Observation of Brewster's effect for transverse-electric electromagnetic waves in metamaterials

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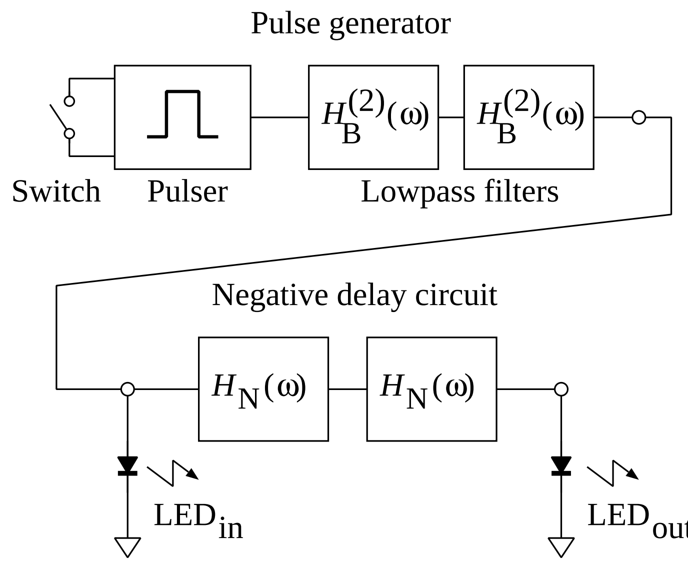
The 10th Japan-US seminar, Breckenridge, CO

2006.8.24

- “Negative group delay and superluminal propagation”
  - Demonstration of negative group delay in a simple electronic circuit



  - The output pulse shows up earlier than the input pulse.
- A question from an audience:

“Is circuit simulation of slow light possible?”

Simulation of slow light with electronic circuits

Nakanishi, Sugiyama, and MK, Am. J. Phys. **73**, 323 (2005)

- Wave equation for envelope

$$\frac{\partial \mathcal{E}}{\partial t} + v_{\mathrm{g}}(t,x)\frac{\partial \mathcal{E}}{\partial x} = 0$$

- The discretized (in space) equation

$$\frac{d}{dt}\left[\frac{v_{n+1}(t) + v_n(t)}{2}\right] + \frac{1}{T(n,t)}[v_{n+1}(t) - v_n(t)] = 0$$

— Cascaded delay circuits with variable time constants

- Movie

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— Cascaded delay circuits with variable time constants

- Movie
- The authors gratefully thank Prof. Vladan Vuletic for helpful discussion at Yatsugatake.

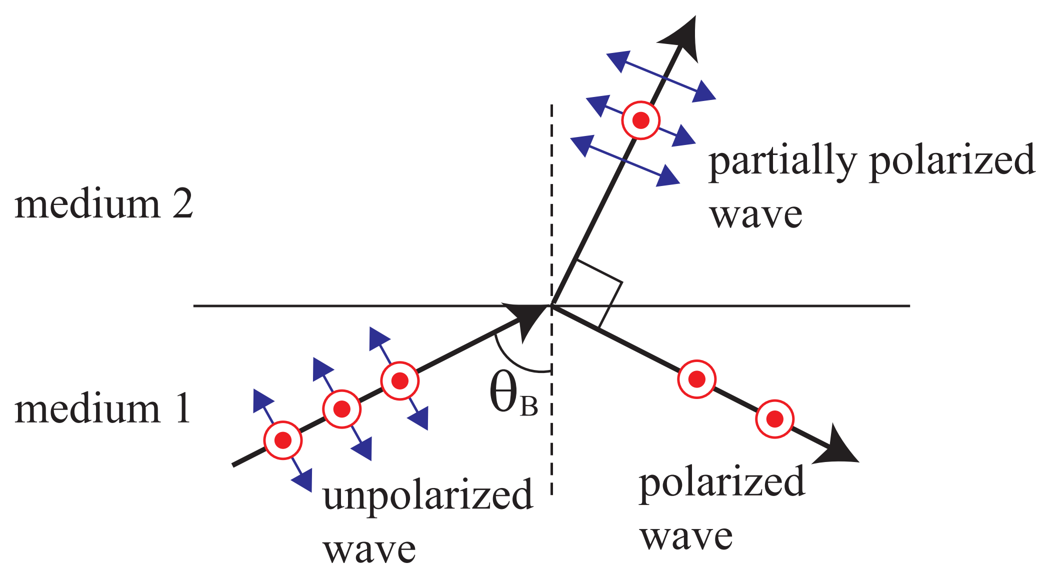
Brewster Effect

D.B. Brewster, Philos. Trans. Roy. Soc. Lond. **105**, 105 (1815).

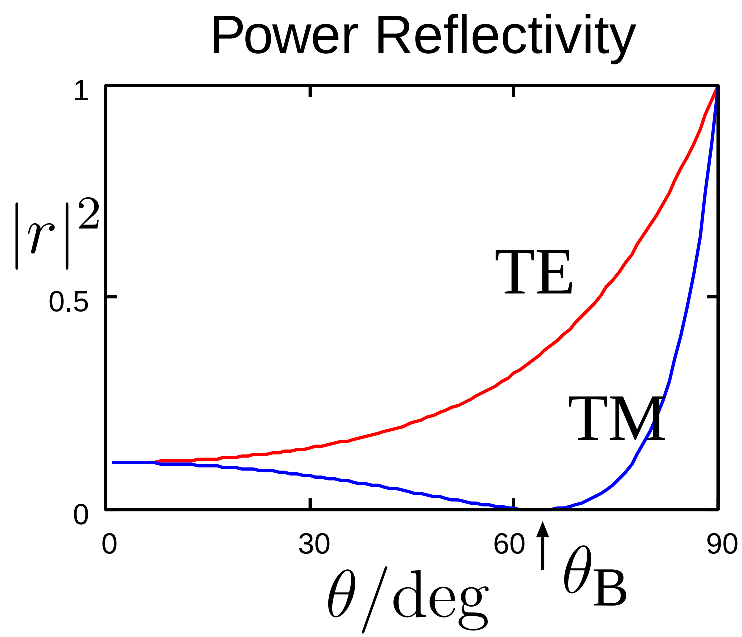
- Brewster's law — No reflection for a specific incident angle:

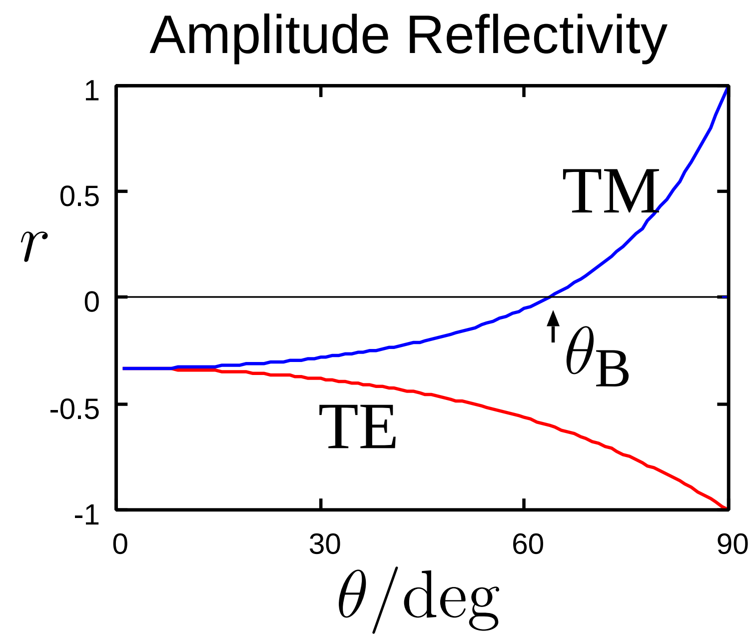
$$\theta_{\mathrm{B}} = \tan^{-1} n \quad (n : \text{refractive index})$$

- Only for TM waves (p waves)
- Practical importance — polarlizers, no reflection surface (for free)



Polarization-dependent reflectivity

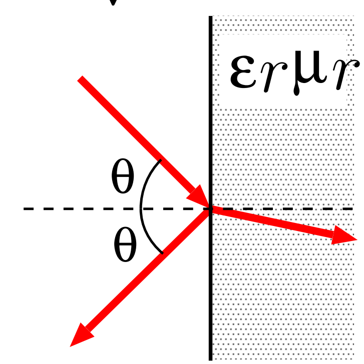




- Electric and magnetic constants.

$$n = \sqrt{\varepsilon_r \mu_r} \; (= 2)$$

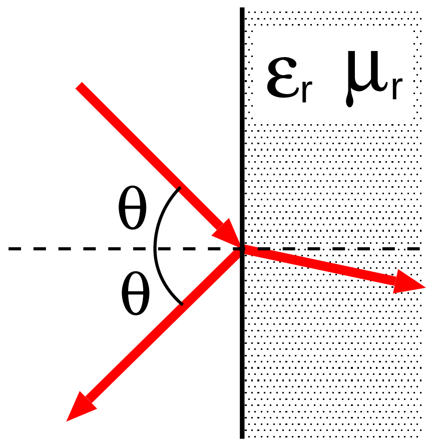
$$\varepsilon_r = \varepsilon/\varepsilon_0, \quad \mu_r = \mu/\mu_0$$

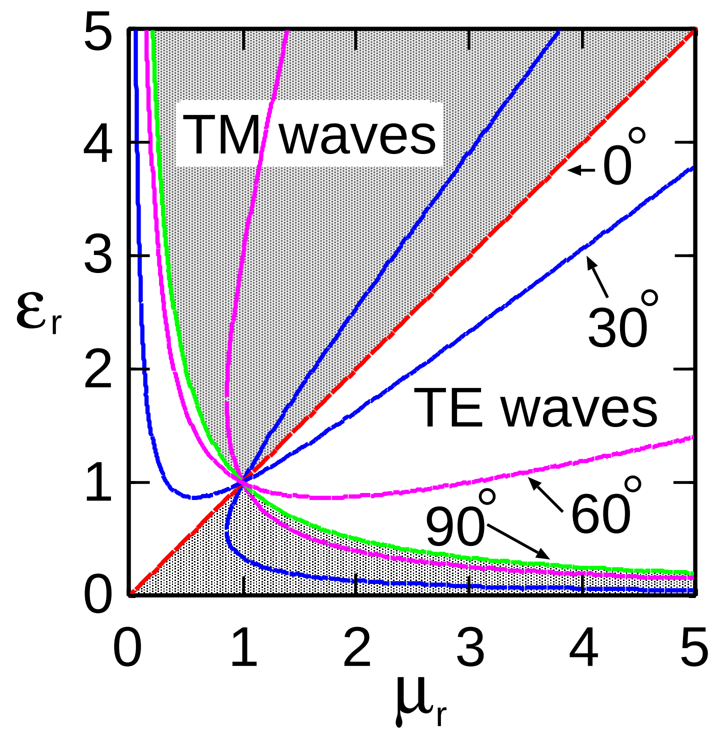


- Asymmetry between TE and TM modes:

$$\epsilon_r \neq 1, \quad \mu_r = 1$$

Brewster condition on μ-ε plane





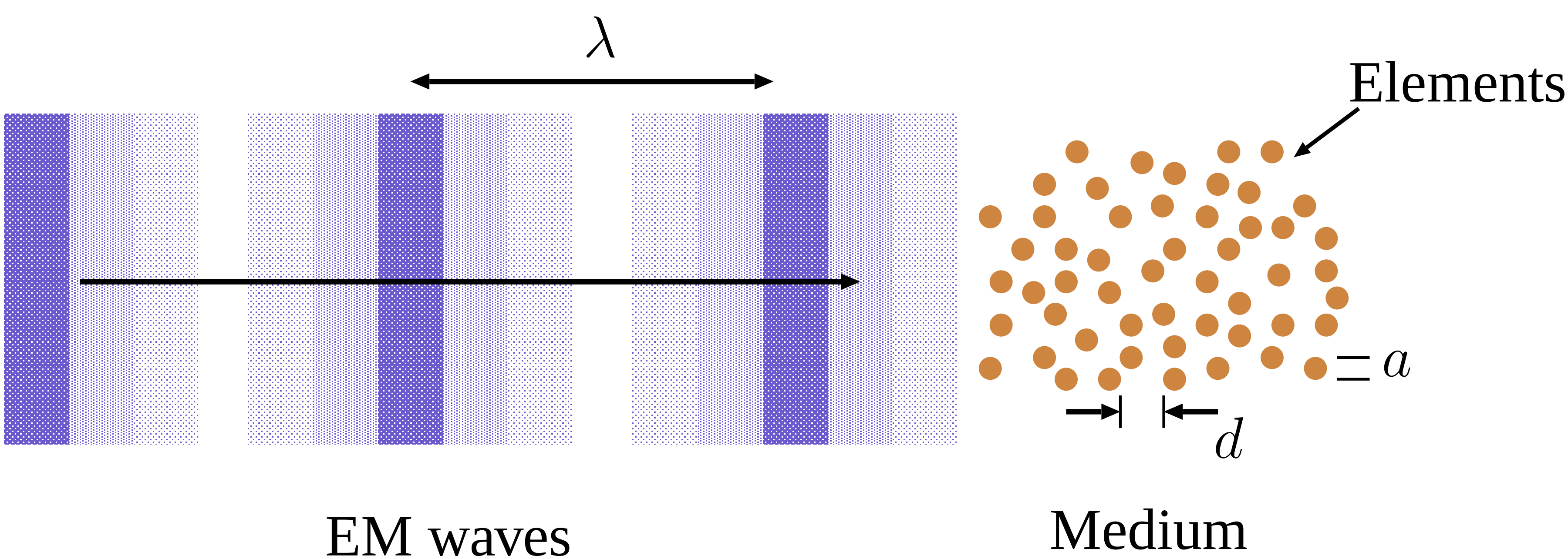
- $\varepsilon_{\rm r} \neq 1$, $\mu_{\rm r} = 1 \longrightarrow$ Brewster effect for TM waves

  $\Updownarrow$ EM-duality

- $\varepsilon_{\rm r} = 1$, $\mu_{\rm r} \neq 1 \longrightarrow$ Brewster effect for TE waves

Metamaterials (1)

- Collection of small conductive elements (coils and rods)
- Each element responds to EM fields and reradiates — like an atom in normal media
- The sizes and separations of elements are much smaller than the wavelengths ($a,\ d \ll \lambda$) — Continious media



Metamaterials (2)

- Owing to the resonant structures of elements, both $\epsilon_{\rm r}$ and $\mu_{\rm r}$ can be shifted from unity significantly.
- Even nagetive $\epsilon_{\rm r}$ or $\mu_{\rm r}$ is possible. — $\epsilon_{\rm r} < 0$, $\mu_{\rm r} < 0$.

$$n = \sqrt{\varepsilon_{\rm r}}\sqrt{\mu_{\rm r}} = ({\rm i}\sqrt{|\varepsilon_{\rm r}|})({\rm i}\sqrt{|\mu_{\rm r}|}) = -\sqrt{\varepsilon_{\rm r}\mu_{\rm r}} < 0$$

Negative refraction

Formula $n = \sqrt{\varepsilon_{\rm r}\mu_{\rm r}}$ must be revised as $n = \sqrt{\varepsilon_{\rm r}}\sqrt{\mu_{\rm r}}$.

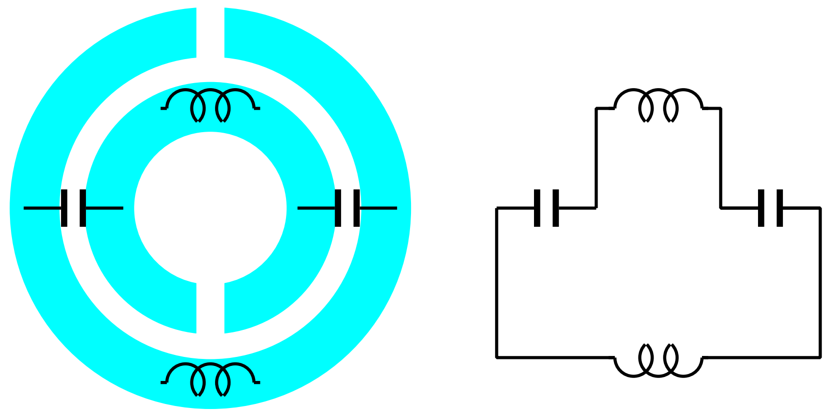
- Magnetic materials ($\mu_{\rm r} \neq 1$) for high frequancies can be made. (In normal media, magnetic response is frozen.)

TE-Brewster effect in metamaterials

Tamayama, Nakanishi, Sugiyama, and MK:
Phys. Rev. B **73**, 193104 (2006).

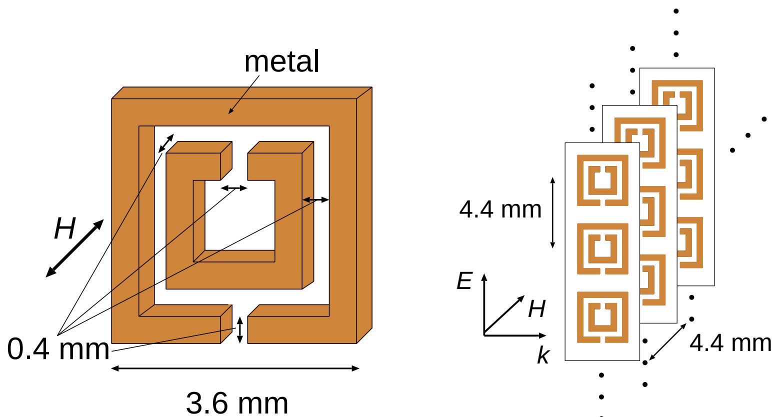
- Synthesize a metamaterial with $\mu_{\mathrm{r}} \neq 1$, $\epsilon_{\mathrm{r}} = 1$. (purely magnetic medium)
- Experimental demonstration of Brewster effect for TE waves (in microwave region)

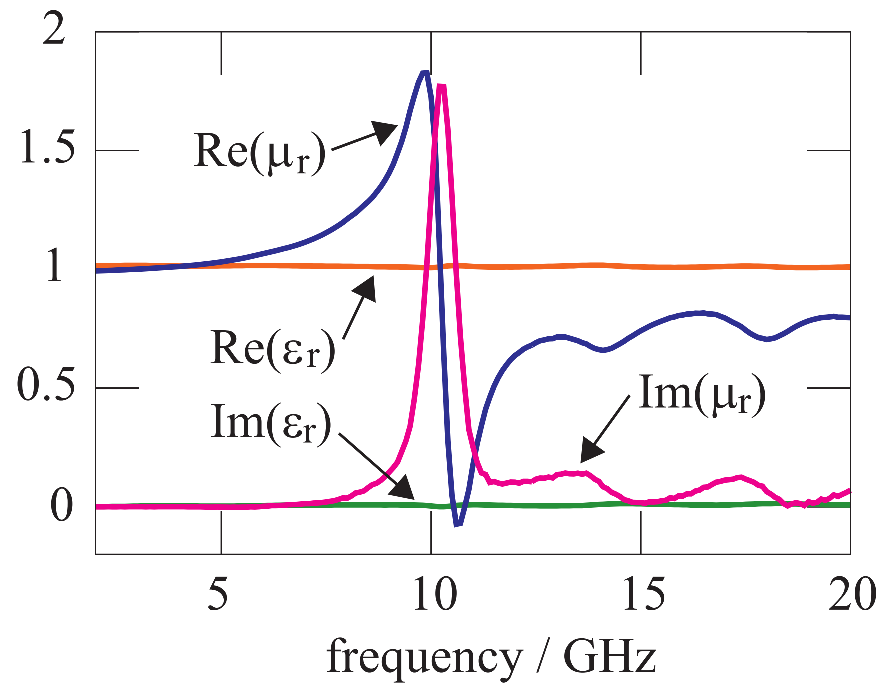
Split ring resonator (SRR) — magnetic atom



- Two planar coils coupled with capacitors
- LC series resonance
- Circular current is induced by time dependent B-field near resonance $\longrightarrow$ induced magnetic moment
- Ensemble of SRRs — magnetic medium

SRR design with FDTD

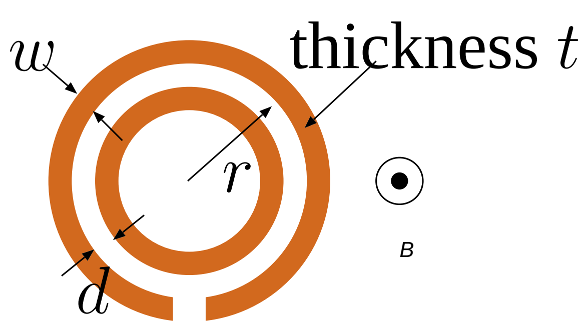


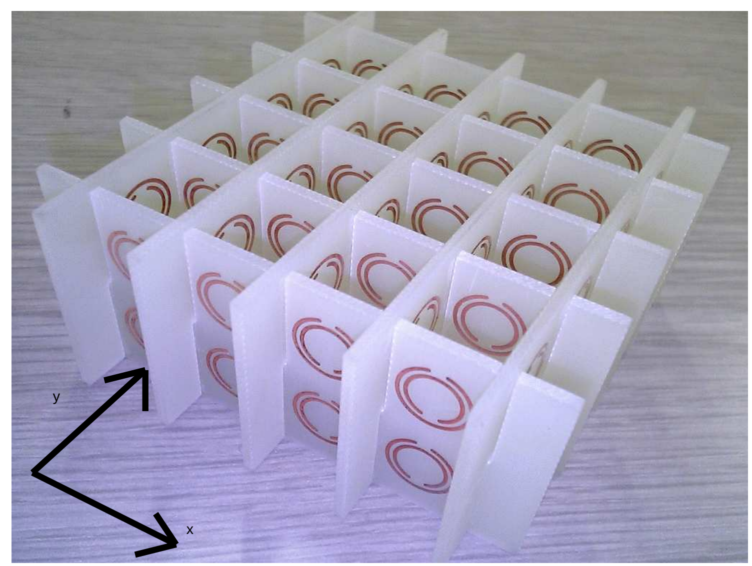


- $\varepsilon_{\mathrm{r}}(f) \simeq 1$, $\mu_{\mathrm{r}}(f)$: Lorentzian
- Resonance frequency $10.3\,\mathrm{GHz}$.
  (The LC equivalent circuit model predicts: $9.54\,\mathrm{GHz}$)

Final design of SRR array

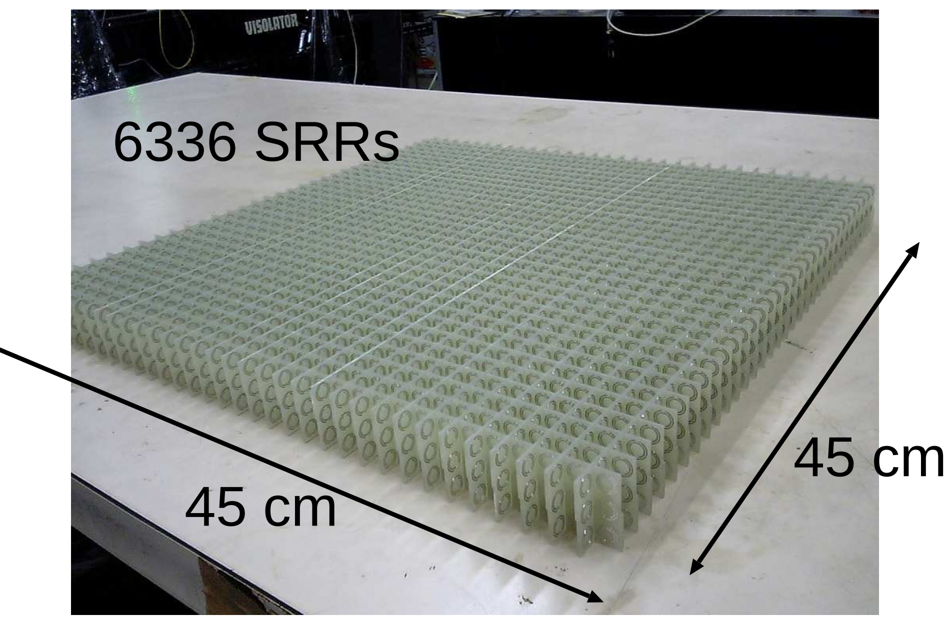
- $r = 4.0\,\mathrm{mm}$, $w = 0.61\,\mathrm{mm}$, $r = 0.48\,\mathrm{mm}$, $t = 35\,\mu\mathrm{m}$
  $\rightarrow f = 3.04\,\mathrm{GHz}$ ($\lambda_0 = 9.9\,\mathrm{cm}$) : LC model value
- 2D crossed array — isotropic response to B-fields in xy-plane
- Lattice constants:
  $$a_x = a_y = 1.4\,\mathrm{cm} = 0.14\lambda_0,\ a_z = 1.3\,\mathrm{cm} = 0.13\lambda_0.$$
- Planer waveguide (separation = $0.38\lambda = 3a_z$) $\longrightarrow$ TE waves

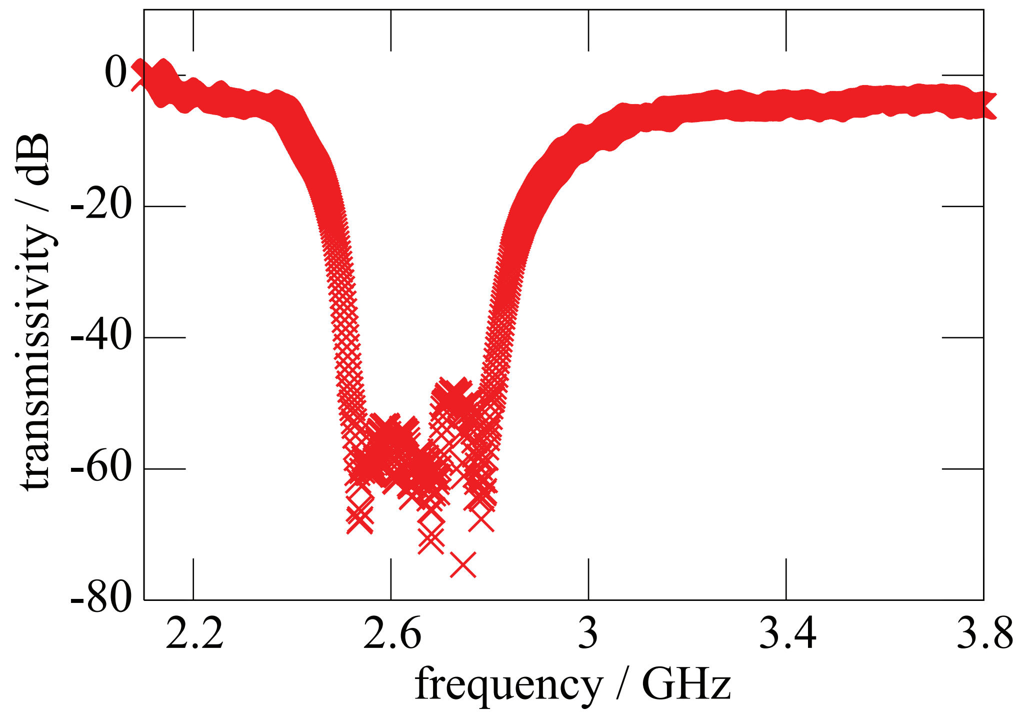




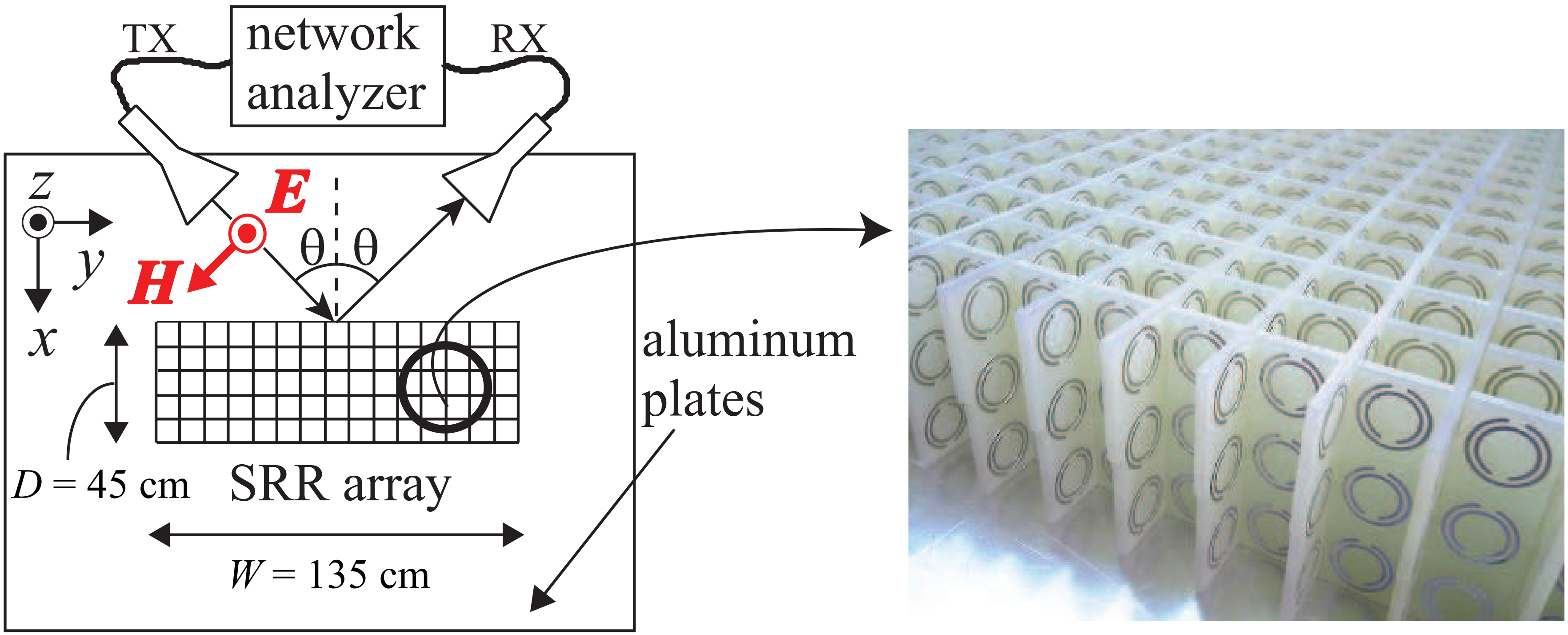
Absorption spectrum

- Absorption due to SRR resonance
- Resonance frequency $2.65\,\mathrm{GHz}$.



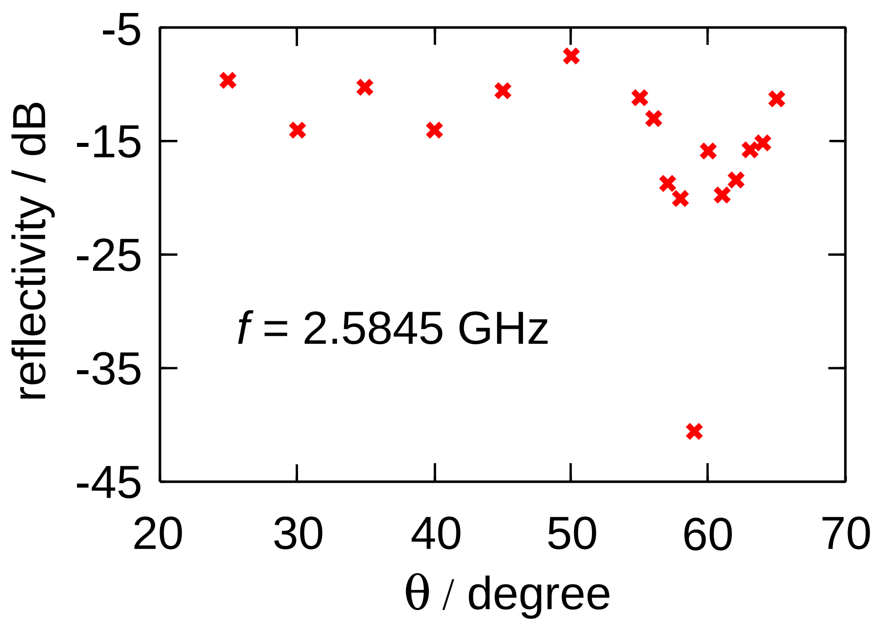


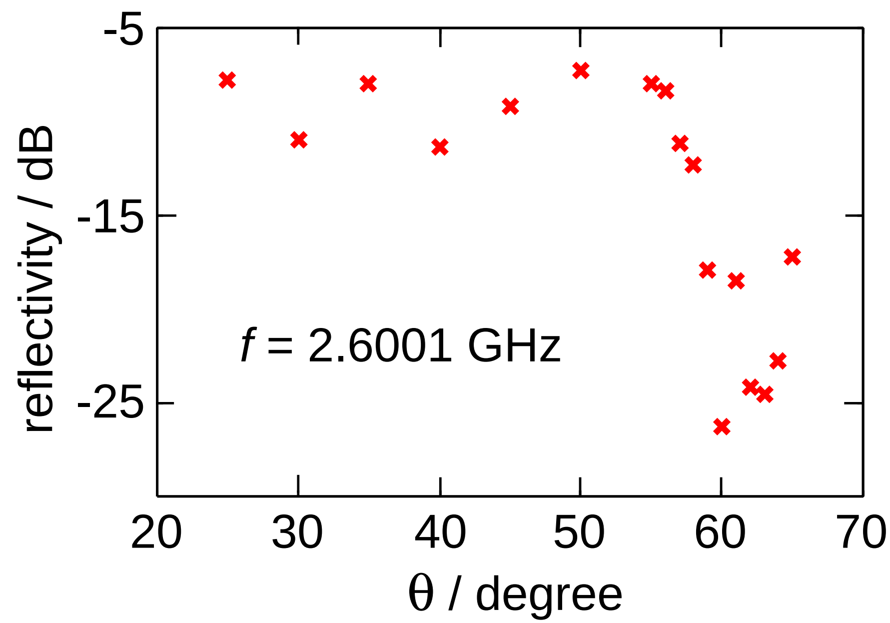
Exprimental setup



- Reflectivity is measured with a network analayzer.

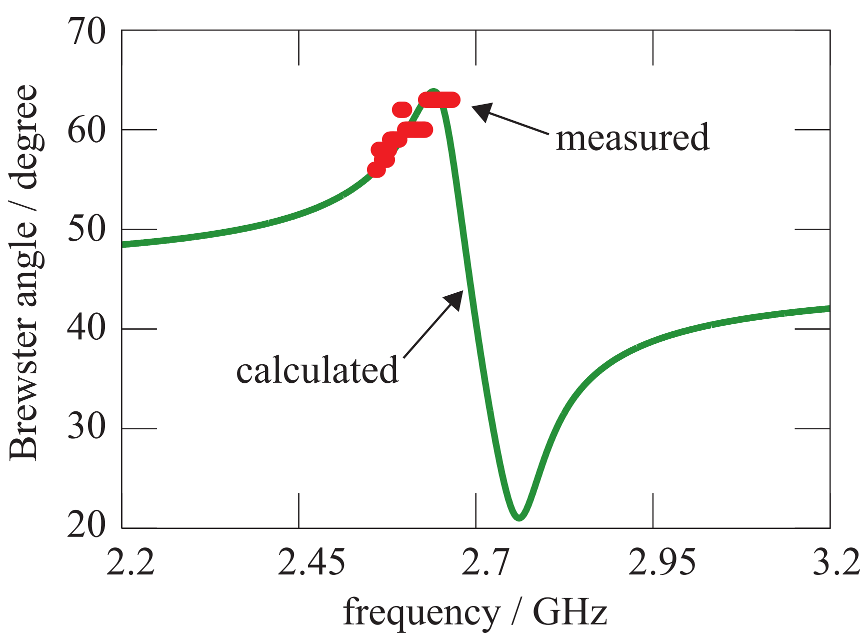
Reflectivity measurement

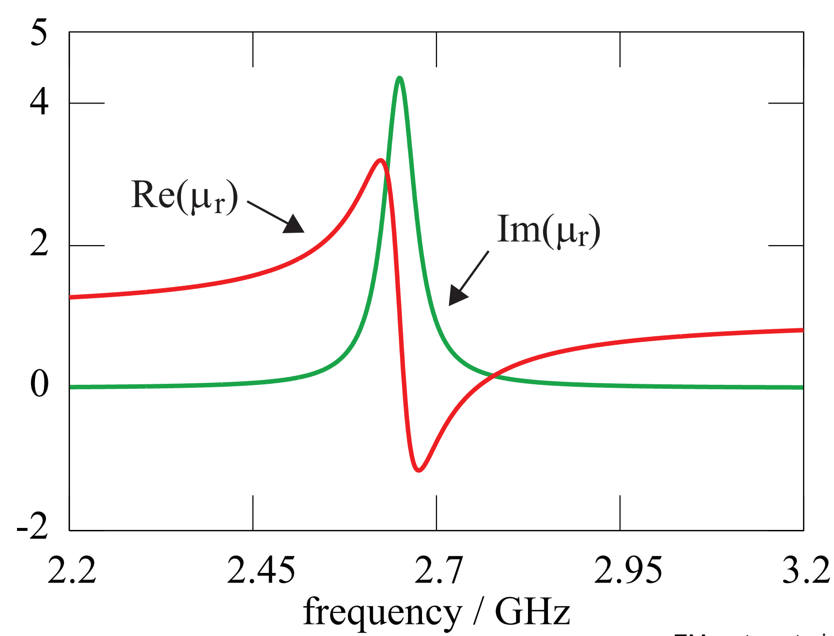




Frequency dependence of Brewster angle

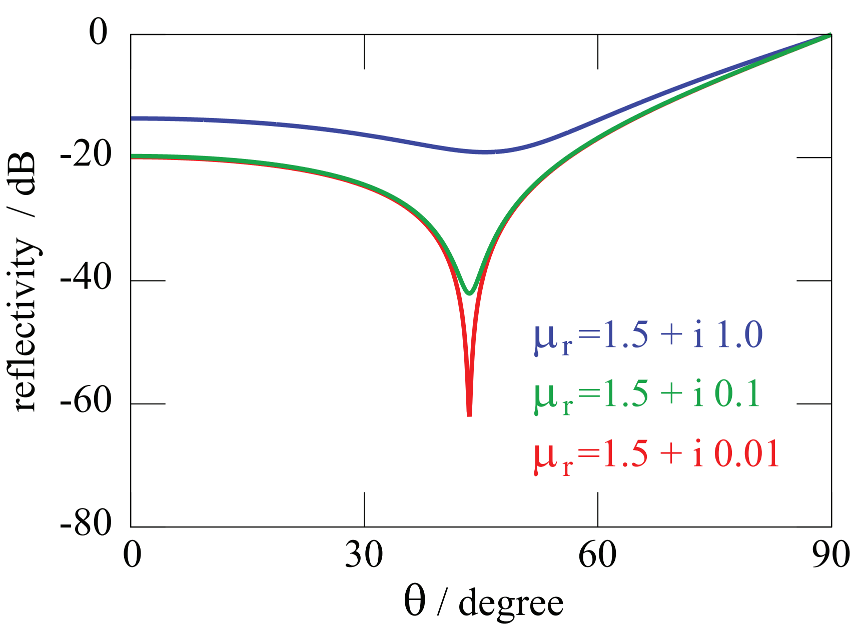
- $\varepsilon_{\mathrm{r}}(f) = 1,\ \mu_{\mathrm{r}}(f) = 1 - \dfrac{F}{f^2 + \mathrm{i}\gamma f - f_0^2}$: model response functions
- $f_0 = 2.65\,\mathrm{GHz}$ (determined by the transmission spectrum of SRR array)
- F, γ: fitting parameters.

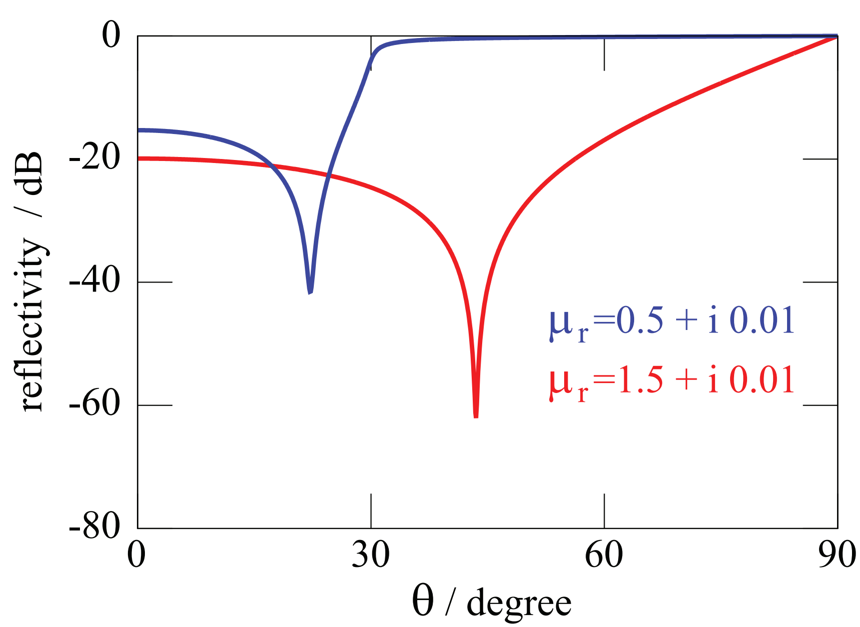




Pseudo-Brewster angle

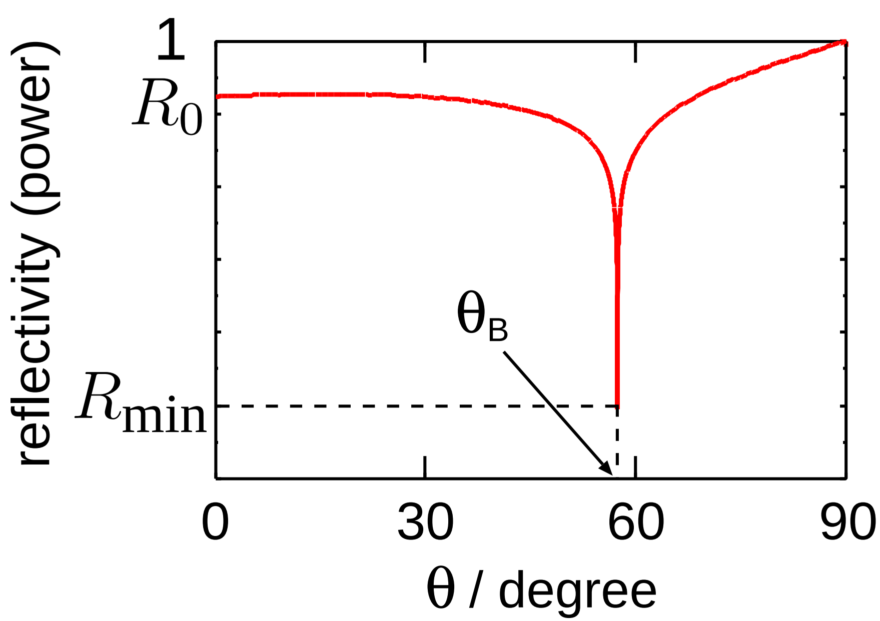
- At SSR resonance wings, we have $\mu_{\rm r} \neq 1$ and $\varepsilon_{\rm r} = 1$.
- Loss ($\mathrm{Im}\,\mu_{\rm r} > 0$) must be considered.
- For lossy media, suppression of reflectivity is incomplete $\longrightarrow$ Pseudo-Brewster angle

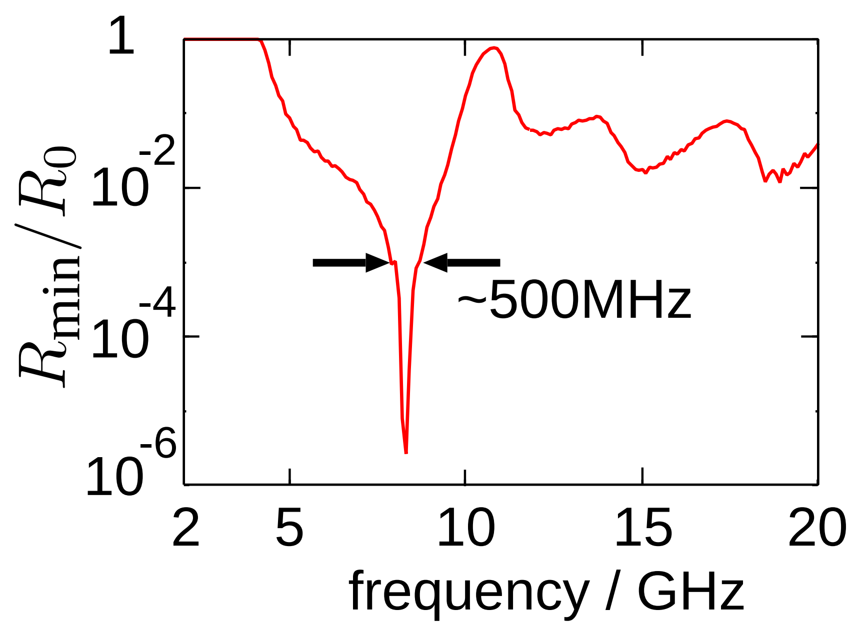




Frequency region

- R_0: reflectivity at $\theta = 0$.
- R_{min}: reflectivity at $\theta = \theta_B$ (Brewster)
- The ratio R_{min}/R_0 becomes smaller in a narrow region blow resonance — easy to find Brewster effect





Brewster condition as impedance matching

- Oblique propagation: (x, z)-plane
  $\boldsymbol{E}(\boldsymbol{r}, t) = \tilde{\boldsymbol{E}}(z)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\beta x}$, $\boldsymbol{H}(\boldsymbol{r}, t) = \tilde{\boldsymbol{H}}(z)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\beta x}$
- TM waves

$$(\mathrm{d}/\mathrm{d}z)H_y = \mathrm{i}\omega\varepsilon E_x, \quad (\mathrm{d}/\mathrm{d}z)E_x = \mathrm{i}\omega\mu_{\text{eff}}(\theta)H_y$$

$$\text{where} \quad \mu_{\text{eff}}(\theta) = \mu(1 - \sin^2\theta), \quad \sin\theta = \beta/\omega\sqrt{\mu\varepsilon}$$

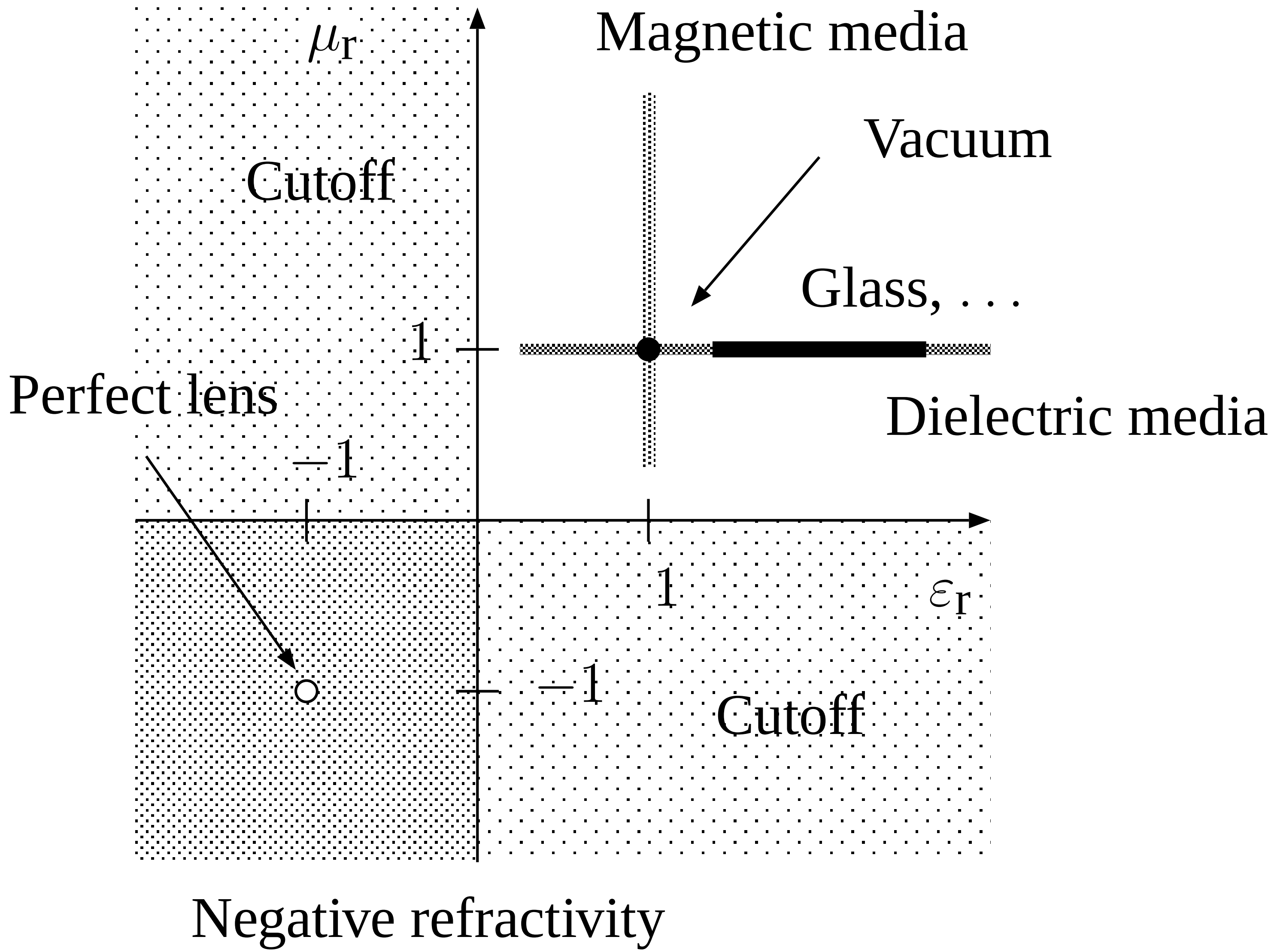
- Impedance matching at the interface (1–2)

$$\frac{\mu_1(1 - \sin^2\theta_1)}{\varepsilon_1} = \frac{\mu_2(1 - \sin^2\theta_2)}{\varepsilon_2}$$

θ_1 : angle of incidence, θ_2 : angle of refraction

- Impedance matching is achieved effectively even for $\varepsilon_1 \neq \varepsilon_2$.
  $\longrightarrow$ Brewster condition

ϵ-μ plane



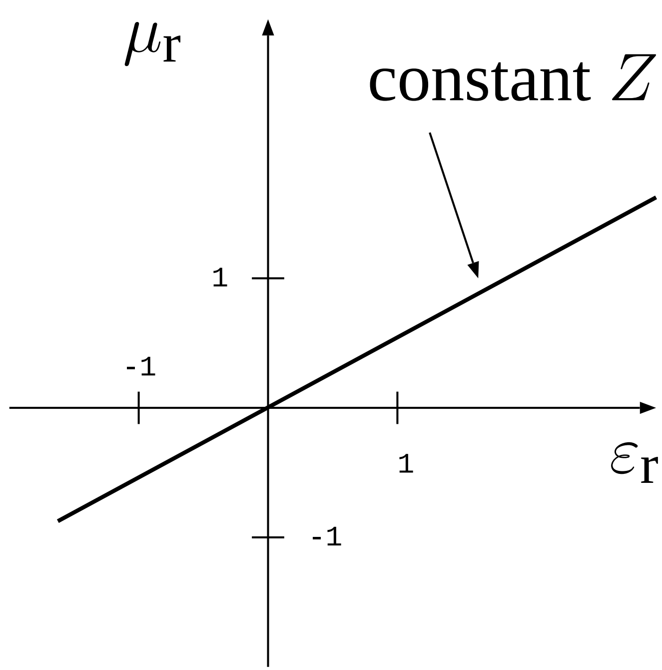
No reflection propagation (1D)

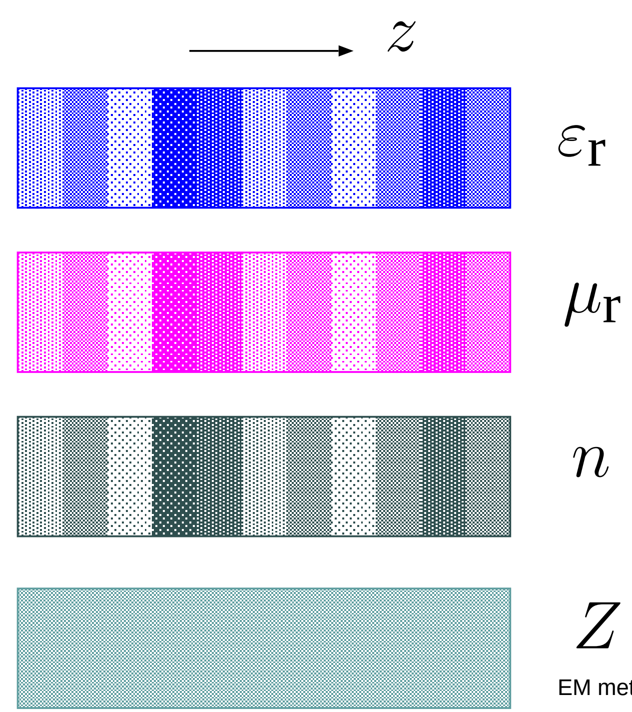
- Constant impedance media:

$$Z(z) = \sqrt{\mu_{\mathrm{r}}(z)/\varepsilon_{\mathrm{r}}(z)}Z_0 = \text{const.}$$

$$n(z) = \sqrt{\mu_{\mathrm{r}}(z)}\sqrt{\varepsilon_{\mathrm{r}}(z)} \neq \text{const.}$$

- No reflection in inhomogenous media (even in negative n)





Constant impedance propagation

- Plane waves propagted in z-direction (ω)

$$\frac{\mathrm{d}}{\mathrm{d}z}\tilde{H}_y = \mathrm{i}\omega\varepsilon(z)\tilde{E}_x, \quad \frac{\mathrm{d}}{\mathrm{d}z}\tilde{E}_x = \mathrm{i}\omega\mu(z)\tilde{H}_y$$

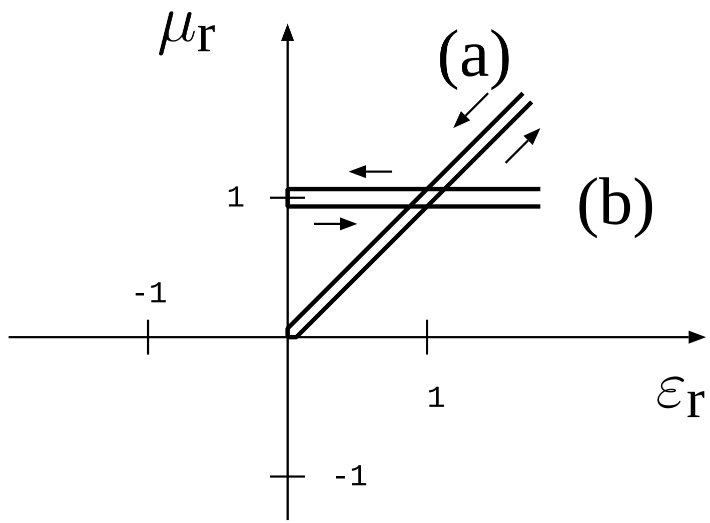
- Impedance condition: $\sqrt{\frac{\mu(z)}{\varepsilon(z)}} = Z$
- Unidirectional wave equation

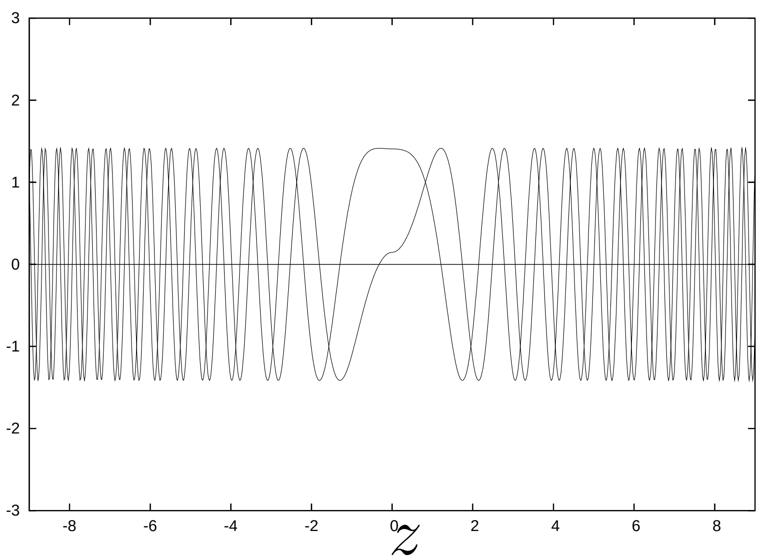
$$\frac{\mathrm{d}}{\mathrm{d}z}\psi_\pm = \pm\mathrm{i}k_0 n(z)\psi_\pm, \quad \psi_\pm = E_x \pm Z H_y$$

where $n(z) = c\sqrt{\mu(z)}\sqrt{\varepsilon(z)}$. $k_0 = \omega/c$.

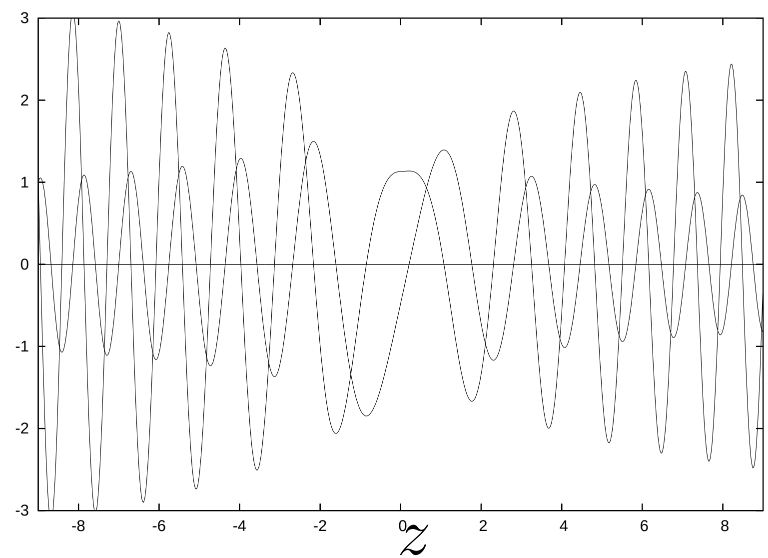
- Solution: $\psi_\pm(z) = \psi_{\mathrm{F}}(0)\exp\left(\pm\mathrm{i}k_0\int_0^z n(z')\mathrm{d}z'\right)$

Constant impedance propagation



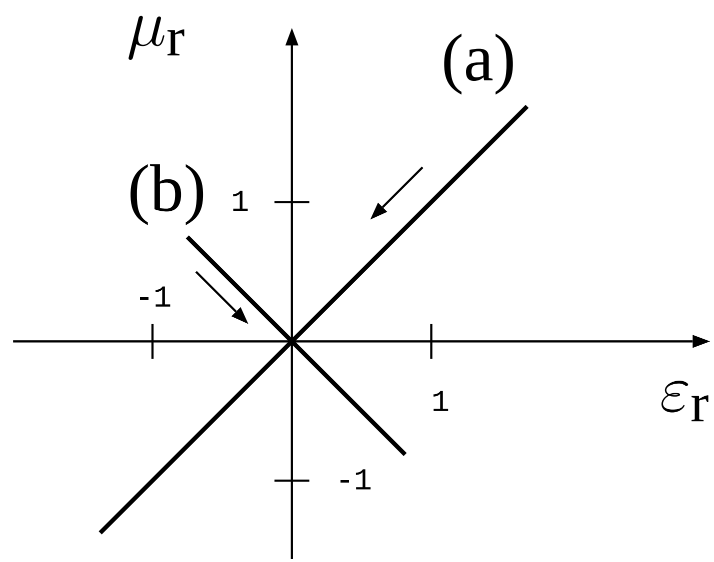


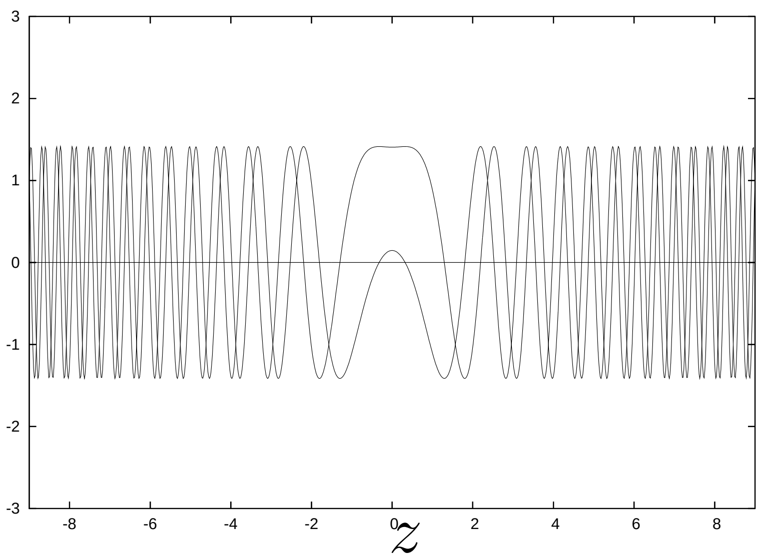
(a)



(b)

Constant impedance propagation





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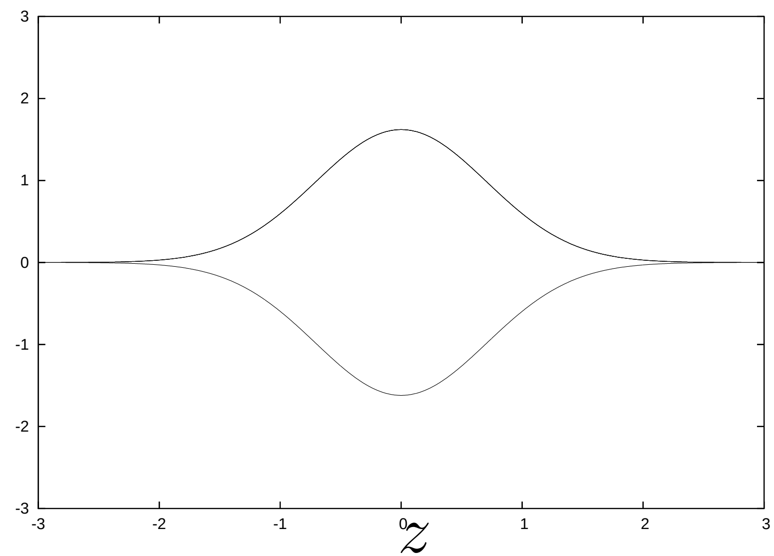

(b)

No reflection propagation in 2D

- $\boldsymbol{\nabla} = (0, \partial_y, \partial_z)$, $\tilde{\boldsymbol{E}} = (\tilde{E}_x, 0, 0)$, $\tilde{\boldsymbol{H}} = (0, \tilde{H}_y, \tilde{H}_z)$

$$\boldsymbol{\nabla} \times \tilde{\boldsymbol{H}} = -\mathrm{i}\omega\varepsilon(\boldsymbol{r})\tilde{\boldsymbol{E}}, \quad \boldsymbol{\nabla} \times \tilde{\boldsymbol{E}} = \mathrm{i}\omega\varepsilon(\boldsymbol{r})\tilde{\boldsymbol{H}}$$

- Impedance condition

$$Z = \sqrt{\mu(\boldsymbol{r})/\varepsilon(\boldsymbol{r})} = \text{const}, \quad n(\boldsymbol{r}) = c\sqrt{\mu(\boldsymbol{r})\varepsilon(\boldsymbol{r})}$$

- Scaler field equation — without approximation

$$\frac{1}{n(\boldsymbol{r})}\boldsymbol{\nabla} \cdot \left(\frac{1}{n(\boldsymbol{r})}\boldsymbol{\nabla}\tilde{E}_x\right) = -k_0^2\tilde{E}_x$$

Constant impedance media

- Wavelength control without reflection
- No adiabatic conditions are required
- Generation of long wave ($k \sim 0$)
  - Difference frequency generation
  - Massive photons

Vacuum impedance Z_0 as universal constant

MK: arXiv: Physics/0607056

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}, \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{(Schelkunoff 1938)}$$

- Constitutive relation for vacuum

$$\begin{bmatrix} c\boldsymbol{D} \\ \boldsymbol{H} \end{bmatrix} = Z_0 \begin{bmatrix} \boldsymbol{E} \\ c\boldsymbol{B} \end{bmatrix}$$

- Fine structure constant α (Hehl *et al.*, arXiv: Physics/0005084)

$$\alpha = \frac{Z_0}{2R_\mathrm{K}}, \qquad R_\mathrm{K} = \frac{h}{e^2} \text{ : von Klitzing constant}$$

- Z_0 is helpful to simplify SI equations.