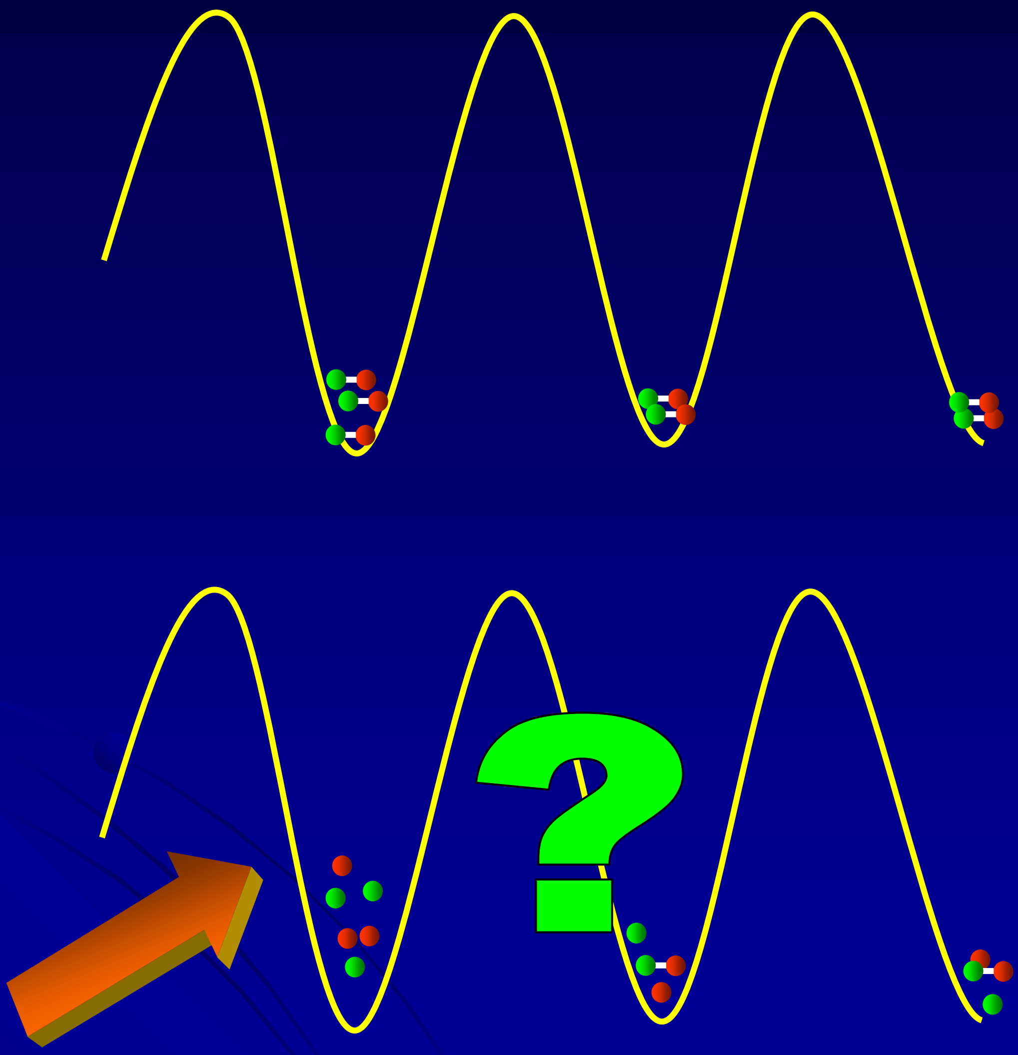
Dissociation dynamics of Bose-Fermi mixtures in lattices

Takahiko Miyakawa and PM
Dept. of Physics & College of Optical Sciences
The University of Arizona

ARO NASA NSF ONR

UAPhysics
THE UNIVERSITY OF ARIZONA
College of Science

The model



$t=0$:

Molecules only,
either MI or SF phase

$t>0$:

Photoassociation/dissociation
laser switched on

Collisions, tunneling, Mott insulator vs. superfluid, ...

Spin-Boson lattice Hamiltonian

$$H_{BF} = H_f + H_b + V_{bf}$$

- Fermionic atoms:

$$H_f = \hbar\omega_f \sum_{i,\sigma} n_{i\sigma}^f - \hbar J_f \sum_{\langle ij\rangle,\sigma} \left(f_{i\sigma}^\dagger f_{j\sigma} + H.c. \right) + \hbar U_f \sum_i n_{i\uparrow}^f n_{i\downarrow}^f$$

tunneling　　　　collisions

- Bosonic molecules:

$$H_b = \hbar\left(\omega_d + \omega_b\right) \sum_i n_i^b - \hbar J_b \sum_{\langle ij\rangle} \left(b_i^\dagger b_j + H.c. \right) + \frac{1}{2}\hbar U_b \sum_i n_i^b \left(n_i^b - 1 \right)$$

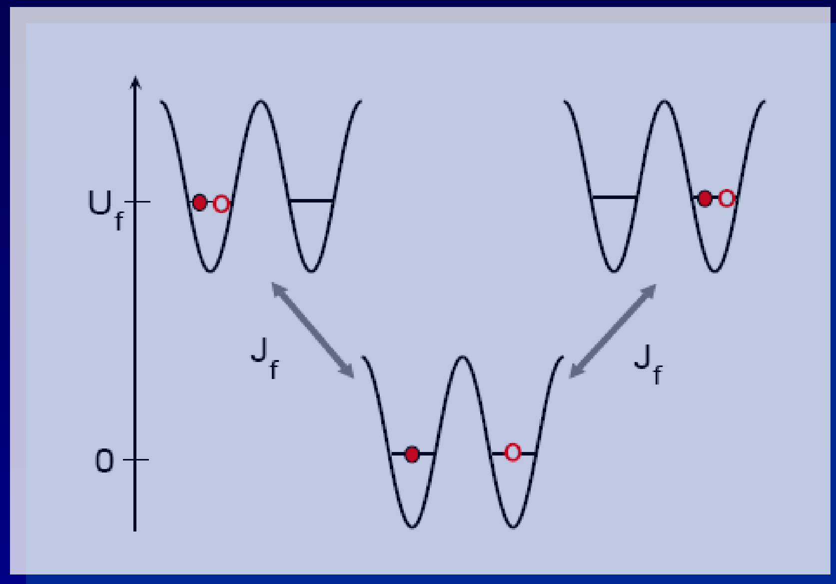
- Photoassociation:

$$V_{bf} = \hbar g \sum_i \left(f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger b_i + H.c. \right) + \hbar U_{bf} \sum_{i\sigma} n_{i\sigma}^f n_i^b$$

photoassociation

Fermions: Strongly confined regime

$$zJ_f \ll U_f$$



- Treat tunneling perturbatively
- But: fermionic pairs created at same site by photodissociation

$\rightarrow$ Eliminate unpaired states adiabatically

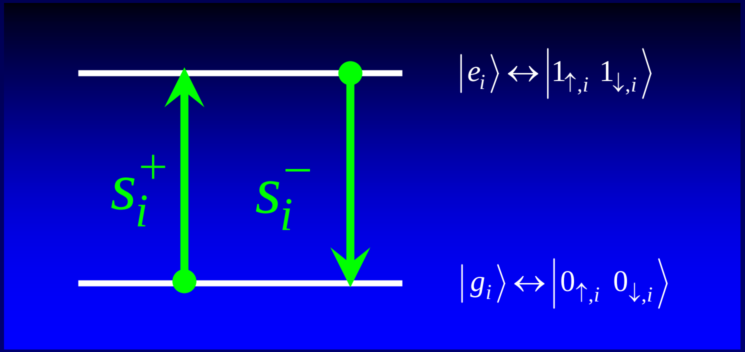
Spin-boson Hamiltonian

Pseudo-spin representation:

$$s_i^+ = f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger$$

$$s_i^- = f_{i\downarrow} f_{i\uparrow}$$

$$s_i^z = \frac{1}{2}\left(n_{i\uparrow}^f + n_{i\downarrow}^f - 1\right)$$



$$H_f \to H_s = \hbar\omega_s \sum_i \left(2s_i^z + 1\right) + \hbar J_s \sum_{\langle ij\rangle} \left(s_i^x s_j^x + s_i^y s_j^y - s_i^z s_j^z\right)$$

$$V_{bf} \to V_{bs} = \hbar U_{bf} \sum_i \left(2s_i^z + 1\right) + \hbar g \sum_i \left(b_i^\dagger s_i^- + H.c.\right)$$

$$\omega_s = \omega_f + U_f / 2$$

Spin-boson Hamiltonian!

$$J_s = 4J_f^2 / U_f$$

Gutzwiller ansatz

$$\left|\Psi(t)\right\rangle = \prod_{i=1}^{N_s}\left(\sum_{n_i=0}^{\infty}\sum_{\sigma_i=-1/2}^{1/2} f_{n_i,\sigma_i}^{(i)}(t)\left|n_i,\sigma_i\right\rangle\right)$$

$$\sum_{n_i,\sigma_i}\left|f_{n_i,\sigma_i}^{(i)}(t)\right|^2 = 1$$

σ_i Fermi pair occupation of site *i*

n_i Bosonic occupation of site *i*

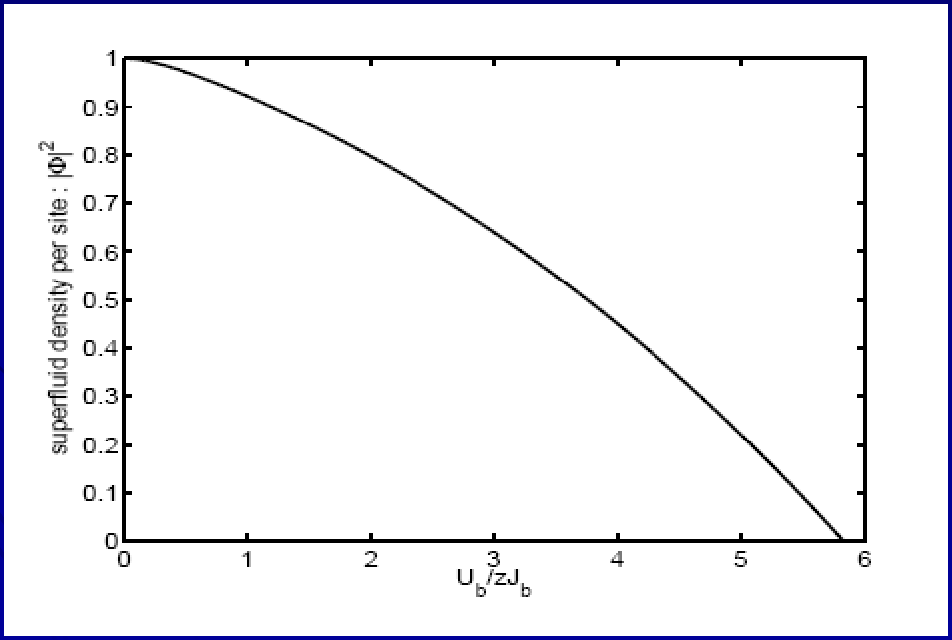
- Time-dependent variational principle:

$$\frac{\partial}{\partial f_{n_i,\sigma_i}^{(i)*}}\left\langle\Psi(t)\left|i\hbar\frac{\partial}{\partial t}-H_{SB}\right|\Psi(t)\right\rangle = 0$$

Initial molecular ground state (No photoassociation)

- Superfluid: $U_b / zJ_b \ll 1$ $\qquad f_{n\downarrow} = e^{-\nu/2} \frac{\left(\sqrt{\nu}\right)^n}{\sqrt{n!}}$

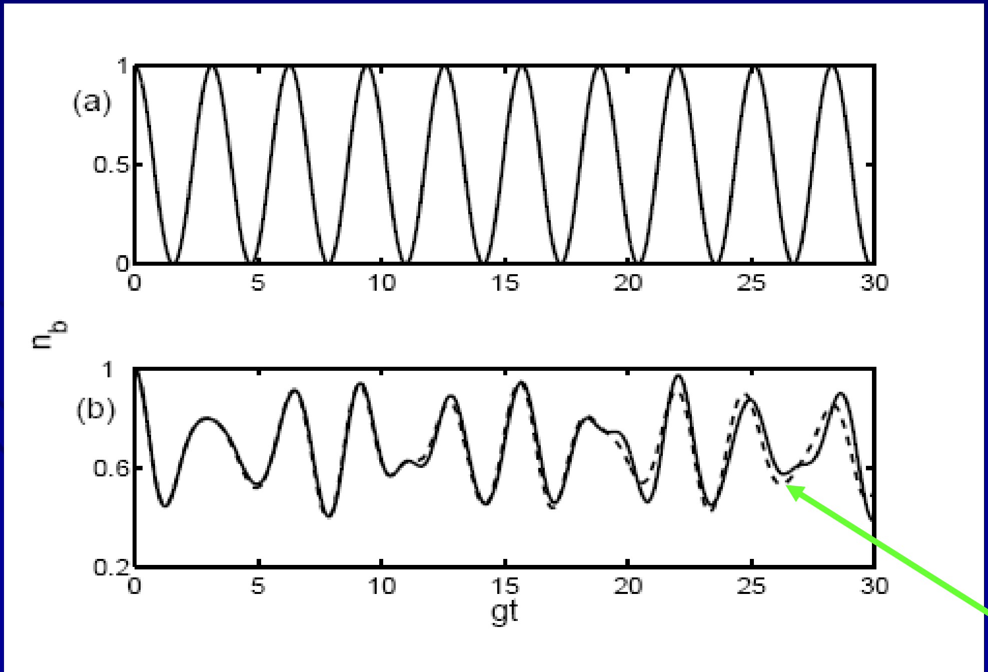
- Mott insulator: $U_b / zJ_b \gg 1$ $\qquad f_{n,\downarrow} = \delta_{n,\nu}$



Generalized Jaynes-Cummings dynamics

$$J_b, J_s \to 0$$

$$H = \hbar\left(\omega_d + \omega_b\right)\sum_i n_i^b + \sum_i \hbar\omega_s\left(2s_{fi}^z + 1\right) + \hbar g\sum_i\left(b_i^\dagger s_{fi}^- + b_i s_{fi}^+\right) + \frac{1}{2}\hbar U_b\sum_i n_i^b\left(n_i^b - 1\right)$$



MI phase

superfluid

Gutzwiller dynamics

$zJ_b / g = .1$

$\delta = 0$ $U_{bf} = 0$

Generalized Tavis-Cummings dynamics

$$zJ_b \gg g$$

$$U_b / zJ_b \lesssim 1$$

- Extreme superfluid regime
- Single-mode approximation for molecular field

$$H \rightarrow \hbar\delta_0 n_0 + \frac{\hbar g}{\sqrt{N_s}}\left[b_0 S_f^- + b_0^\dagger S_f^+\right] + \frac{\hbar U_0}{2N_s} n_0^2$$

$S_f^+ = \sum_i s_{fi}^+$ Collective spin operator

$$\delta_0 = \delta - zJ_b - 2\nu U_{bf}$$

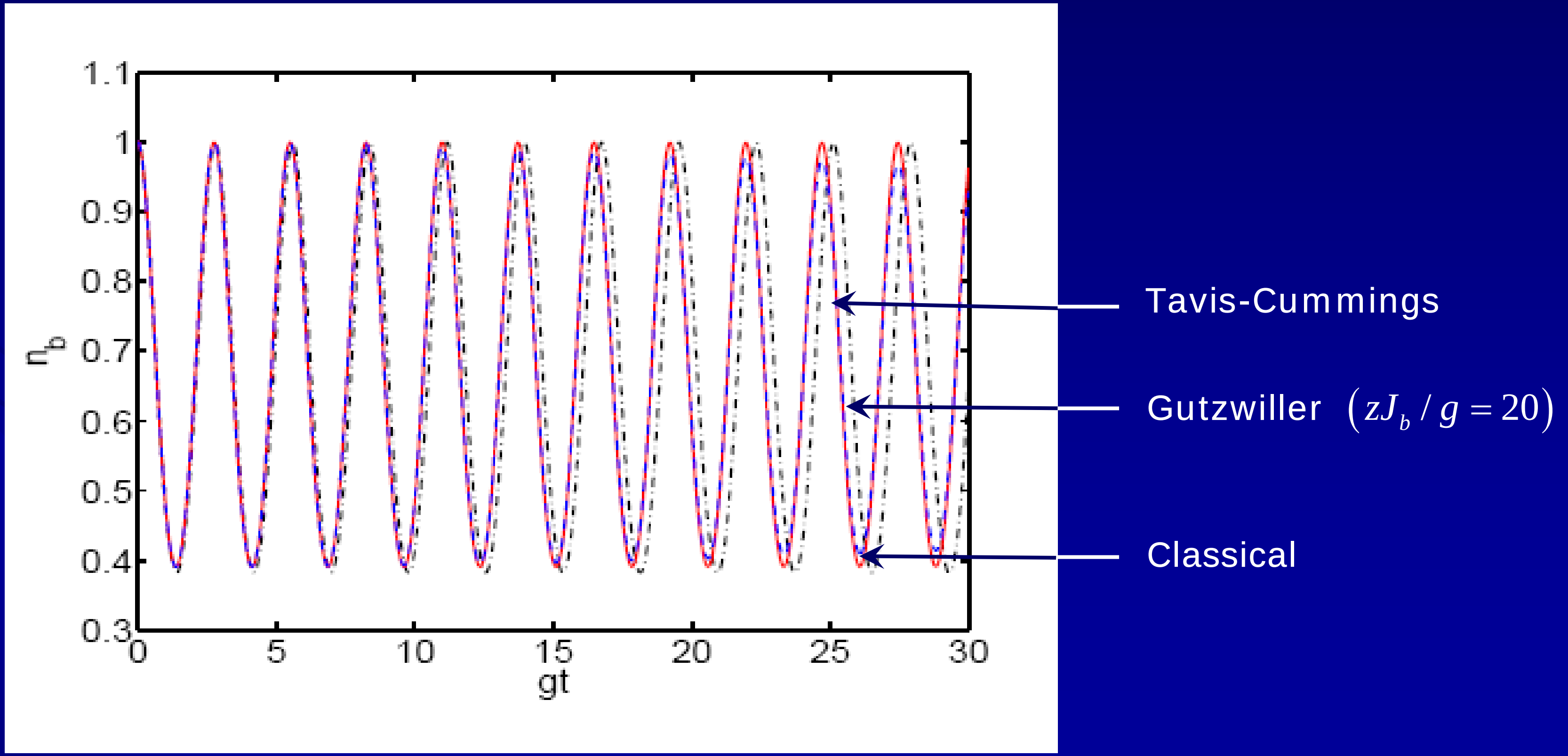
$$U_0 = U_b - 4U_{bf}$$

→ Collective oscillations between atoms and molecules

Classical limit

$$n_0 \approx \langle n_0 \rangle \;\rightarrow\; \boxed{\frac{d^2}{dt^2}\langle n_0 \rangle = -\frac{d}{d\langle n_0 \rangle} V\left(\langle n_0 \rangle\right)}$$

$$V\left(\langle n_0 \rangle\right) = \frac{U_0^2}{8N_s^2}\langle n_0 \rangle^4 + (\cdots)\langle n_0 \rangle^3 + (\cdots)\langle n_0 \rangle^2 + (\cdots)\langle n_0 \rangle \quad ; \qquad U_0 = U_b - 4U_{bf}$$



Gutzwiller mean-field dynamics

$$\left|\Psi(t)\right\rangle = \prod_{i=1}^{N_s}\left(\sum_{n_i=0}^{\infty}\sum_{\sigma_i=-1/2}^{1/2} f^{(i)}_{n_i,\sigma_i}(t)\left|n_i,\sigma_i\right\rangle\right)$$

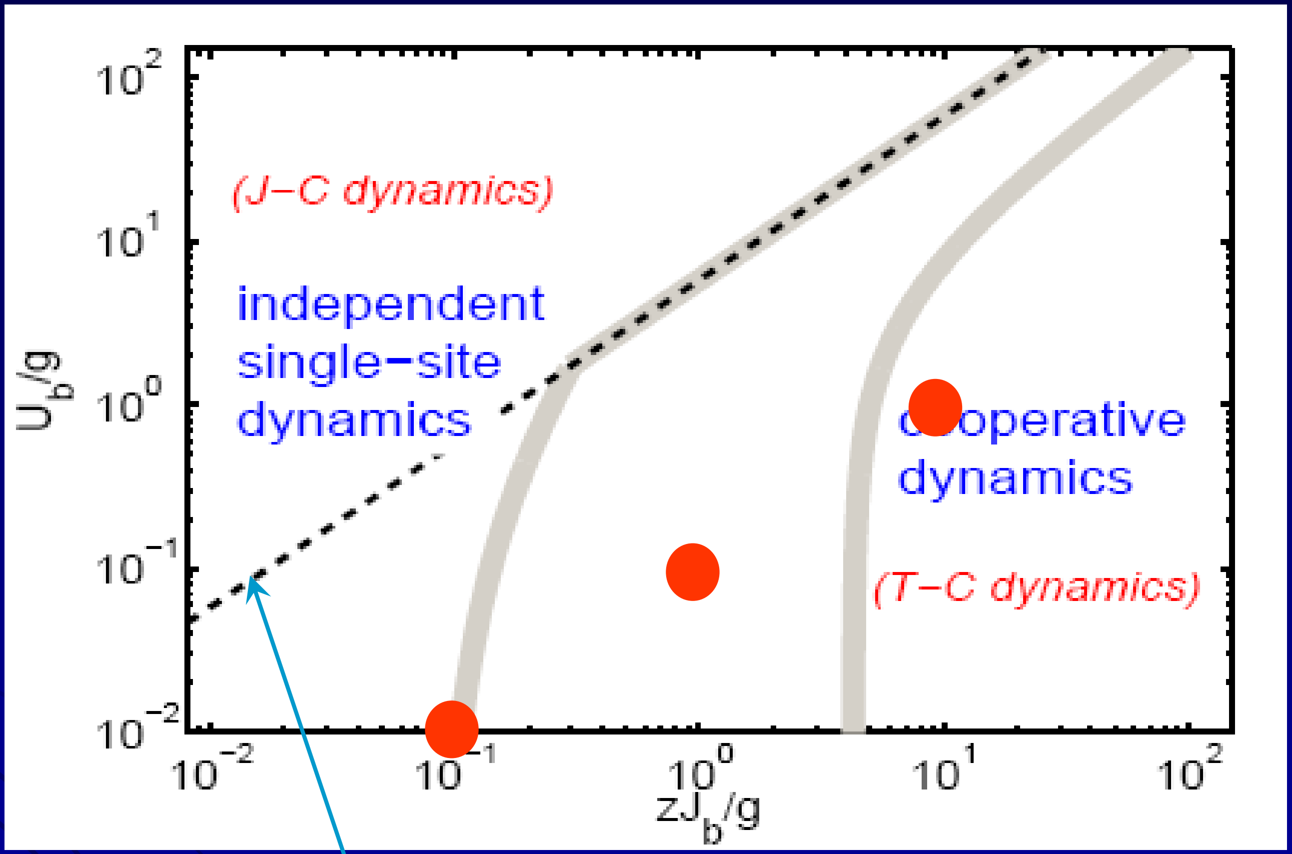
$$\begin{aligned}
i\frac{\partial f_{n,\sigma}}{\partial t} &= h_{n,\sigma} f_{n,\sigma} + g\left[\sqrt{n}\, f_{n-1,\sigma+1} + \sqrt{n+1}\, f_{n+1,\sigma-1}\right] \\
&\quad - zJ_b\left[\Phi^*\sqrt{n+1}\, f_{n+1,\sigma} + \Phi\sqrt{n}\, f_{n-1,\sigma}\right] \\
&\quad + (z/2)\, J_s\left[\Delta^* f_{n,\sigma+1} + \Delta f_{n,\sigma-1} - 2\sigma M f_{n,\sigma}\right]
\end{aligned}$$

$\Phi(t) = \left\langle b(t)\right\rangle$ bosonic order parameter

$\Delta(t) = \left\langle s_f^-(t)\right\rangle$ fermionic pairs order parameter

$M(t) = \left\langle s_f^z(t)\right\rangle$ pseudo-spin magnetization

Gutzwiller dynamics $\left(J_s = 0\right)$



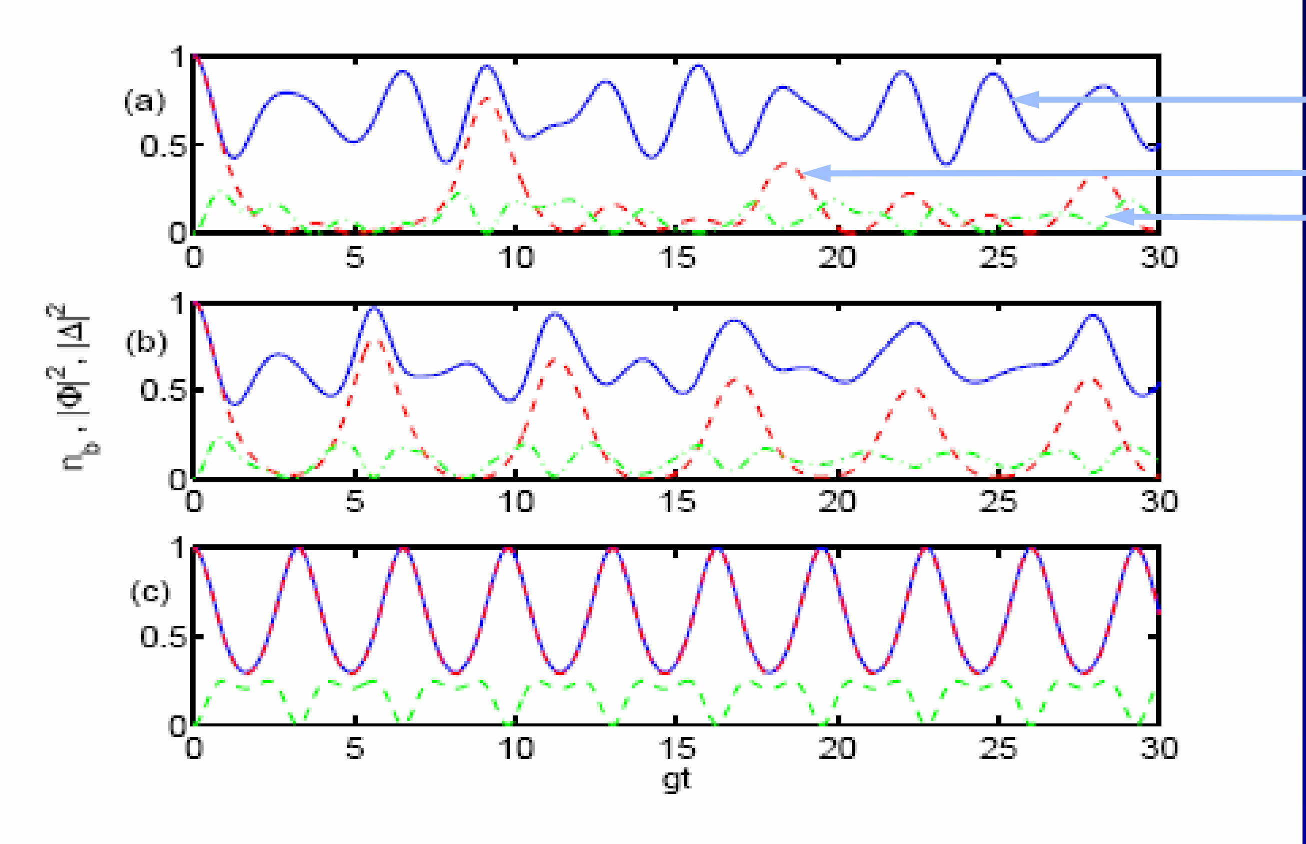
Separation between superfluid and MI initial conditions

Gutzwiller dynamics

$zJ_b / g = 0.1$

$zJ_b / g = 1$

$zJ_b / g = 10$



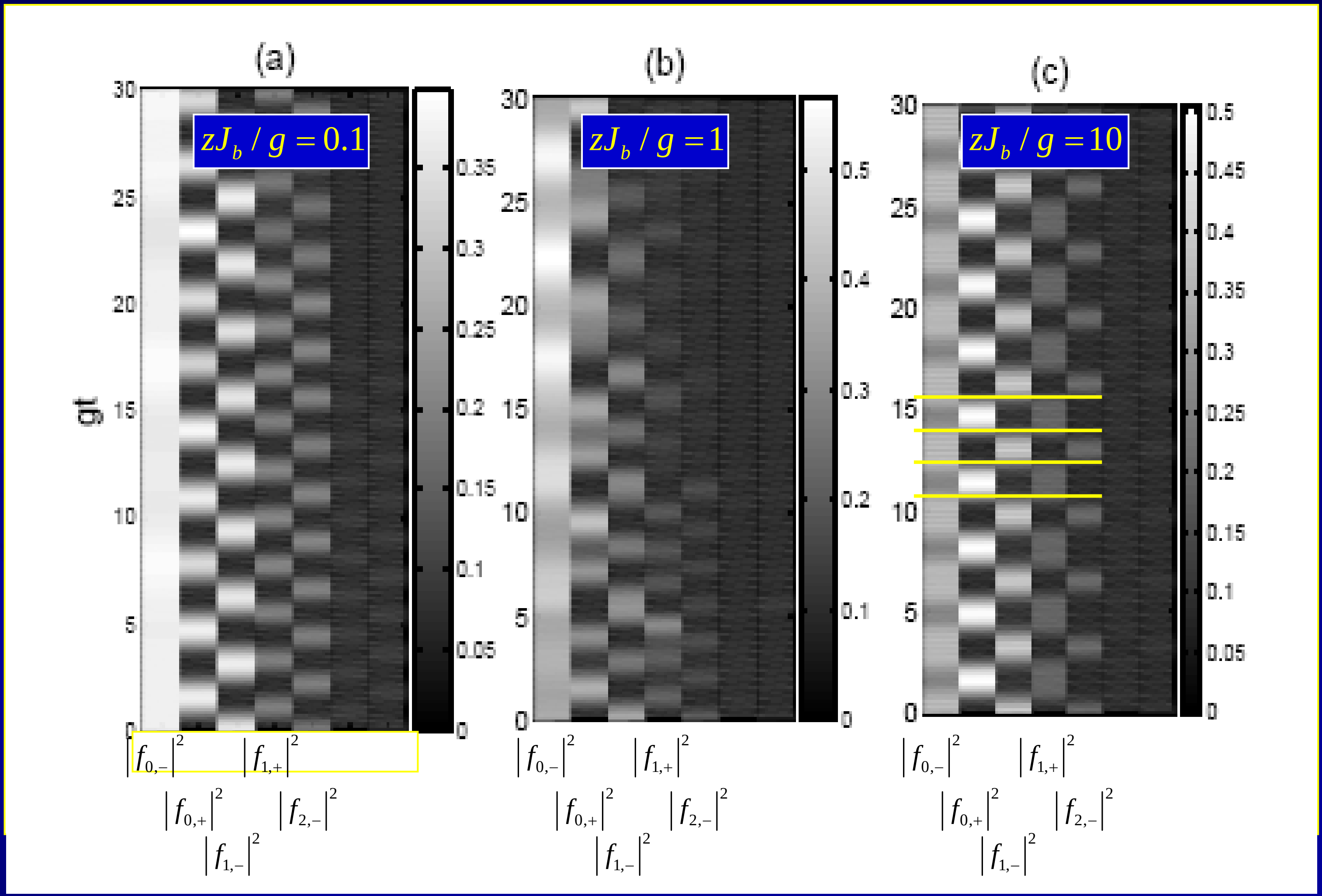
n_b

$|\Phi|^2$

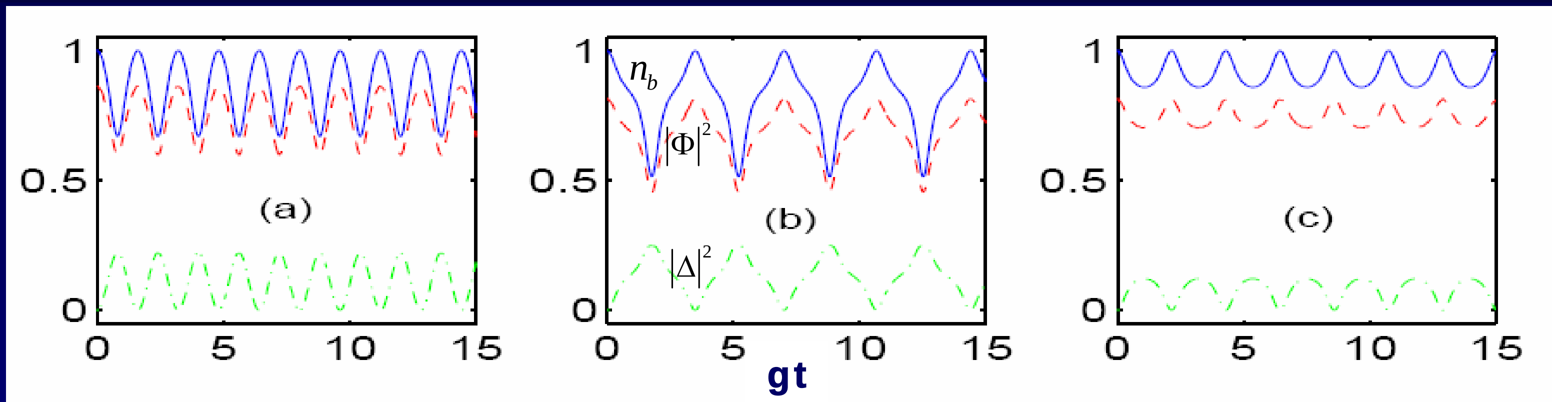
$|\Delta|^2$

Gutzwiller probabilities

Remember: $\left|\Psi(t)\right\rangle = \prod_{i=1}^{N_s} \left(\sum_{n_i=0}^{\infty} \sum_{\sigma i=-1/2}^{1/2} f_{n_l,\sigma_l}^{(l)}(t) \left| n_i, \sigma_i \right\rangle \right)$

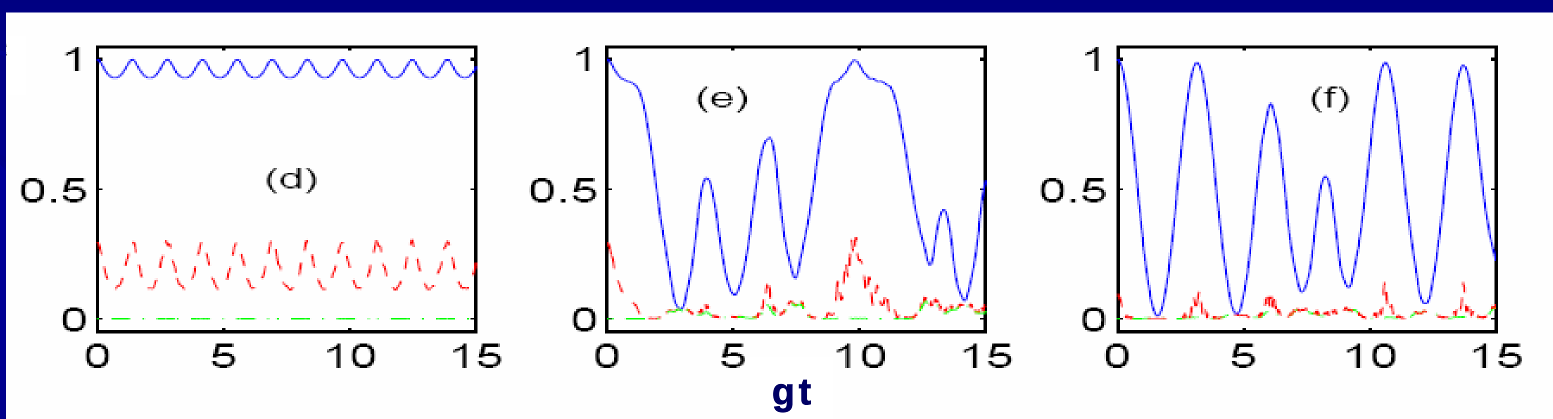


Self-trapping transitions



$U_b / zJ_b = 1.50$ $U_b / zJ_b = 1.86$ $U_b / zJ_b = 1.87$

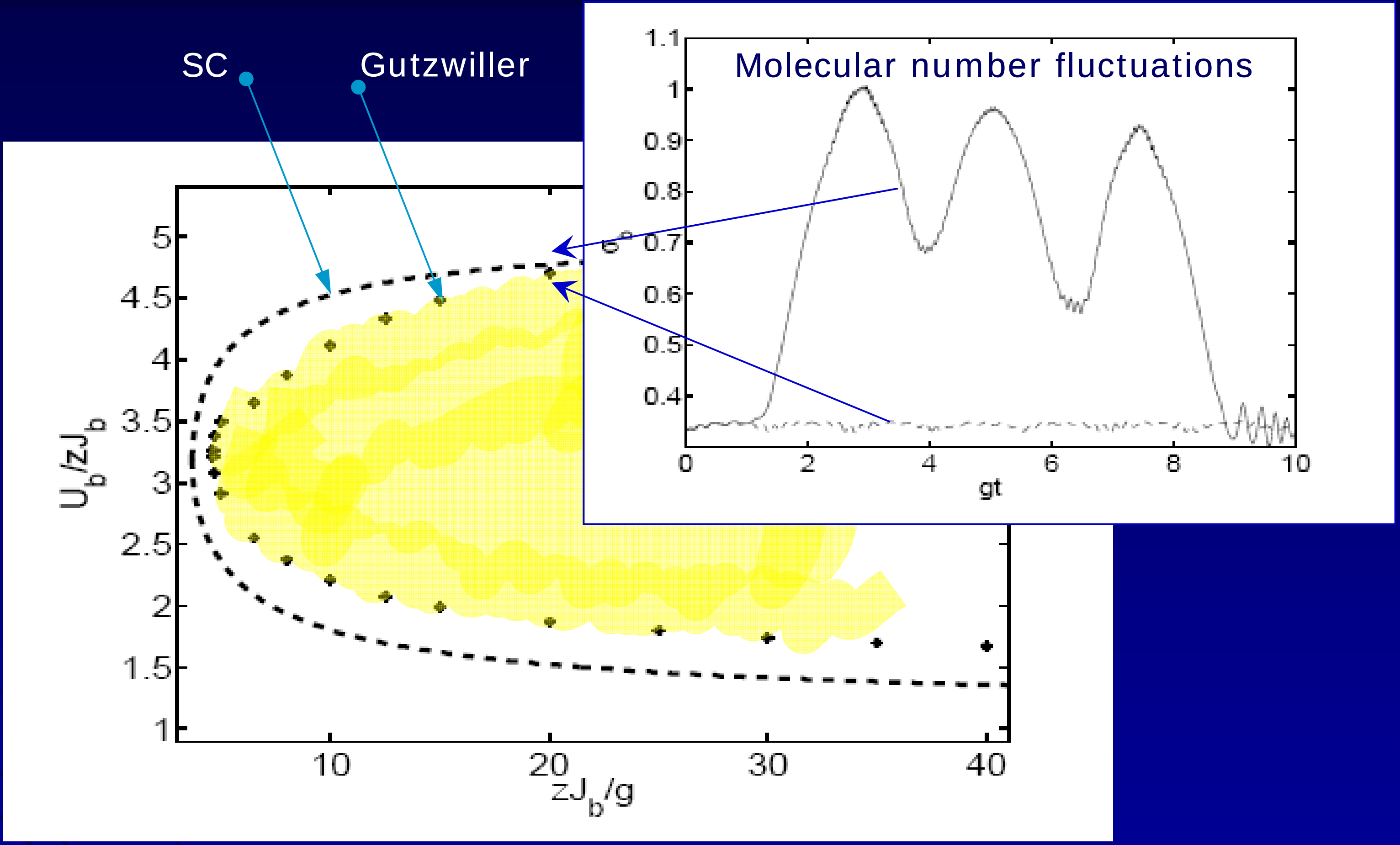
"coherent" self-trapping transition



$U_b / zJ_b = 4.70$ $U_b / zJ_b = 4.71$ $U_b / zJ_b = 5.50$

"incoherent" self-trapping transition

Self-trapping solutions



Physical origin: quartic term in classical potential

$$V\left(\langle n_0\rangle\right)=\frac{U_0^2}{8N_s^2}\langle n_0\rangle^4+(\cdots)\langle n_0\rangle^3+(\cdots)\langle n_0\rangle^2+(\cdots)\langle n_0\rangle \quad ; \qquad U_0=U_b-4U_{bf}$$

Summary and outlook

- Weak molecular tunneling regime: Jaynes-Cummings dynamics
- Strong molecular tunneling regime: Tavis Cummings dynamics
- Coherent and incoherent self-trapping transitions as function of U_b / zJ_b

NEXT:

- XXZ terms (intersite spin-spin coupling)
- Strong fermionic tunneling
- ...