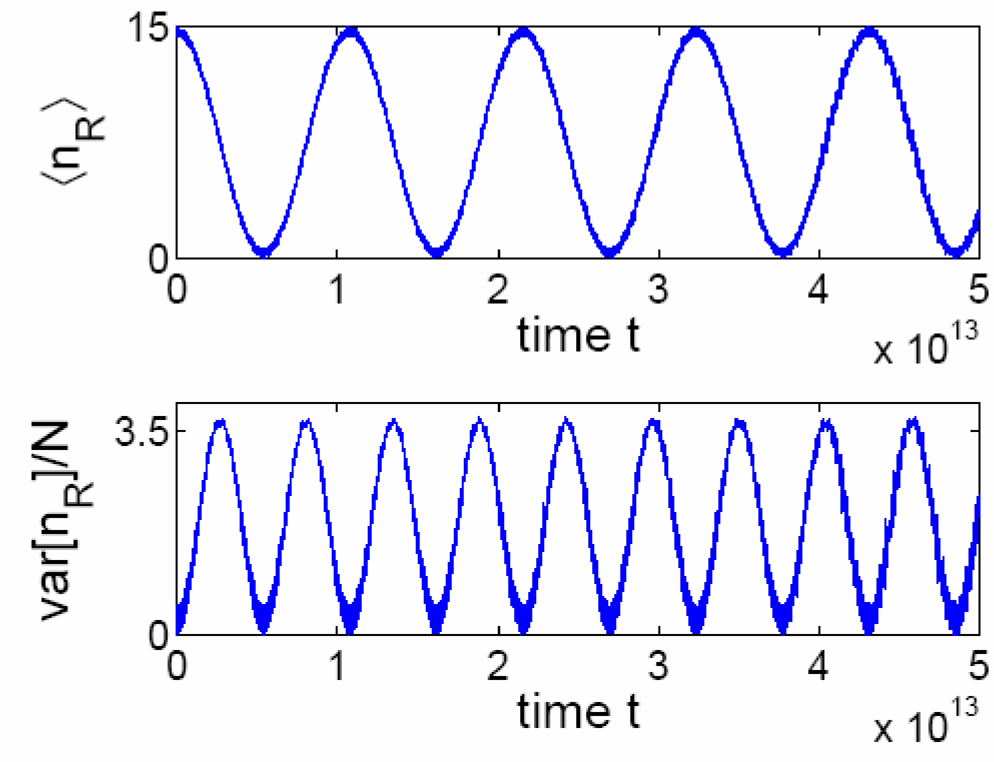
Weak Interactions:Long Time Oscillations

- $\langle \hat{n}_R \rangle = N \left(1 + \cos(2Jt/\hbar)\cos(Ut/\hbar)^{N-1}\right)$
  - Tunneling ➔ carrier frequency
  - Interactions ➔ envelope frequency



Strong Interactions/High Barrier (J/NU << 1)

- $\langle \hat{n}_R \rangle = N\left(1 + \cos(\Delta E_N t/\hbar)\right)$
  - ΔE_N is the splitting between antisymmetric and symmetric extreme cat states $|0, N\rangle \pm |N, 0\rangle$

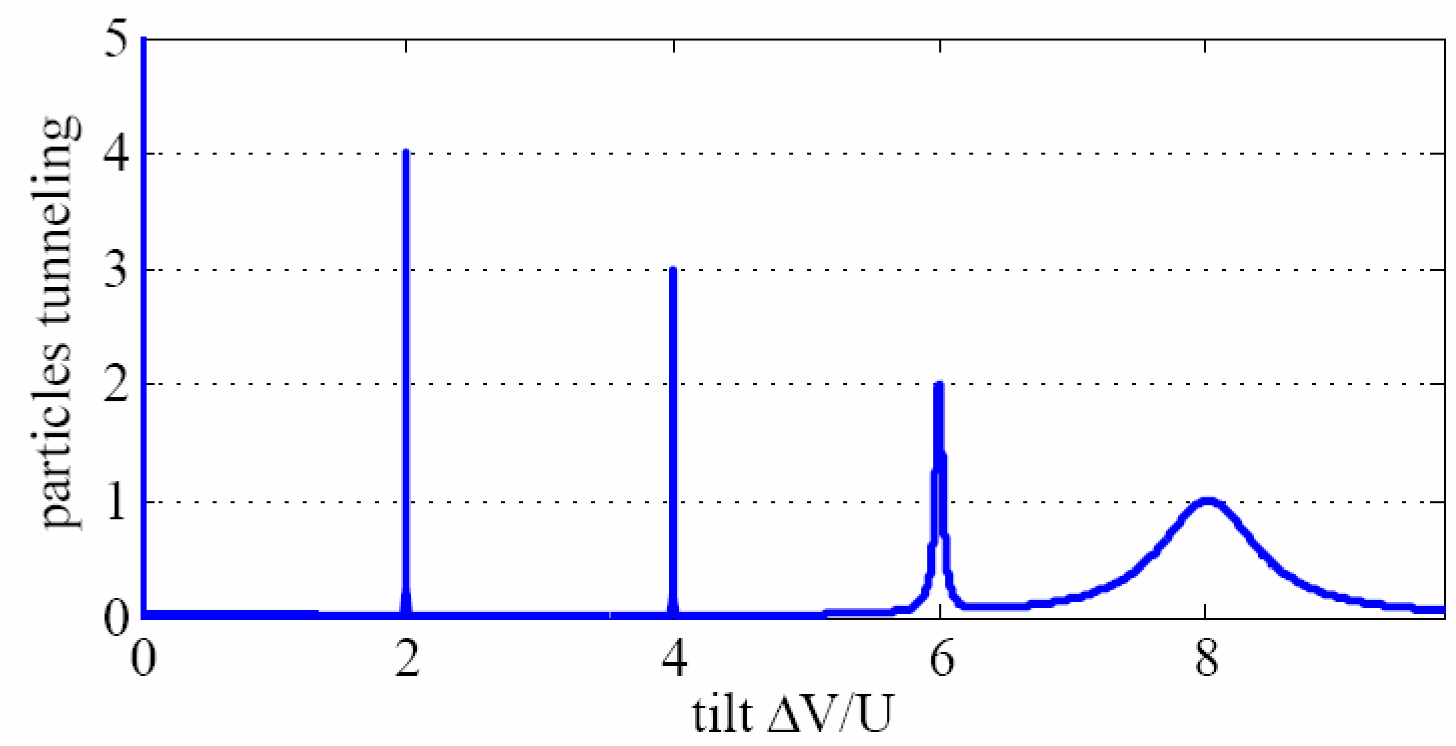


Time in units of $\hbar/NU$

$$T_{osc} \propto (U/J)^{N-1} \pi\hbar/J$$

Tunneling Resonances

- Tilt 2Uk ➔ N-k particles slosh



Conclusions

- Higher mode effects cannot be neglected in many experiments

- Entangled states are fragile to potential decoherence
  - Reappear for tilt proportional to interaction strength

- MQT tunneling times exponential in number of particles in high barrier limit