US-Japan Seminar, Breckenridge, Aug23-25, 2006

Superradiant light scattering from condensed and non-condensed atoms

Aug 23, 2006

Yoshio Torii, Yutaka Yoshikawa ,and Takahiro Kuga

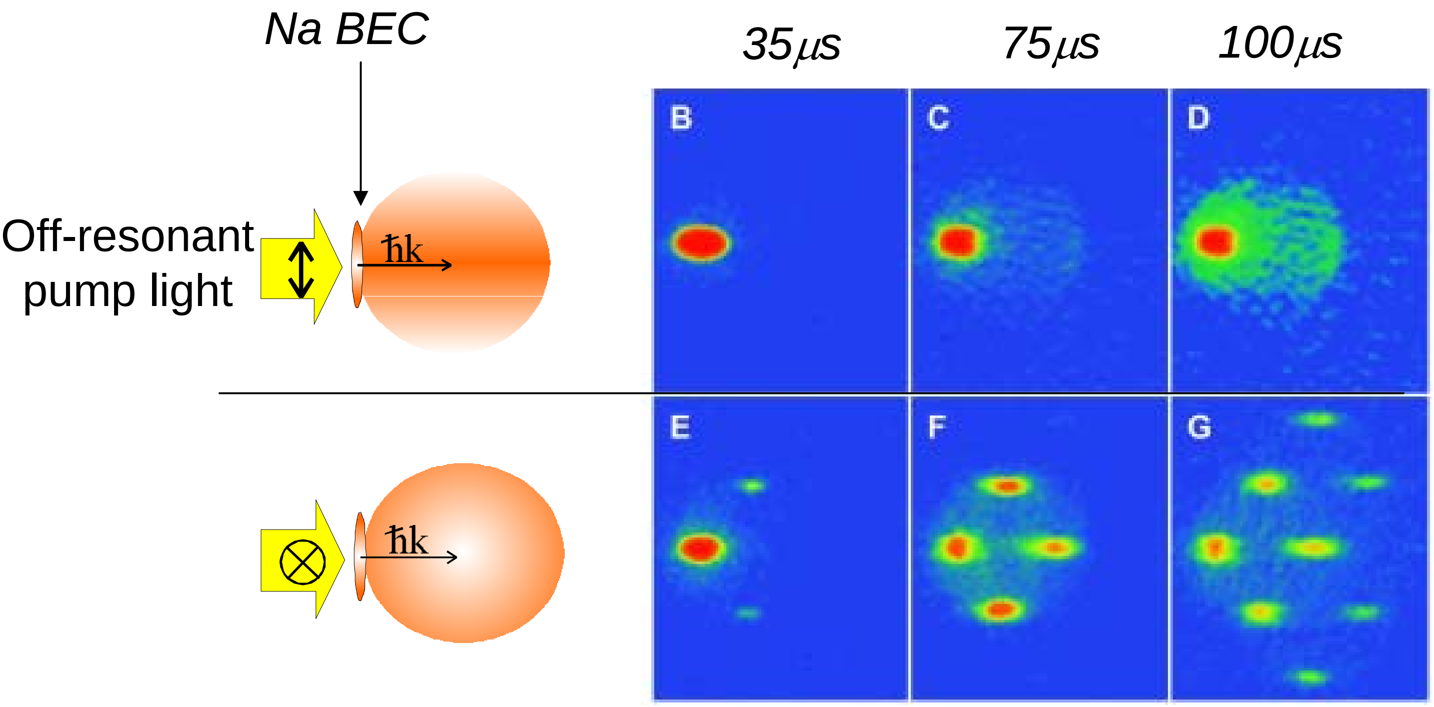
Institute of Physics, University of Tokyo, Komaba

Outline

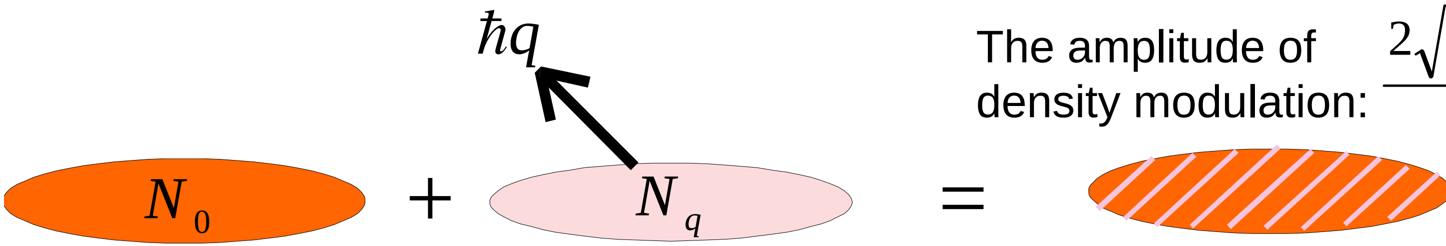
- Review of superradiance in a BEC (MIT99)
- Superradiance in the short and strong pulse regime (MIT03)
- Raman superradiance (Tokyo04, MIT04)
- Superradiance in a thermal atom cloud (Tokyo05)

Superradiant Rayleigh scattering from a Bose-Einstein condensate

S. Inouye, et. al., Science **285**, 571 (1999)

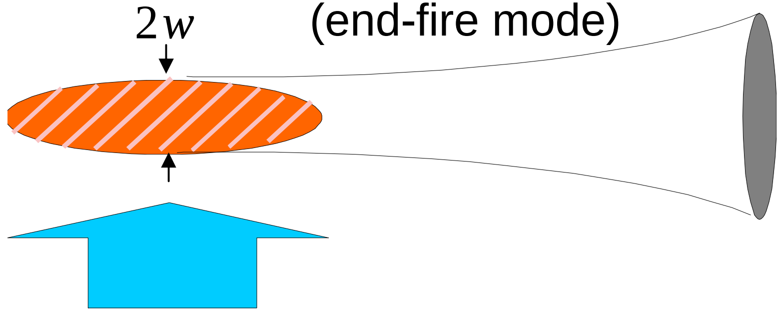


Semi-classical explanation



The amplitude of density modulation: $\dfrac{2\sqrt{N_0 N_q}}{V}$

Bragg scattered Light (end-fire mode)



Phase-matching solid angle

$$\Omega \approx \left(\frac{\lambda}{\omega}\right)^2$$

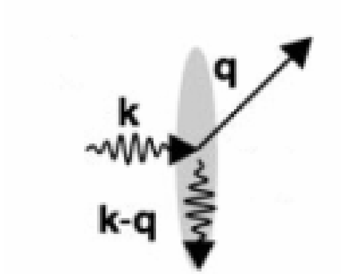
Power in the end-fire mode $P = \hbar\omega \dfrac{\sin^2\theta}{8\pi/3} R N_0 N_q \Omega$

$$\boxed{\dot{N}_q = R \frac{\sin^2\theta}{8\pi/3} N_0 N_q \Omega}$$

R : single-atom Rayleigh scattering rate

Fully-quantum picture (Fermi's Golden Rule)

Scattering process:



Condensate

Recoiling atoms

$$|N_0, n_k; N_q, n_{k-q}> \xrightarrow[\hat{H} = \hat{a}_q^+ \hat{c}_{k-q}^+ \hat{a}_0 \hat{c}_k]{} |N_0 - 1, n_k - 1; N_q + 1, n_{k-q} + 1>$$

Pump beam

Endfire mode

Scattering rate:

$$W \propto |< N_0 - 1, n_k - 1; N_q + 1, n_{q-k} + 1 | \hat{H} | N_0, n_k; N_q, n_{k-q} >|^2$$

$$= N_0 n_0 \left(N_q + 1\right)\left(\cancel{n_{k-q}} + 1\right)$$

neglect

Summing over Ω

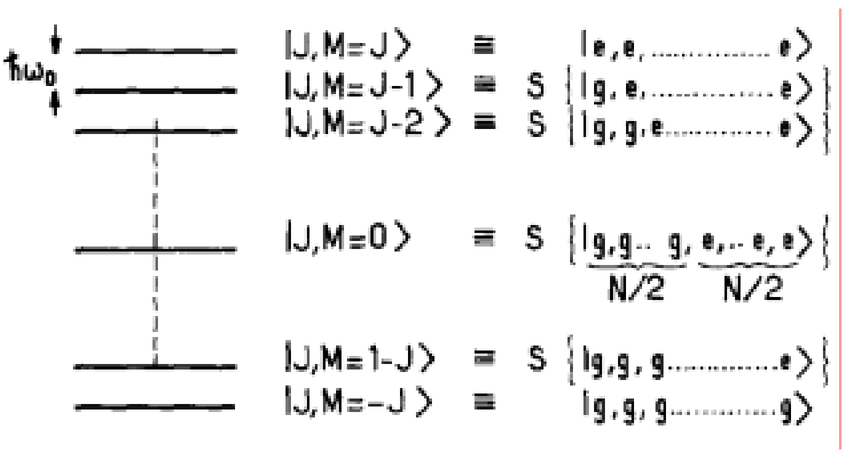
$$\boxed{\dot{N}_q = R \frac{\sin^2\theta}{8\pi/3} N_0 \left(N_q + 1\right) \Omega}$$

Stimulated scattering (Bosonic enhancement)

Spontaneous scattering

Dicke's picture

N-atom system ⇔ N spin-1/2 system with the total spin J = N/2 (assumption: *Indiscernability* of the atoms with respect to photon emission)



R. H. Dicke, Phys. Rev. **93**, 99 (1954)
M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982)

Spontaneous emission rate

$$W_N = \Gamma \langle J, M \mid J_+ J_- \mid J, M \rangle$$
$$= \Gamma (J+M)(J-M+1)$$
$$= \Gamma N_e (N_g + 1)$$

$$\Gamma \to R \frac{\sin^2\theta}{8\pi/3} \Omega$$

$$N_g = N_0$$
$$N_e = N_q$$

$$\dot{N}_j = R \frac{\sin^2\theta}{8\pi/3} N_0 \left(N_q + 1 \right) \Omega$$

Three different pictures for superradinace in a BEC

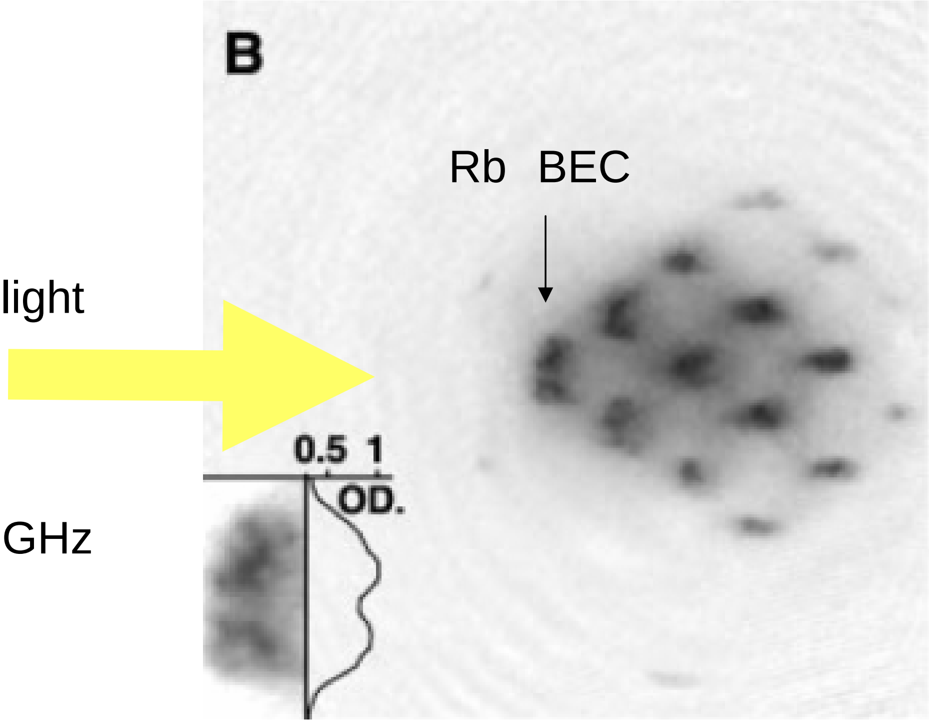
- Semi-classical picture (Bragg diffraction of a pump beam off a matter wave grating)
- Full-quantum picture (Bosonic enhancement by the recoiling atoms)
- Dicke’s picture (enhanced radiation from a symmetric cooparative state)

Superradiant Rayleigh scattering in a Rb BEC

Week pump light

63 mW/cm^2

detuning: -4.4 GHz



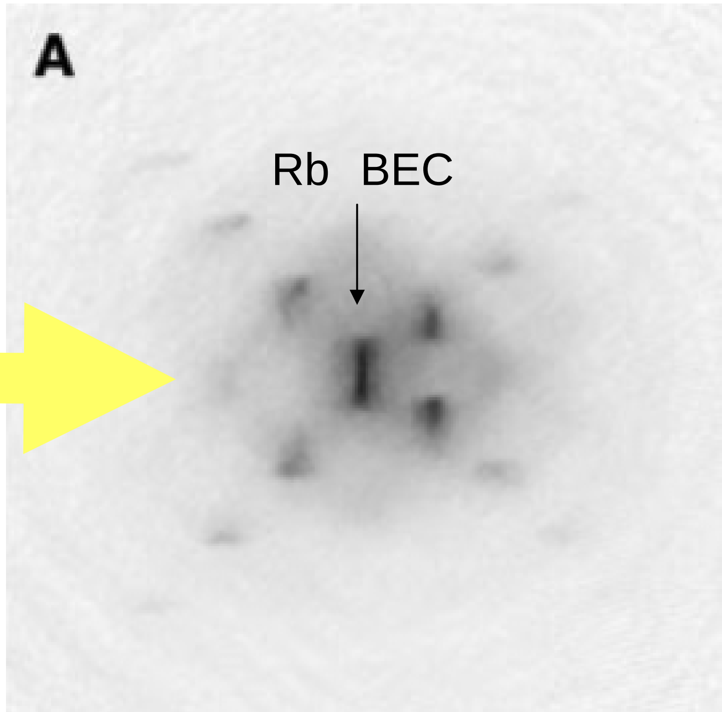
25 μs-3200 μs

30 ms TOF

D. Schneble, Y.T., M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003)

Superradiant Rayleigh scattering in the short (strong) pulse regime

Strong pump light



1 μs-10 μs

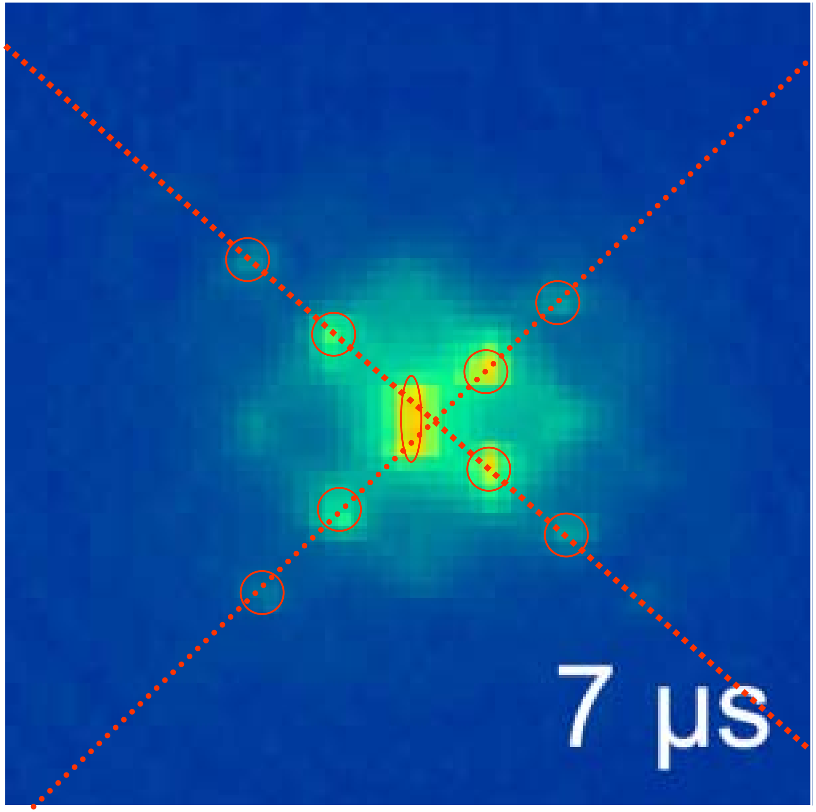
63 mW/cm^2

detuning: -440 MHz

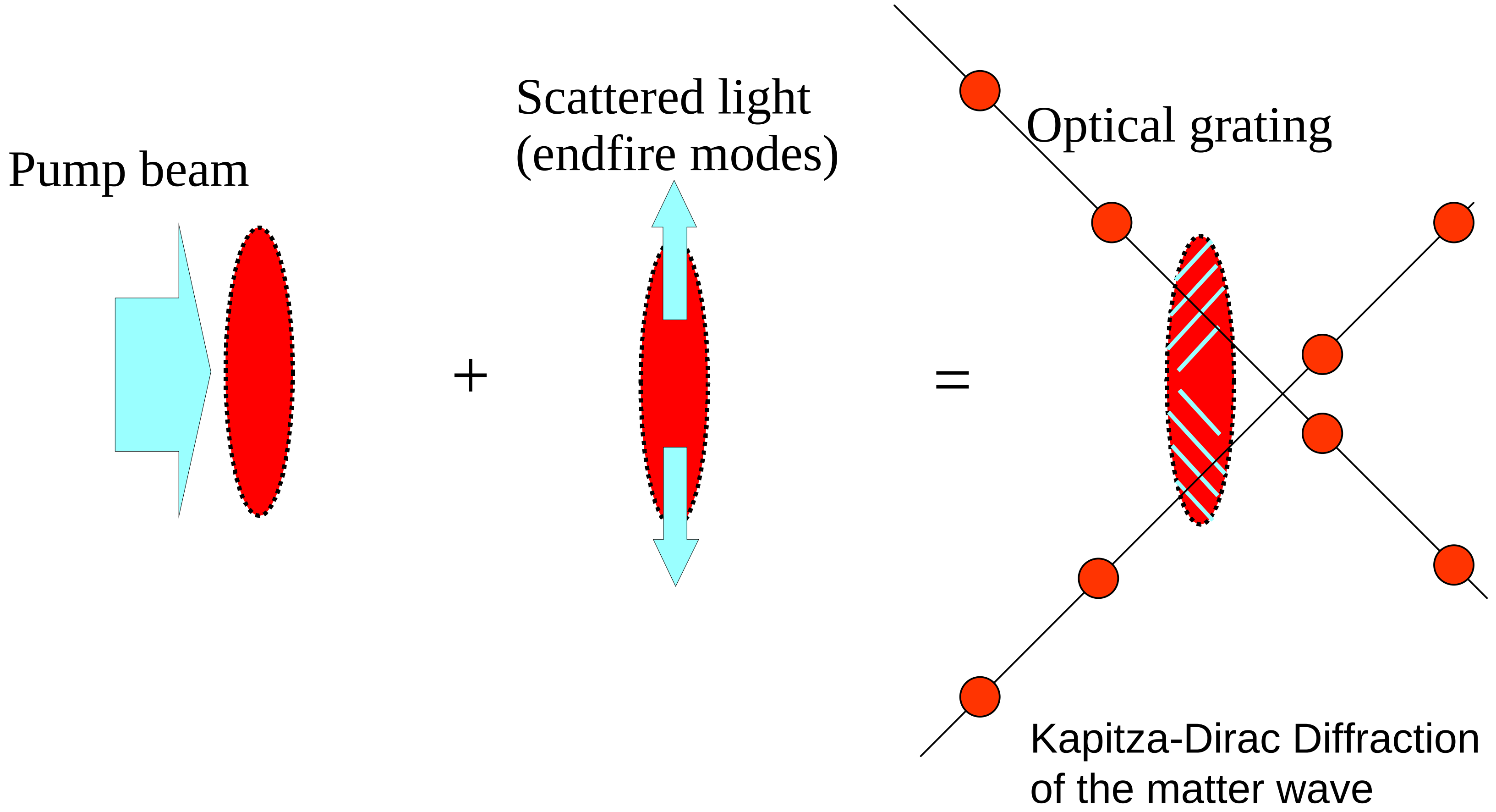
30 ms TOF

D. Schneble, Y.T., M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003)

Asymmetry of the X-shaped pattern

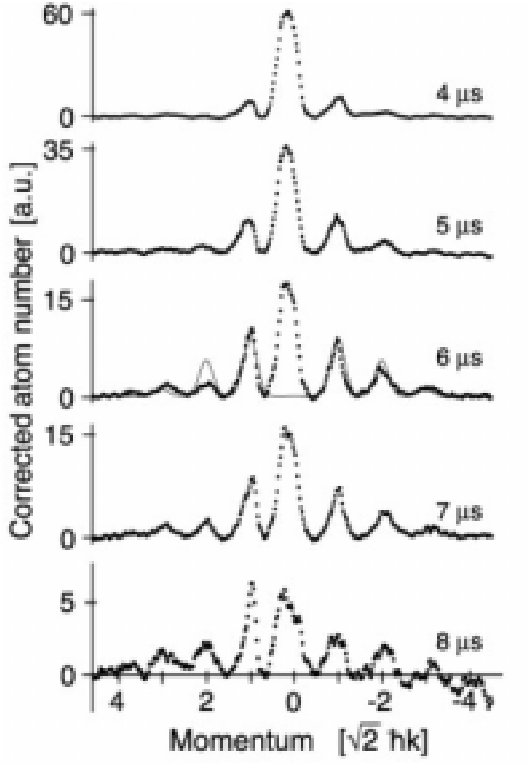


Explanation for the asymmetric X-shape

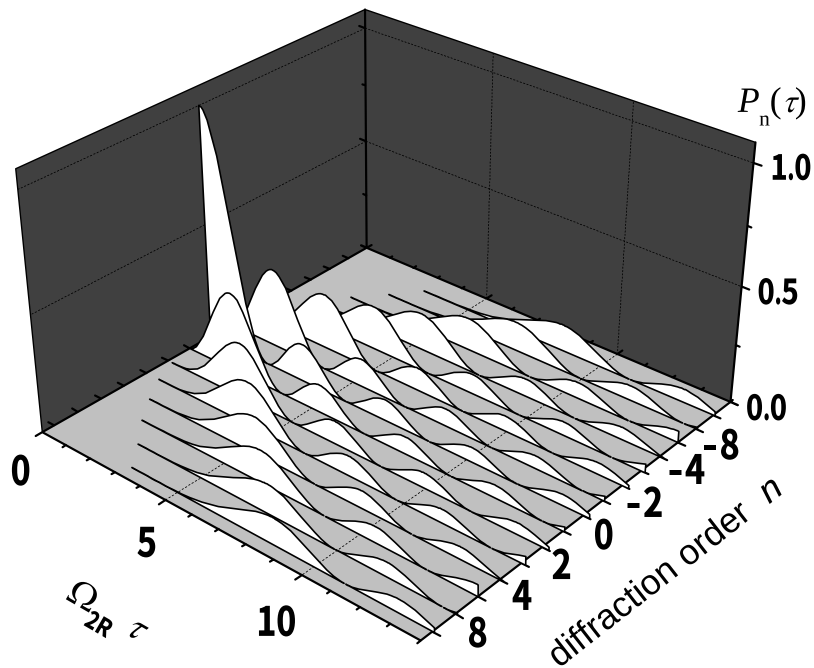


Intensity of the endfire mode

experiment



theory



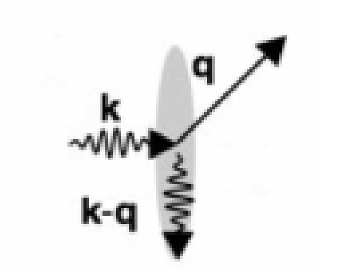
$$P_n(\tau) = J_n^2(\Omega_{2R}\tau),\ \Omega_{2R} = \frac{\Omega_p \Omega_e}{2\Delta}$$

Intensity of the Endfire mode $I_e = 2\frac{\Omega_e^2}{\Gamma^2} I_s = 0.8\ \mathrm{mW/cm^2}\left(I_s = 1.6\ \mathrm{mW/cm^2}\right)$

Scattering rate when the endfire photon n_{k-q}>>1

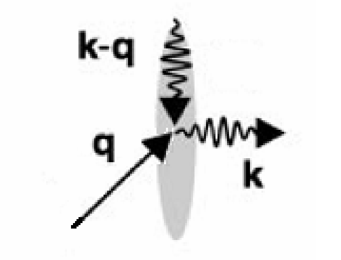
Scattering process:

$$|N_0, n_k; N_p, n_{k-q}> \to |N_0 - 1, n_k - 1; N_p + 1, n_{k-q} + 1>$$



Reverse scattering process:

$$|N_0, n_k; N_p, n_{k-q}> \to |N_0 + 1, n_k + 1; N_p - 1, n_{k-q} - 1>$$



Net scattering rate:

$$\begin{aligned} W &\propto N_0 n_k (N_q + 1)(n_{k-q} + 1) - N_q n_{k-q} (N_0 + \cancel{1})(n_k + \cancel{1}) \\ &= N_0 n_k (N_q + n_{k-q} + 1) \end{aligned}$$

Bosonic stimulation by the *sum* (not the product) of N_q and n_{k-q}

Bosonic stimulation by the recoiling atoms N_q or the endfire photon n_{k-q}?

$$W \propto N_0 n_0 \left(N_q + n_{k-q} + 1\right)$$

$n_{k-q} \ll 1$

$N_q \ll 1$

$$W \propto N_0 n_0 \left(N_q + 1\right)$$

$n_{k-q} = N_q$

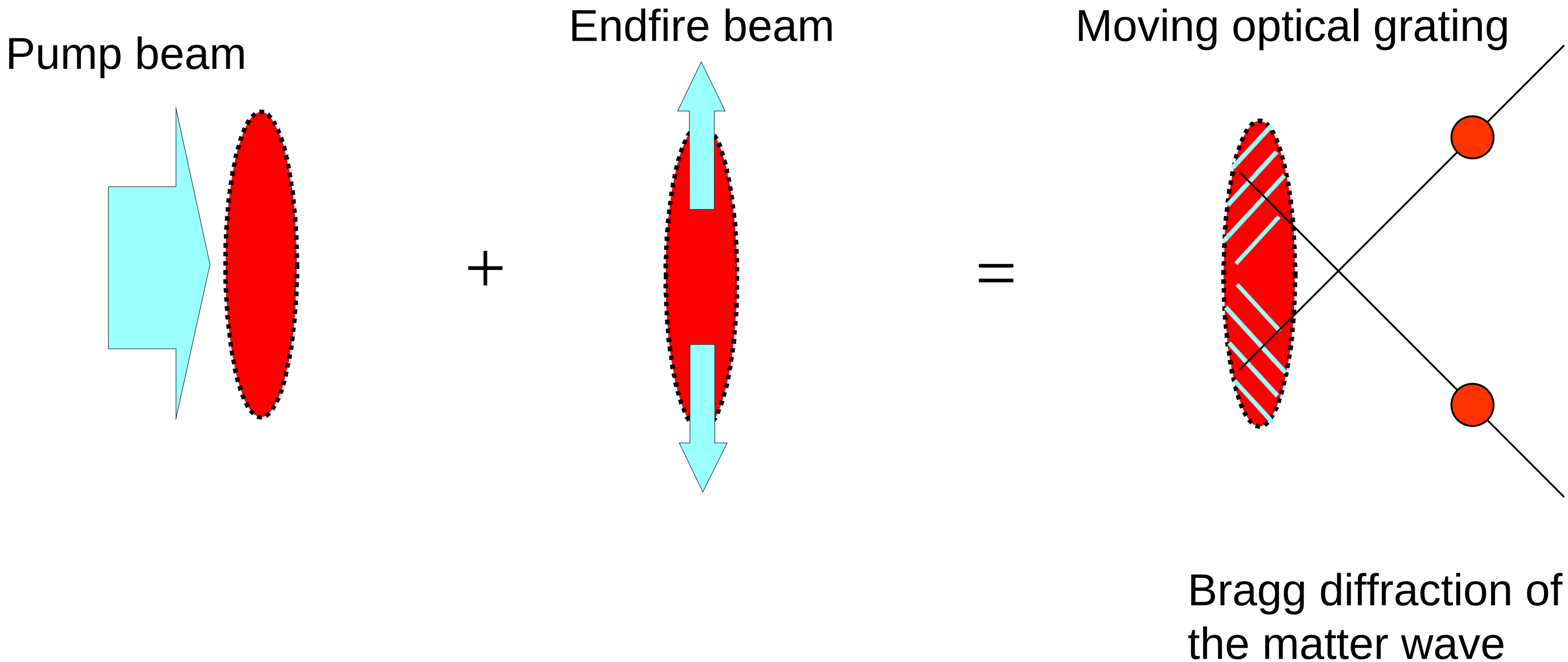
$$W \propto N_0 n_0 \left(n_{k-q} + 1\right)$$

Stimulation by N_q (atom)

Stimulation by n_{k-q} (photon)

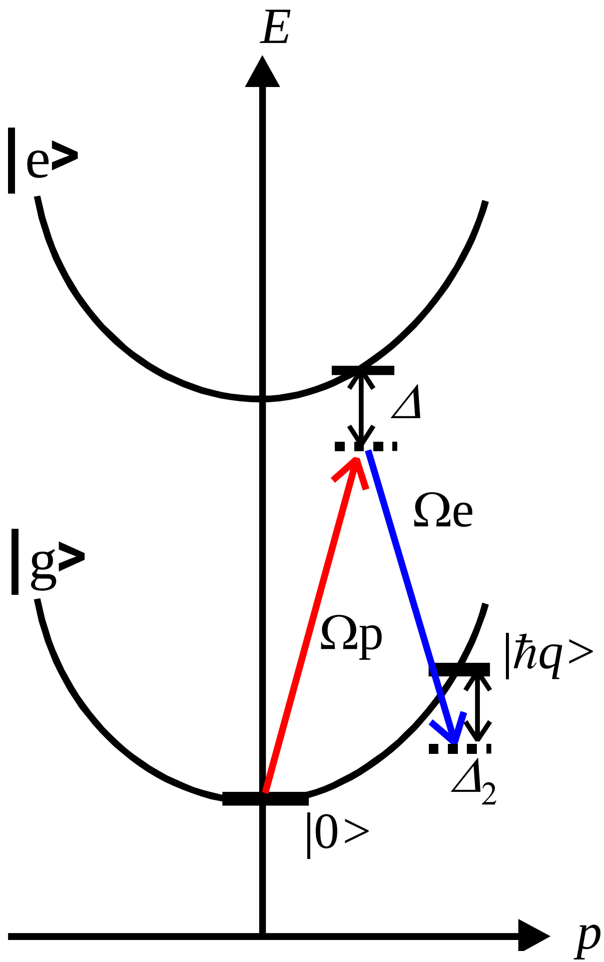
Both pictures would give the same scattering rate!

New interpretation of superradiance (in the long pulse regime)



Superradiant Rayleigh scattering regarded as (self-stimulated)
Bragg diffraction of a matter wave off a moving optical grating

Semi-classical derivation of the Bragg scattering rate



Fermi's Golden Rule

$$W = \frac{2\pi}{\hbar^2} |\hbar\Omega_{2R}/2|^2 \, \delta(\Delta_2)$$

Normalized Lorentzian $\dfrac{(\Gamma_2/2)/\pi}{\Delta_2^2 + (\Gamma_2/2)^2}$

$$\Gamma_2 \equiv 1/\tau_c$$

Width of the two-photon (Bragg) resonance

Coherence time of the condensate

At two-photon resonance (Δ_2=0)

$$W = N_0 \Omega_{2R}^2 / \Gamma_2 = N_0 \frac{\Omega_p^2 \Omega_e^2}{4\Delta^2} / \Gamma_2$$

How to express the rate W in terms of R and $n_{k\text{-}q}$?

$$W = N_0 \frac{\Omega_p^2 \Omega_e^2}{4\Delta^2} / \Gamma_2$$

Single-atom Rayleigh scattering rate:

$$R = \Gamma \rho_{ee} \cong \Gamma \cdot \frac{1}{2} s_0 \frac{1}{(2\Delta/\Gamma)^2} = \Gamma \frac{\Omega_p^2}{4\Delta^2} \qquad \left(s_0 \equiv \frac{2\Omega_p^2}{\Gamma^2} \right)$$

Saturation parameter of the pump beam

Intensity of the endfire mode:

$$I_e = I_s \frac{2\Omega_e^2}{\Gamma^2} \qquad \left(I_s \equiv \frac{\pi \hbar \omega \Gamma}{3\lambda^2} \right)$$

Saturation intensity
(I_s = 1.6 mW/cm^2 for Rb D_2 line)

Number of photons emitted in the coherence time $\tau_c = 1/\Gamma_2$:

$$n_{q-k} = \frac{I_e A \tau_c}{\hbar \omega} = \frac{2\pi A}{3\lambda^2} \frac{\Omega_e^2}{\Gamma \Gamma_2}$$

…continued

$$\Omega_p^2 = R\frac{4\Delta^2}{\Gamma} \qquad \Omega_e^2 = n_{k-q}\frac{3\lambda^2}{2\pi A}\Gamma\Gamma_2$$

$$W = N_0\frac{\Omega_p^2\Omega_e^2}{4\Delta^2}/\Gamma_2 = R\frac{3\lambda^2}{2\pi A}N_0 n_{k-q}$$

$$\Omega \approx \left(\frac{\lambda}{w}\right)^2 \approx \frac{\lambda^2}{A}$$

$$\boxed{W = R\frac{3}{2\pi}N_0 n_{k-q}\Omega} \approx \boxed{\dot{N}_j = R\frac{\sin^2\theta}{8\pi/3}N_0 N_q\Omega}$$

Semi-classical expression based on the matter wave grating

Four different pictures for superradinace in a BEC

- Semi-classical picture (Bragg diffraction of a pump beam off a matter wave grating)

- Full-quantum picture (Bosonically enhanced scattering by the recoiling atoms)

- Dicke’s picture (enhanced radiation from a symmetric cooparative state)

- self-stimulating Bragg diffraction of the matter wave off the optical grating

Analysis including propagation effects

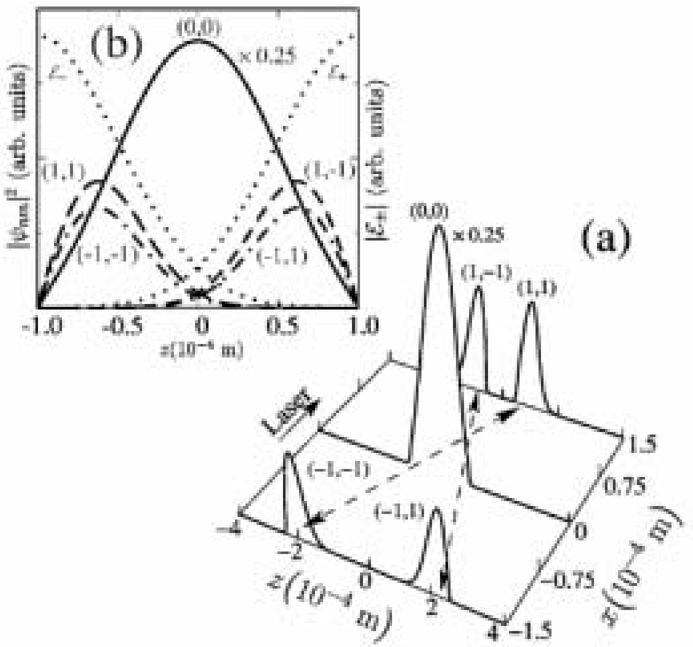


FIG. 1. Strong-pulse regime. (a) Spatial distribution of the first-order forward $(1, \pm 1)$ and backward $(-1, \pm 1)$ atomic side modes, after applying a laser pulse of duration $t_f = 14$ μs and strength $g = 2 \times 10^6$ s^{-1} to the condensate followed by a free propagation for a time $t_p = 25$ ms. (b) Spatial distributions of the atomic side modes and the optical endfire modes ($\mathcal{E}_\pm$), at time t_f. For the sake of illustration the BEC population (0, 0) has been divided by 4.

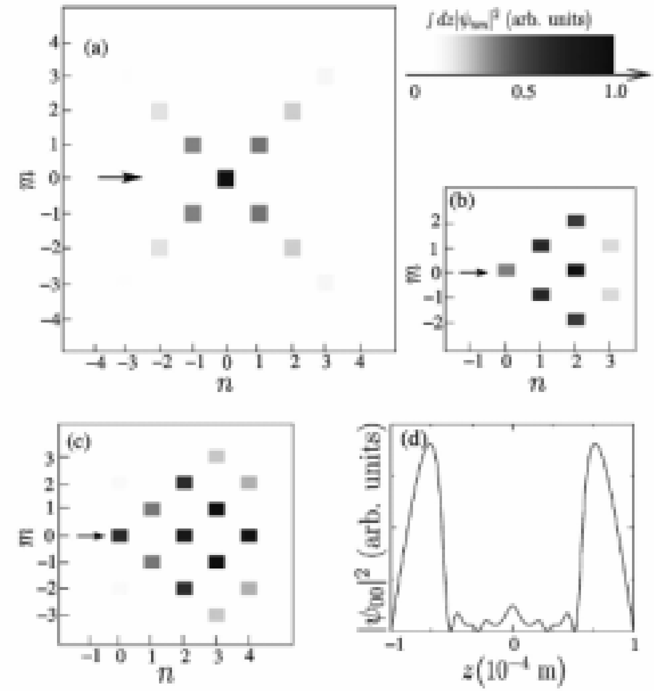
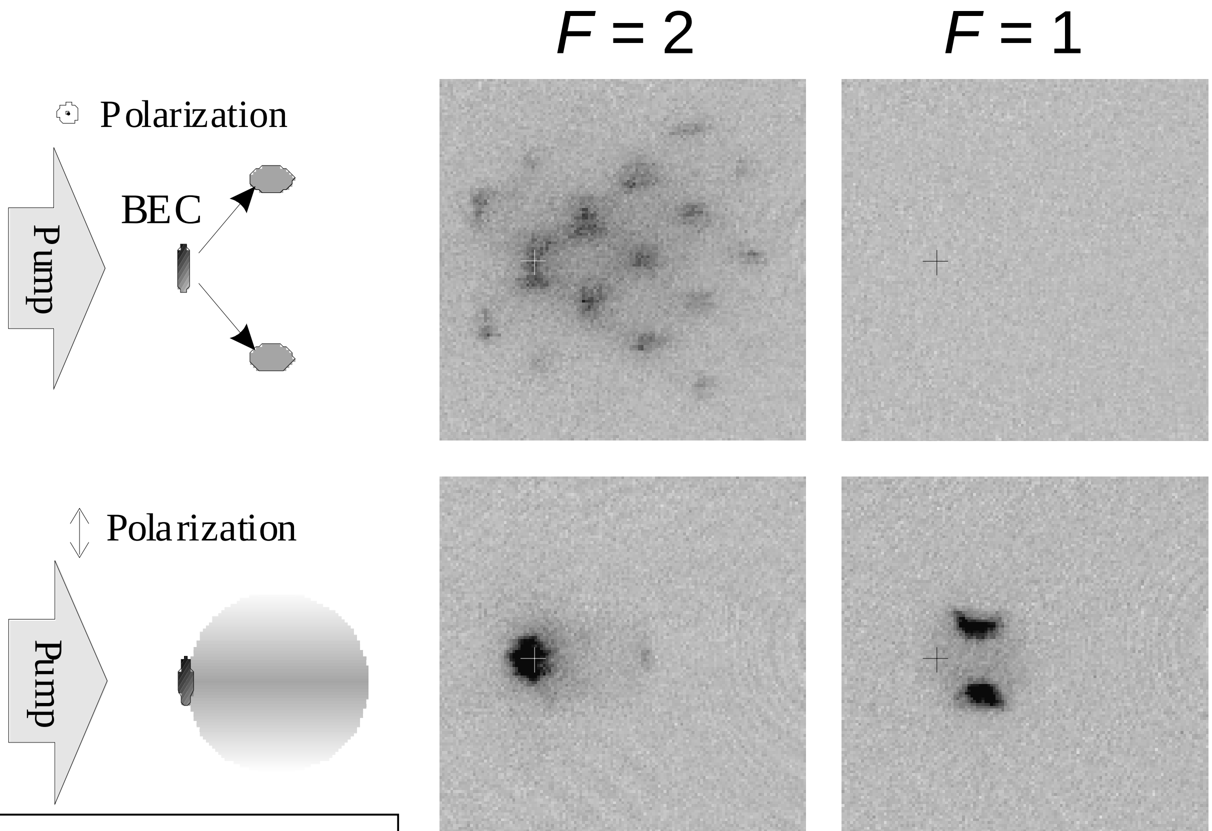


FIG. 2. Atomic side-mode distributions. Each square represents an integrated probability $p_{nm} = \int dz |\psi_{nm}(z,t)|^2$. (a) Strong-pulse regime: $t_f = 10.6$ μs and $g = 2.6 \times 10^6$ s^{-1}. (b) Weak-pulse regime: $t_f = 232$ μs and $g = 5.0 \times 10^5$ s^{-1}. (c) Weak-pulse regime: $t_f = 291$ μs and $g = 6.5 \times 10^5$ s^{-1}. (d) Spatial distribution of the condensate along the axis z corresponding to (c).

O. Zobay and Georgios M. Nikolopoulos, PRA **72**, 041604R (2005)

Changing the polarization of the pump beam



Detuning: -2.6 GHz
Intensity: 40 mW/cm^2
Pulse duration: 100 μs

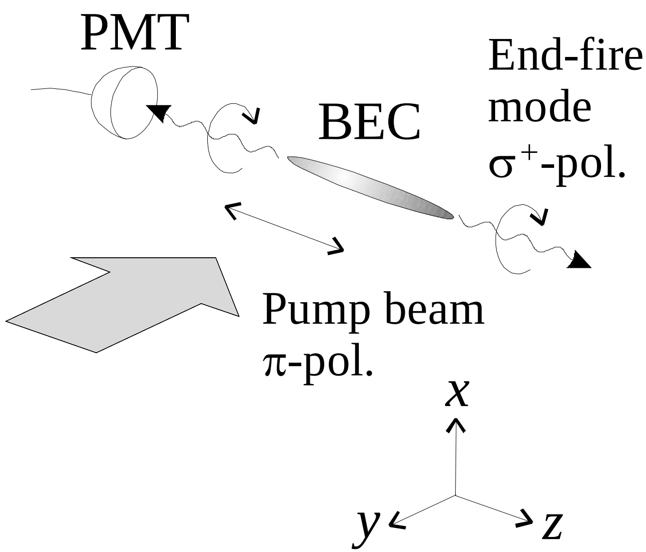
Y. Yoshikawa, *et al*., PRA **69** 041603 (2004)
D. Schneble, *et at*., PRA **69** 041601 (2004)

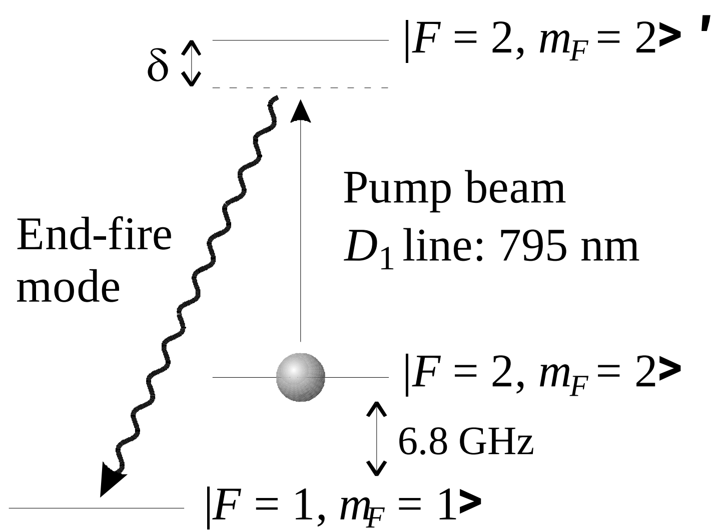
Raman superradiance

The only condition for Raman superradiance:

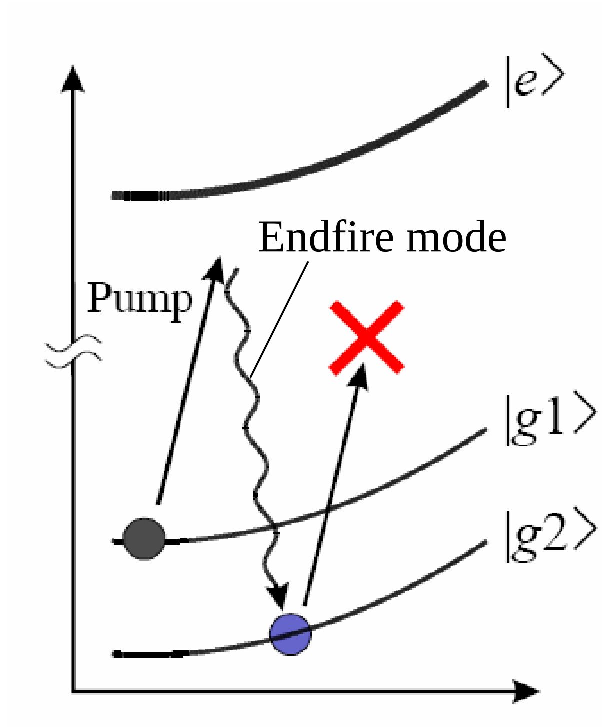
Raman scattering gain > Rayleigh scattering gain

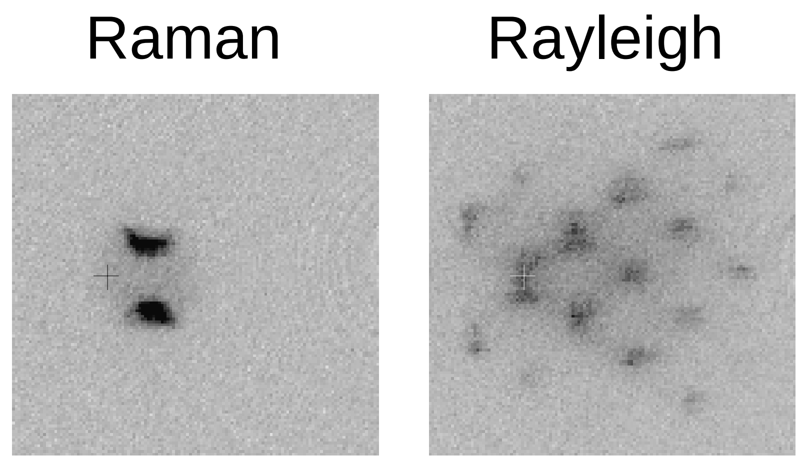
$$R_{\text{Raman}} \frac{3}{16\pi(1+\cos^2\theta)} > R_{\text{Rayleigh}} \frac{\sin^2\theta}{8\pi/3}$$





Merits of Raman superradiance over Rayleight superradiance

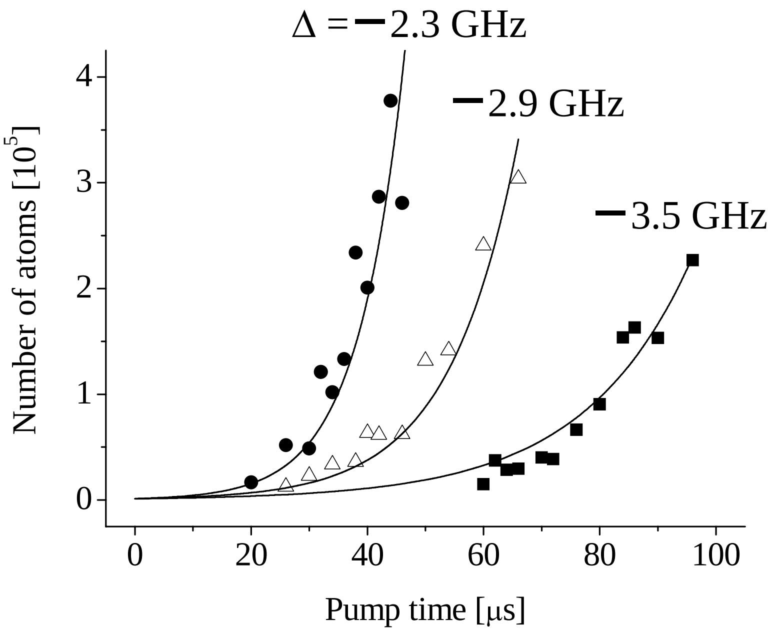


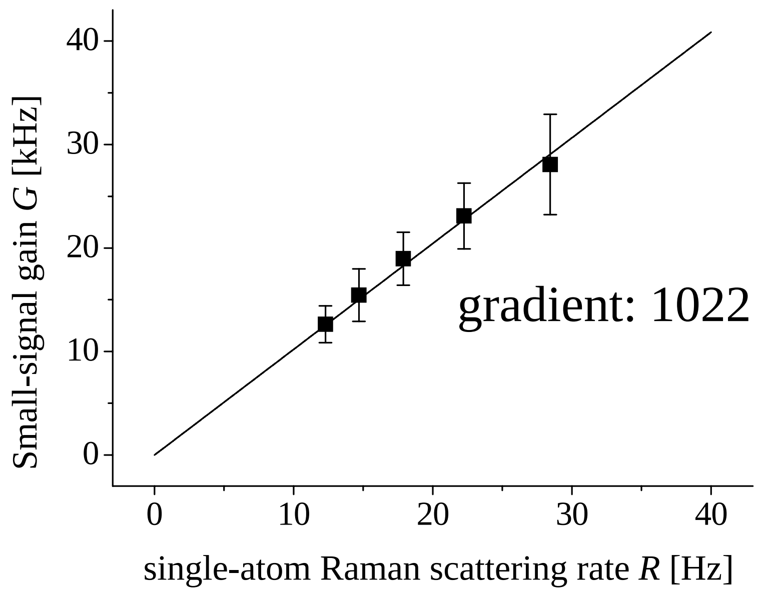


- No backward scattering (K-D scattering)

- No interaction with the pump bean once scattered

Exponential growth of the Raman scattered atoms





$$\dot{N}_q \approx \frac{3}{8\pi} R N_0 N_q \Omega \xrightarrow{N_0 << N_q} N_q \approx e^{Gt}$$

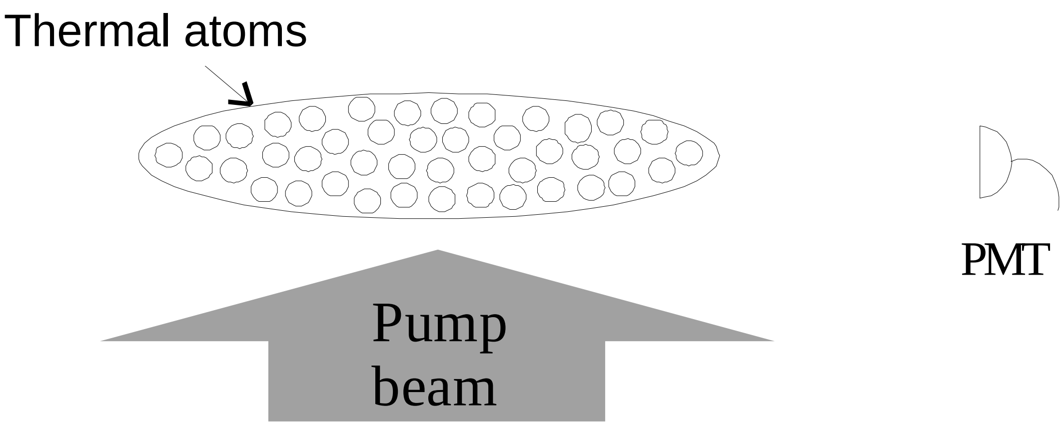
Small-signal gain:

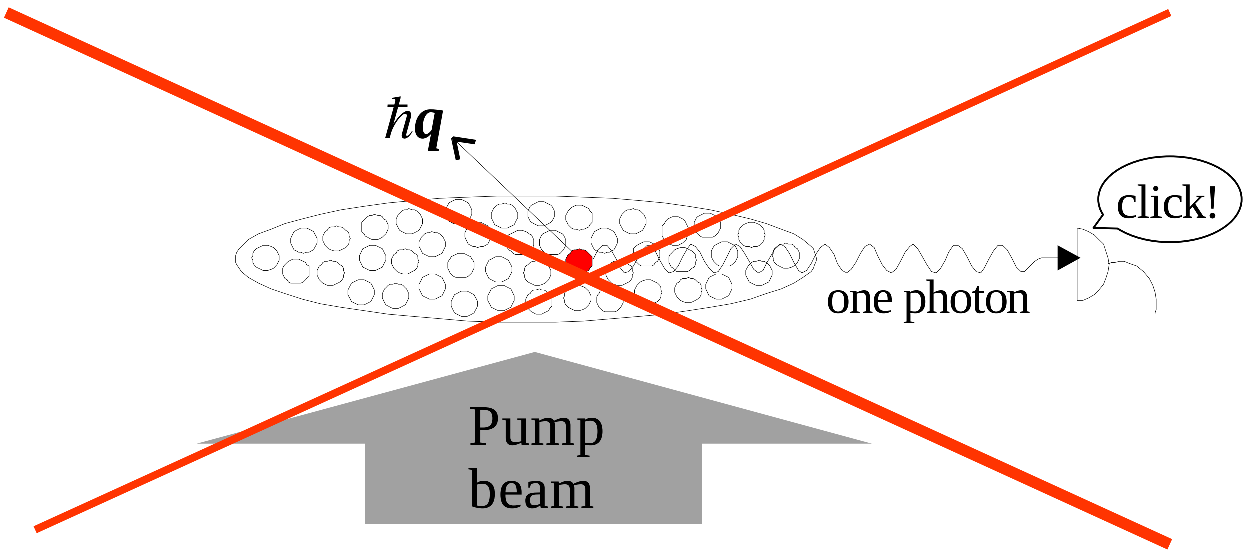
$$G = \frac{3}{8\pi} R N_0 \Omega = 890 R$$

R: single-atom Raman scattering rate

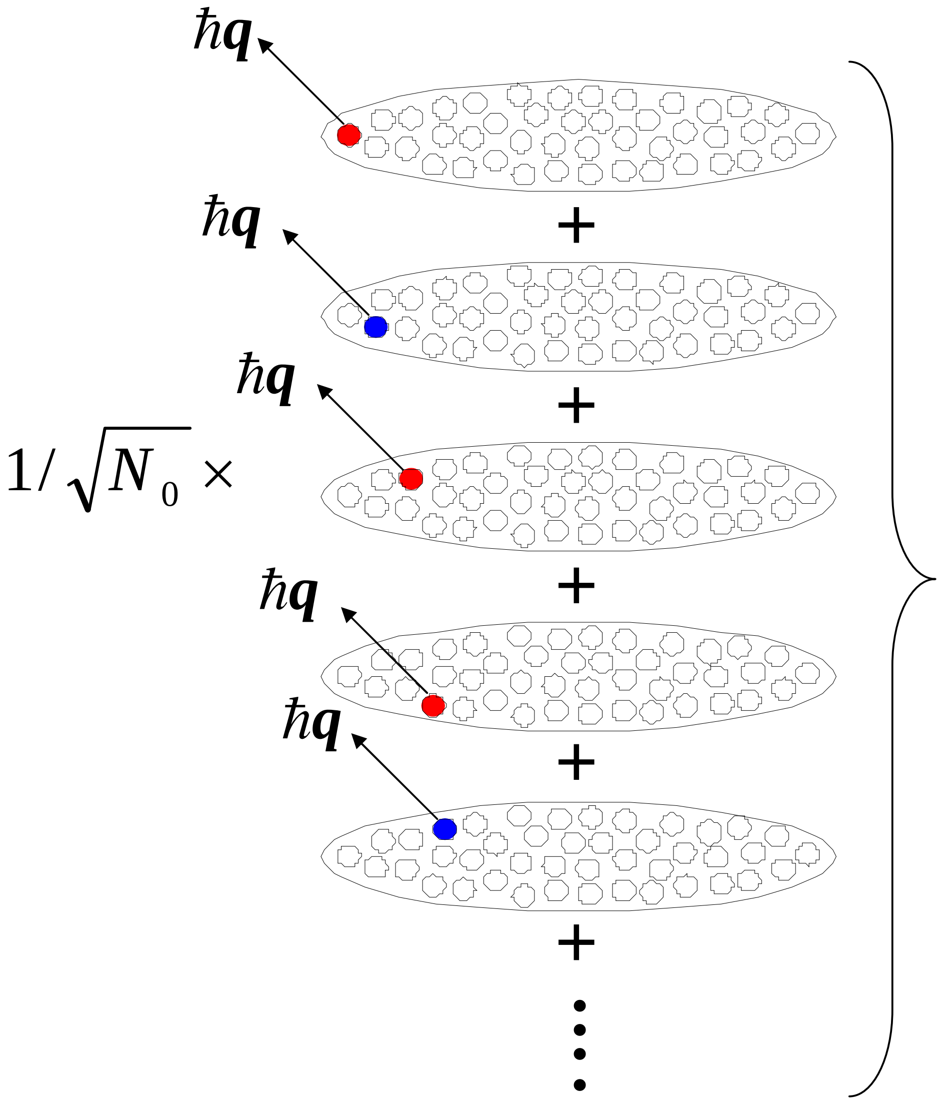
Y. Yoshikawa, *et al.*, PRA **69** 041603 (2004)

Where is a grating?

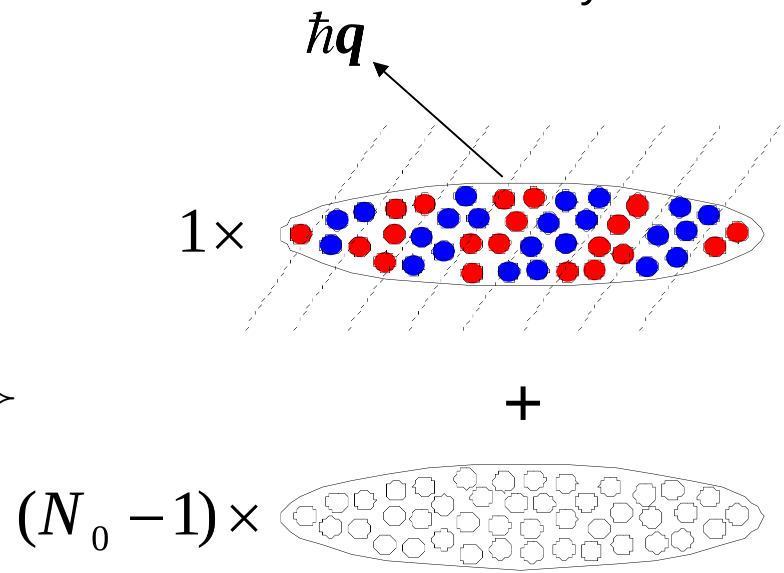




The origin of a grating (Collective mode excitation)



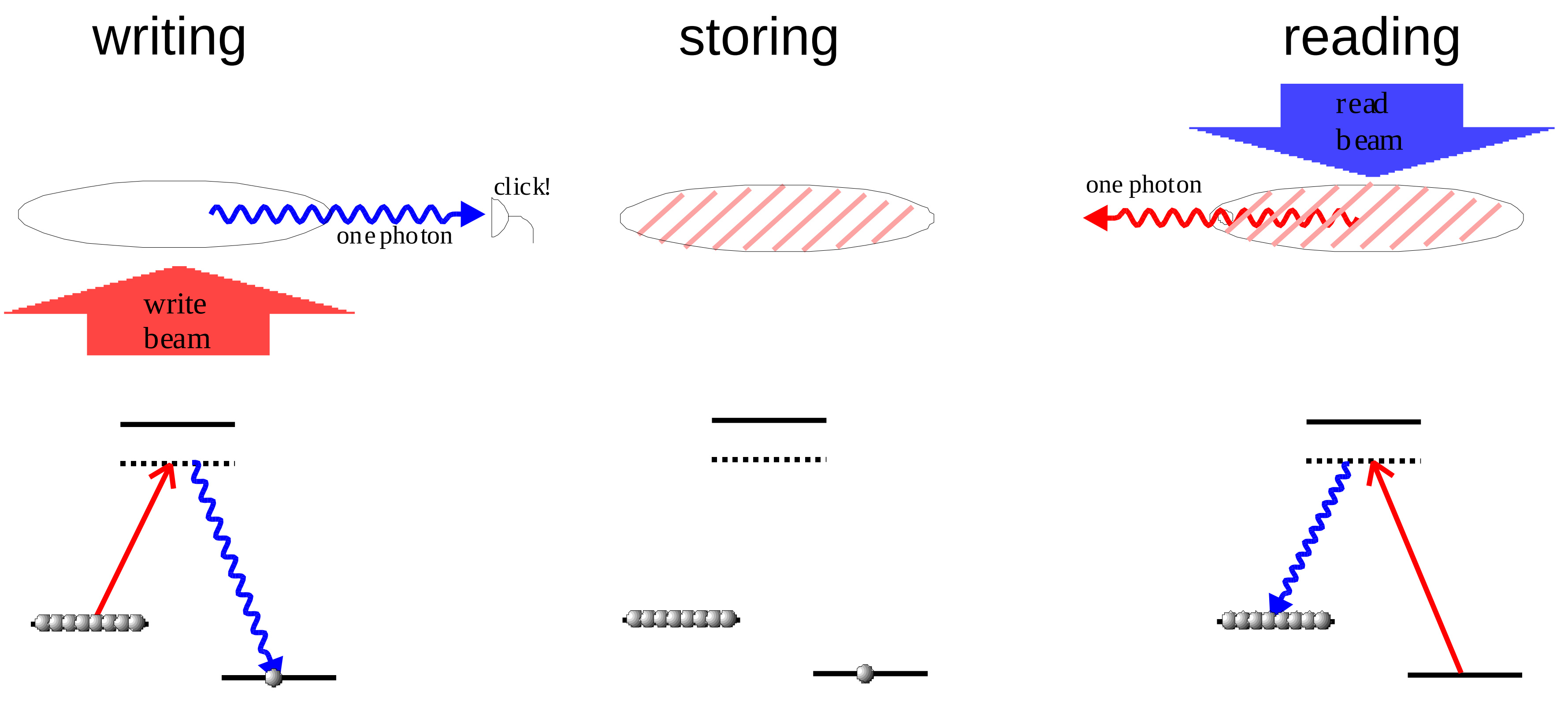
One atom is excited to the collective atomic mode defined by S^+



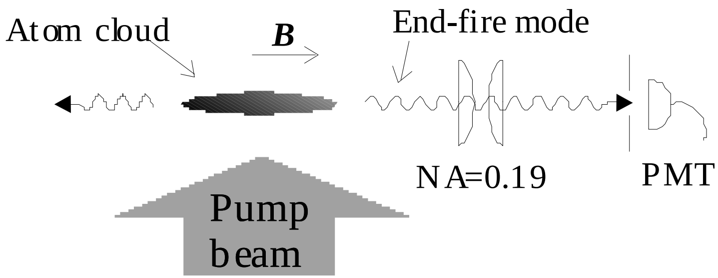
$$|J, M=-J+1> = S^+ |J, M=-J>$$

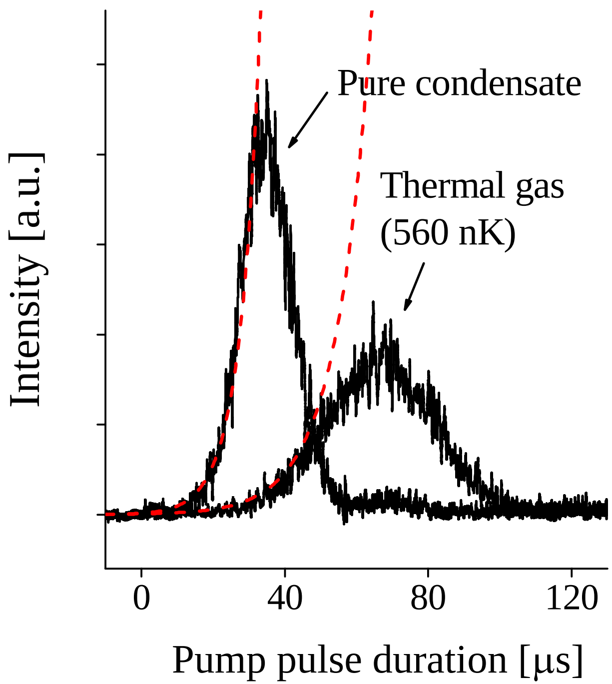
$$\left(S^+ \equiv \frac{1}{\sqrt{N_0}} \sum_{i=1}^{N_0} |\hbar q>_i < 0| \right)$$

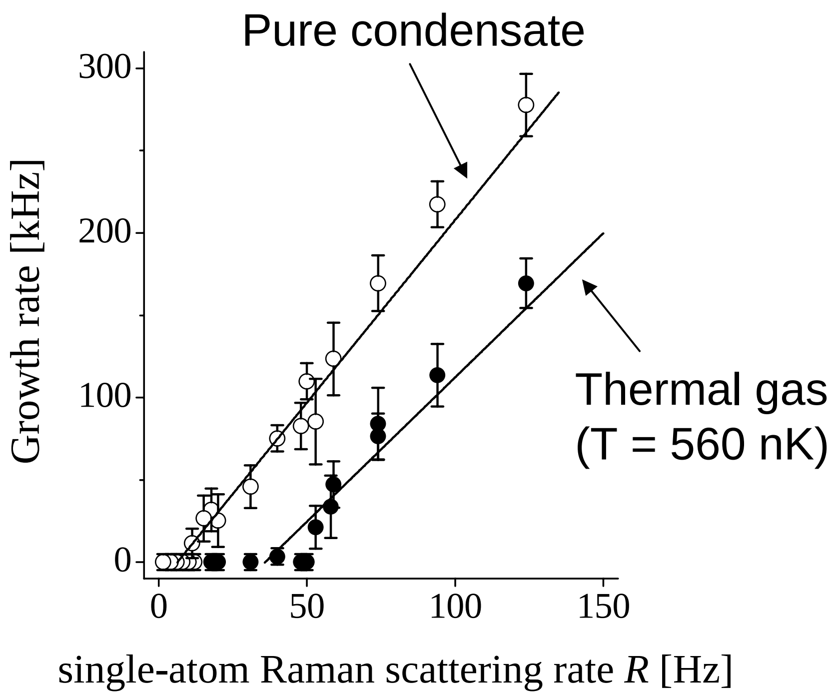
Writing, storing, and reading of a single photon



Superradiance in a Thermal gas







Y. Yoshikawa, Y. T. and T. Kuga, PRL **94** 083602 (2005)

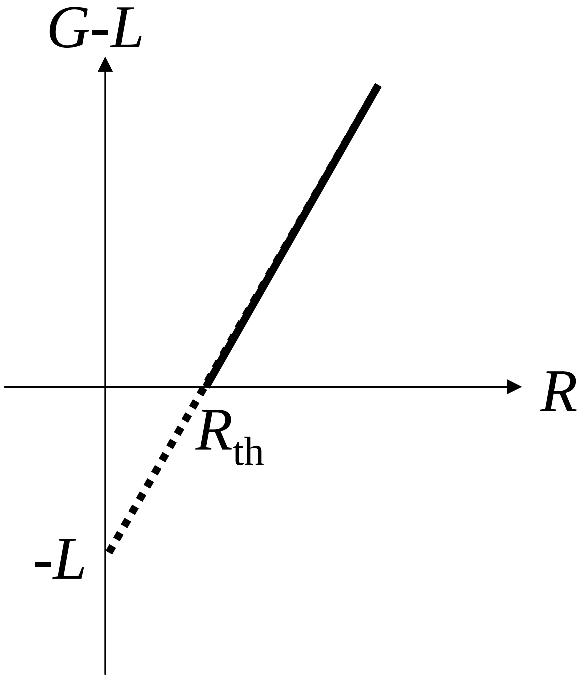
The origin of the threshold

Loss term

$$\dot{N}_q = (G - L)N_q \rightarrow N_q \propto e^{(G-L)t}$$

$L > G$: No exponential growth

$G > L$: exponential growth with a growth rate of *G-L*



$$L = 1/\tau_c$$

coherent time of the system

What determines the coherence time?

a) The endfire mode

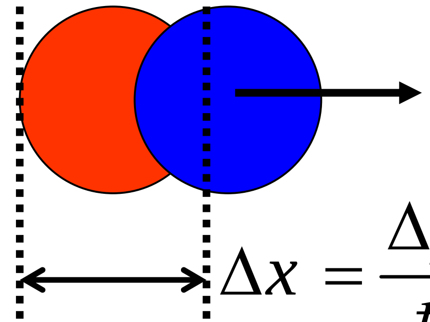
Doppler width:

$$\Delta\omega_{\mathrm{D}} = q\bar{v} \qquad \left(\bar{v} = \sqrt{\frac{k_{\mathrm{B}}T}{m}}\right)$$

RMS velocity

$$\tau_{\mathrm{c}} = \frac{1}{\Delta\omega_{\mathrm{D}}} = \frac{1}{q\bar{v}}$$

b) matter wave grating (overlap of the wave packets)



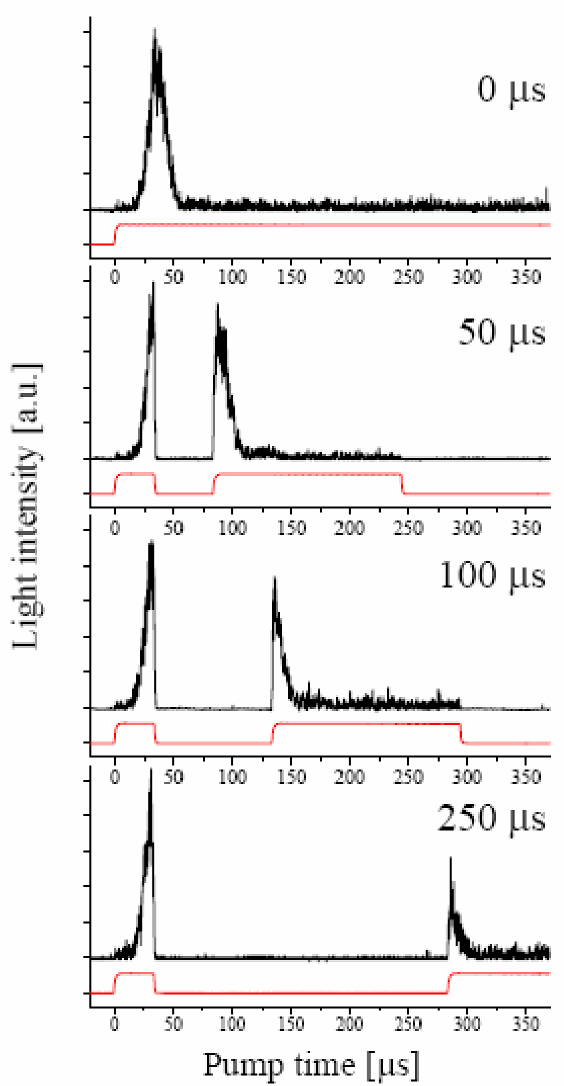
$$v = \frac{\hbar q}{m}$$

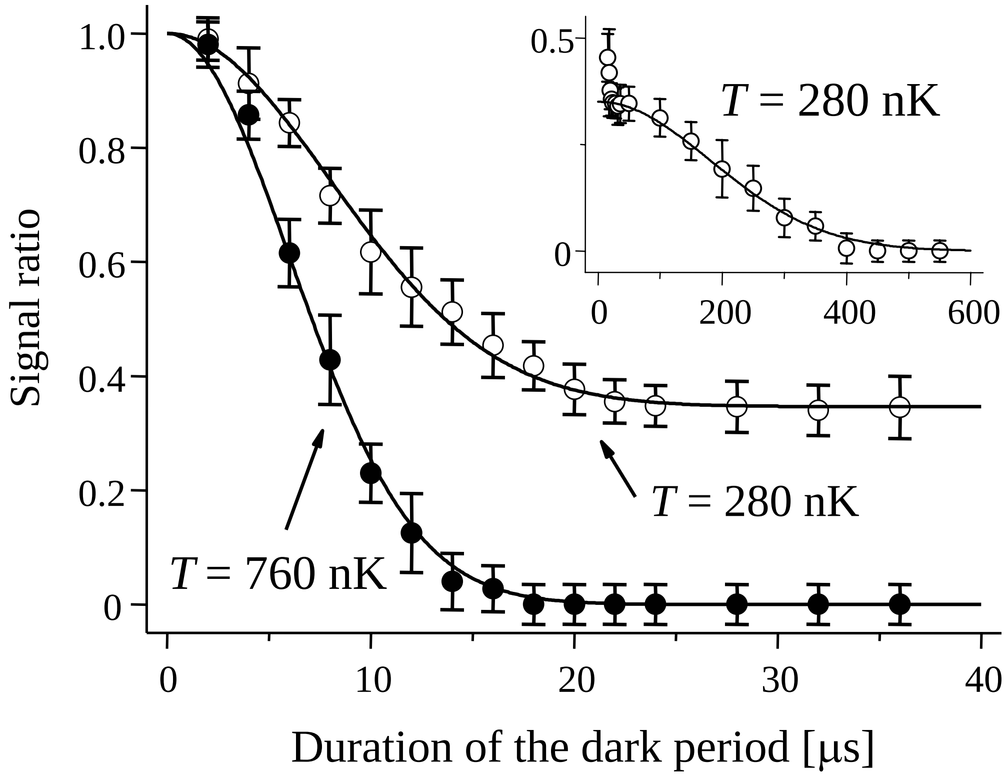
$$\Delta x = \frac{\Delta p}{\hbar} = \frac{m\bar{v}}{\hbar}$$

$$\tau_{\mathrm{c}} = \frac{\Delta x}{v} = \frac{1}{q\bar{v}}$$

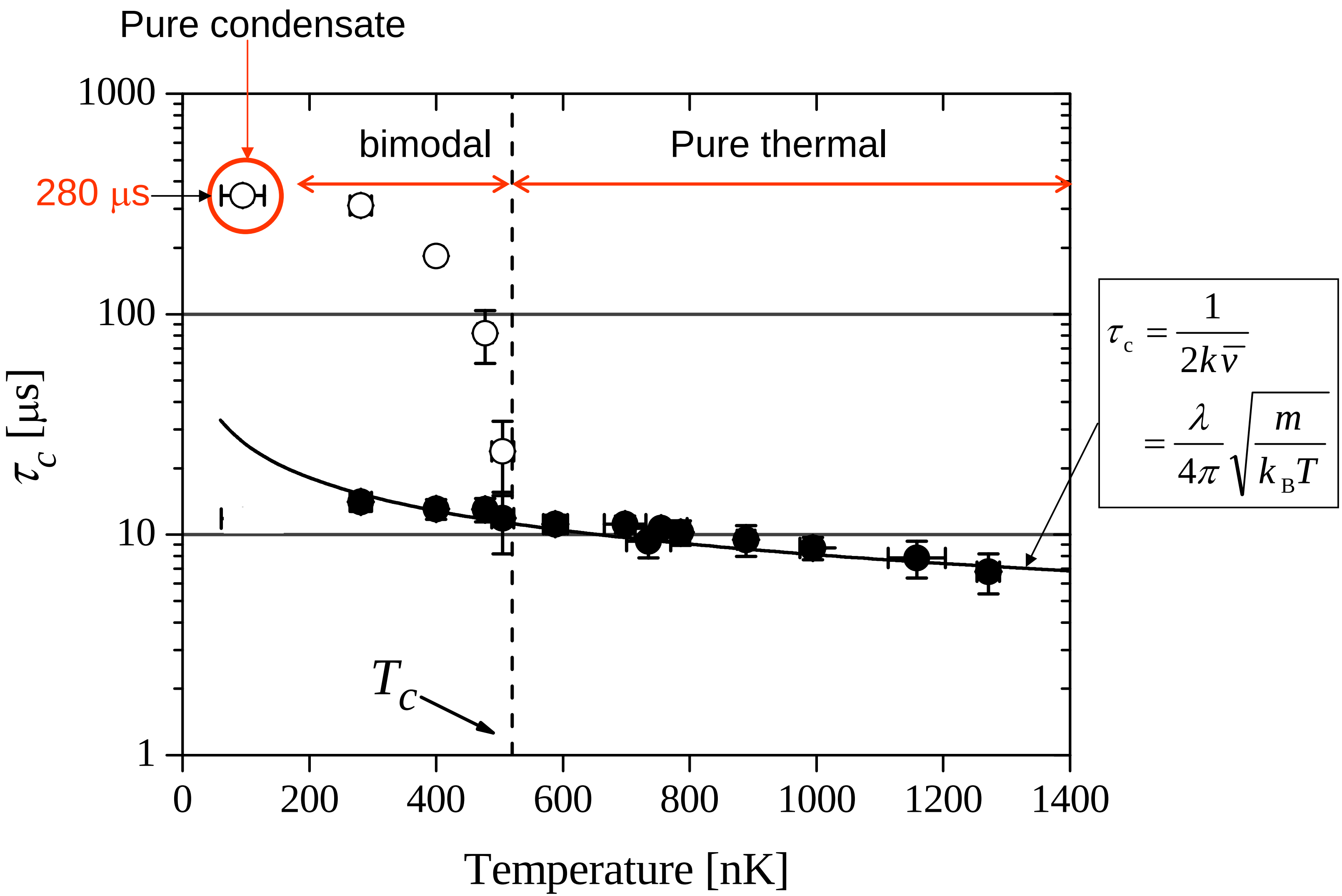
Coherence time is given by the inverse of the Doppler width

Measurement of the coherence time





Coherence time vs. temperature



Conclusion

- The behavior of superradiance in the short and strong pulse regime has led to a new picture of superradiance (optical stimulation)

- The study of superradiance in a thermal gas showed that a thermal gas will act as a pure condensate within a time scale shorter than the coherence time, which is determined by the Doppler effect.