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Schrödinger cat and EPR state with quantum optics

Akira Furusawa
Department of Applied Physics
University of Tokyo
CREST, JST

Collaborators

A. Furusawa Univ. of Tokyo

T. Aoki, H. Yonezawa, *K. Wakui*, *H. Takahashi*,
Y. Takeno, J. Yoshikawa, T. Kajiya, N. Lee,
M. Yukawa, Y. Miwa, H. Uchigaito,
J. S. Neergaard-Nielsen (NBI), N. Takei (ERATO)
A. Huck (Erlangen)

M. Sasaki NICT

M. Fujiwara, M. Takeoka, J. Hayase, A. Kitagawa,
K. Tsujino

M. Ban Hitachi

S. L. Braunstein, P. van Loock, U. L. Andersen

Quantum optics

annihilation operator $\hat{a}$

$$[\hat{a},\hat{a}^{\dagger}]=1 \quad \left(\hbar=\frac{1}{2}\right)$$

Photon-number units

quantum complex amplitude

$$\hat{a}=\hat{q}+i\hat{p}$$

q: cosine component
p: sine component

$$[\hat{x},\hat{p}]=\frac{i}{2} \longleftrightarrow [\hat{q},\hat{p}]=\frac{i}{2}$$

x: position
p: momentum

Coherent states **Laser**

Minimum uncertainty state

$$\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$$

$$\left|\alpha\right\rangle = e^{-\frac{\left|\alpha\right|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \left|n\right\rangle$$

Squeezed vacuum

Minimum uncertainty state

$$\hat{S}(r)|0\rangle = e^{\frac{r}{2}\left(\hat{a}^2 - \hat{a}^{\dagger 2}\right)}|0\rangle$$

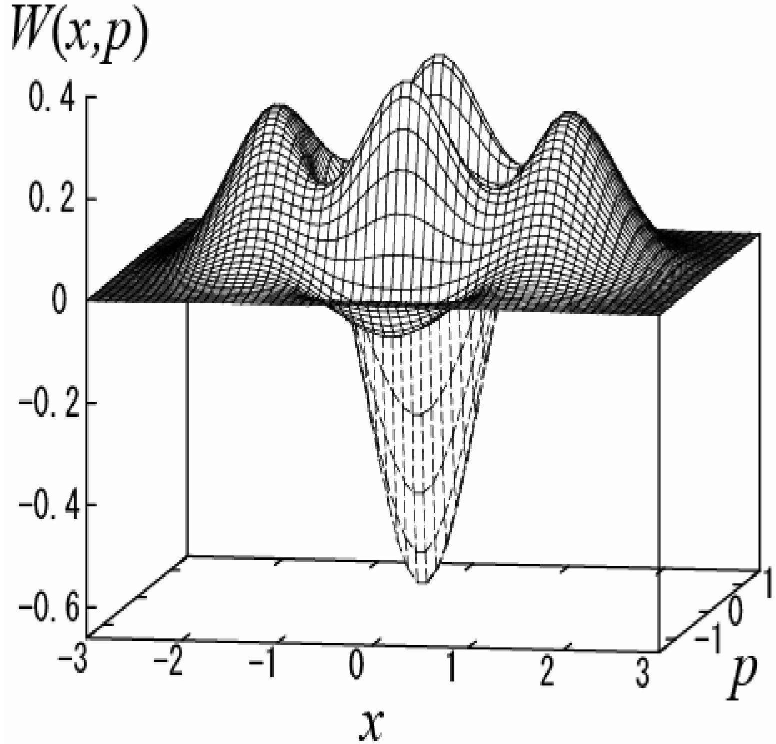
$$= \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} \tanh^n r |2n\rangle$$

$$\hat{S}^\dagger(r)\hat{a}\,\hat{S}(r) = \hat{a}\cosh r - \hat{a}^\dagger \sinh r$$

$$= e^{-r}\hat{x} + ie^{r}\hat{p}$$

Schrödinger cat state

$$|\alpha\rangle - |-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

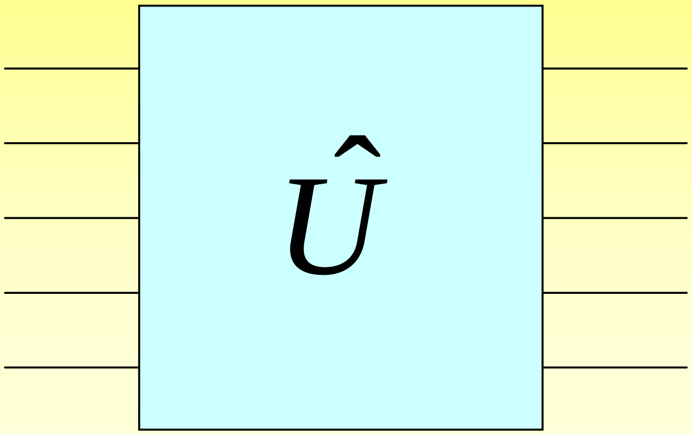


$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$|-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}} |n\rangle$$

Quantum information processing

Unitary transformation

$|\psi\rangle$  $|\varphi\rangle$

$$|\varphi\rangle = \hat{U}|\psi\rangle = e^{-i\frac{\hat{H}}{\hbar}t}|\psi\rangle$$

Arbitrary Hamiltonians （polynomials of $\hat{x}, \hat{p}$）

S. Lloyd, S.L. Braunstein PRL **82**, 1784 (1999)

Gaussian operations

$\hat{p}_1\hat{x}_2 - \hat{x}_1\hat{p}_2$	$i\left(\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_1\hat{a}_2^\dagger\right)$	Beam splitters
$\hat{x}, \hat{p}$	$i\left(\alpha^*\hat{a} - \alpha\hat{a}^\dagger\right)$	Displace in phase space
$\hat{x}^2 + \hat{p}^2$	$\hat{a}^\dagger\hat{a}$	Phase shifters
$\hat{x}\hat{p} + \hat{p}\hat{x}$	$i\left(\hat{a}^{\dagger 2} - \hat{a}^2\right)$	Squeezers $\chi^{(2)}$
$\left(\hat{x}^2 + \hat{p}^2\right)^2$	$\left(\hat{a}^\dagger\hat{a}\right)^2$	Kerr effect $\chi^{(3)}$

$$\hat{a} = \hat{x} + i\hat{p}$$

Non-Gaussian operations

Toward universal QIP

Quantum teleportation of non-Gaussian states

A non-Gaussian input state

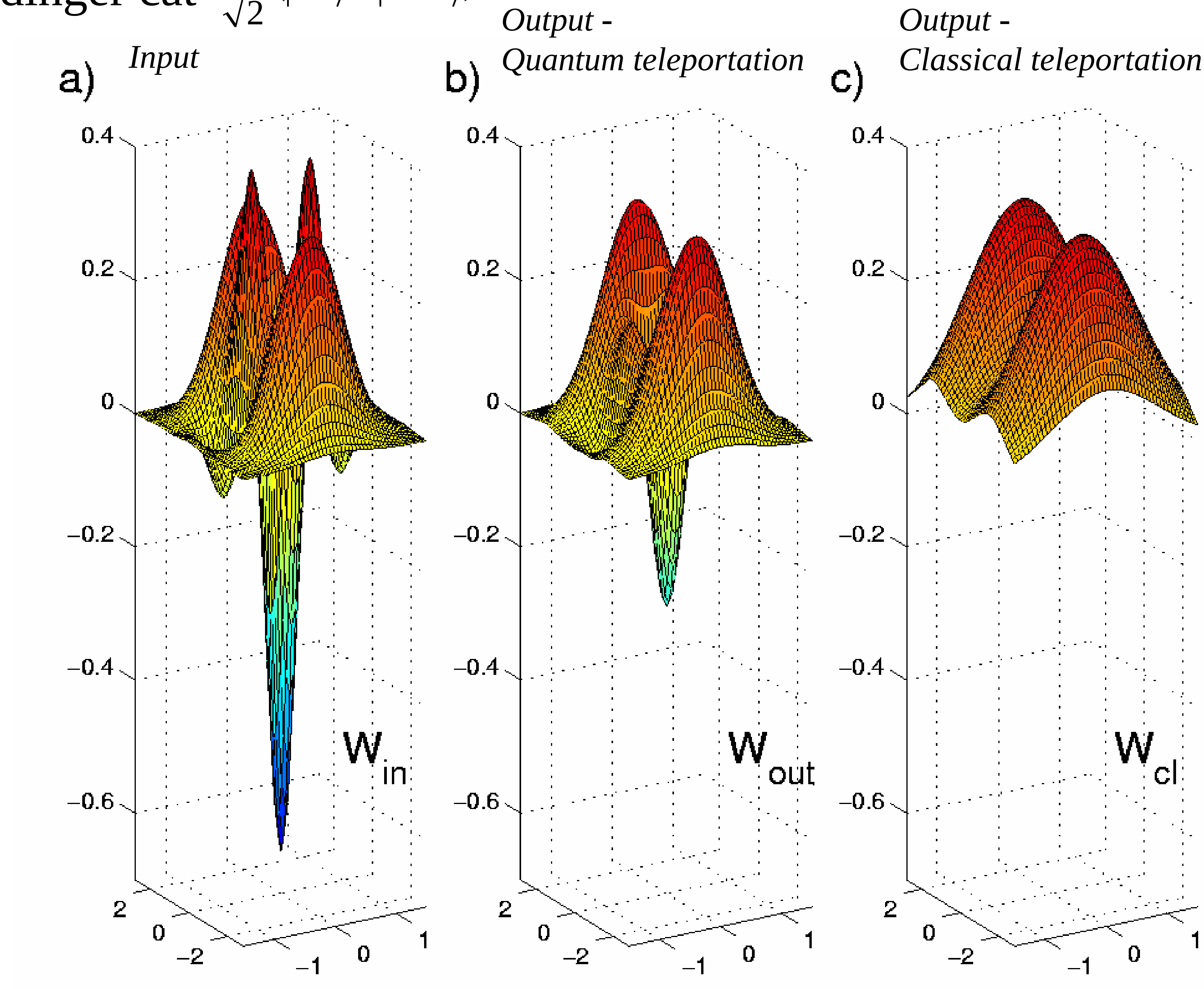
Schrödinger cat state

$$\left|\psi_{cat}\right\rangle \propto \left|\alpha\right\rangle - \left|-\alpha\right\rangle$$

Resource for quantum teleportation

Time domain EPR correlation

Schrödinger cat $\frac{1}{\sqrt{2}}(|1.5\rangle - |-1.5\rangle)$



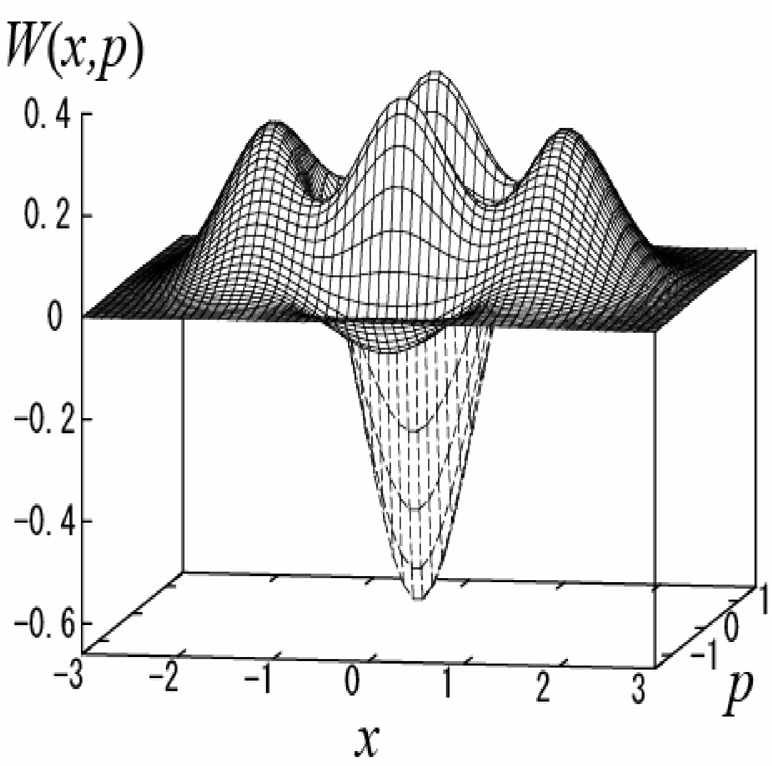
S. L. Braunstein & H. J. Kimble, PRL **80**, 869 (1998).

Creation of Schrödinger cat state with photon subtraction

K. Wakui, H. Takahashi, A. Furusawa, & M. Sasaki, CQIQCII-2006

Schrödinger cat state

$$|\alpha\rangle - |-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

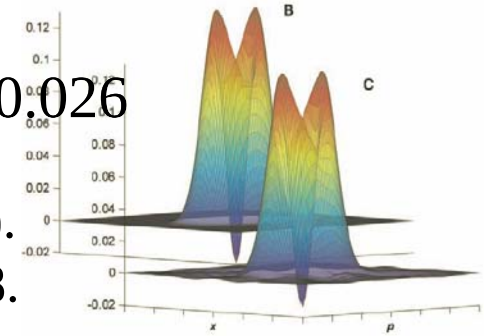


$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

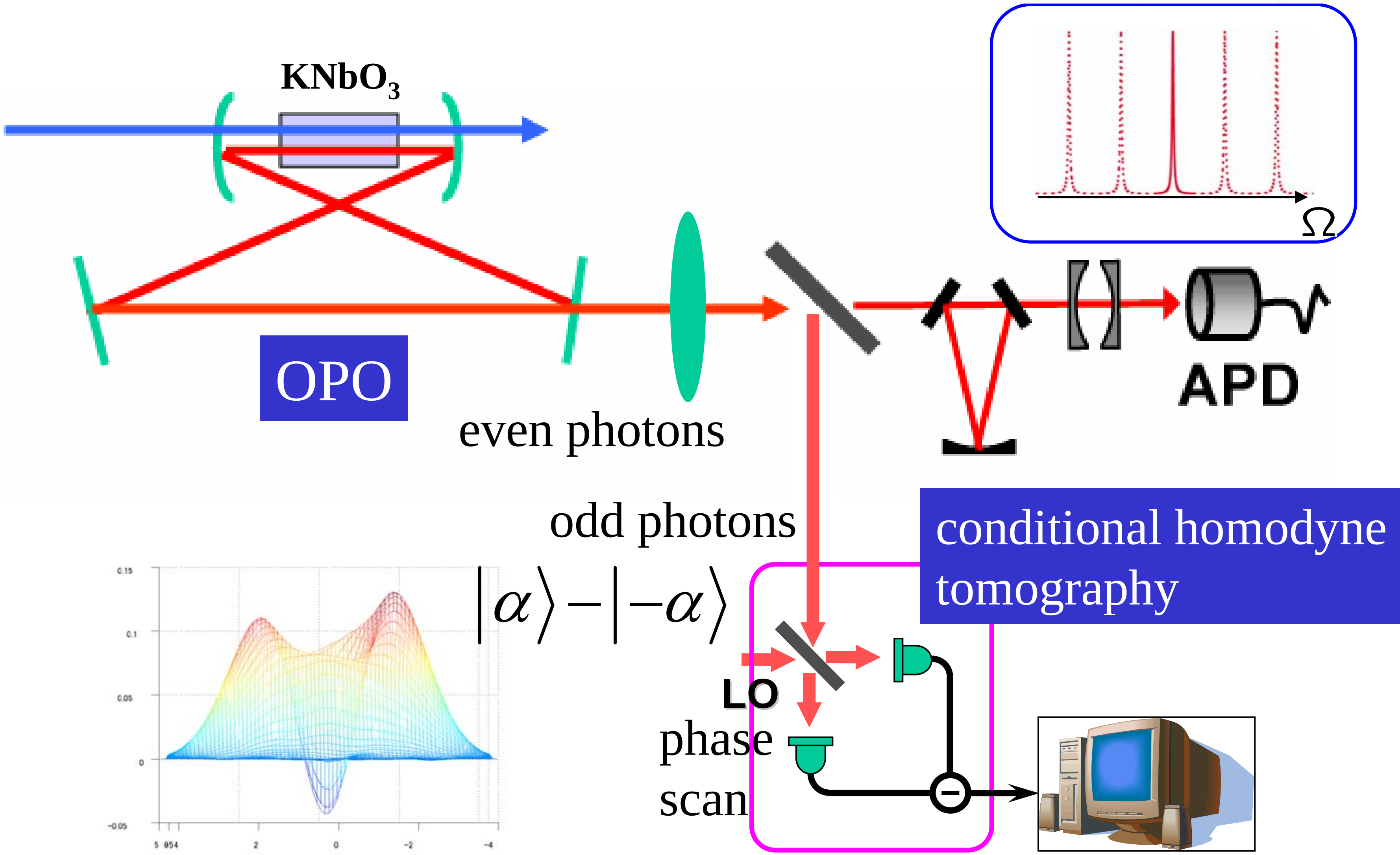
$$|-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}} |n\rangle$$

Photon subtraction

$W(0,0) = -0.026$

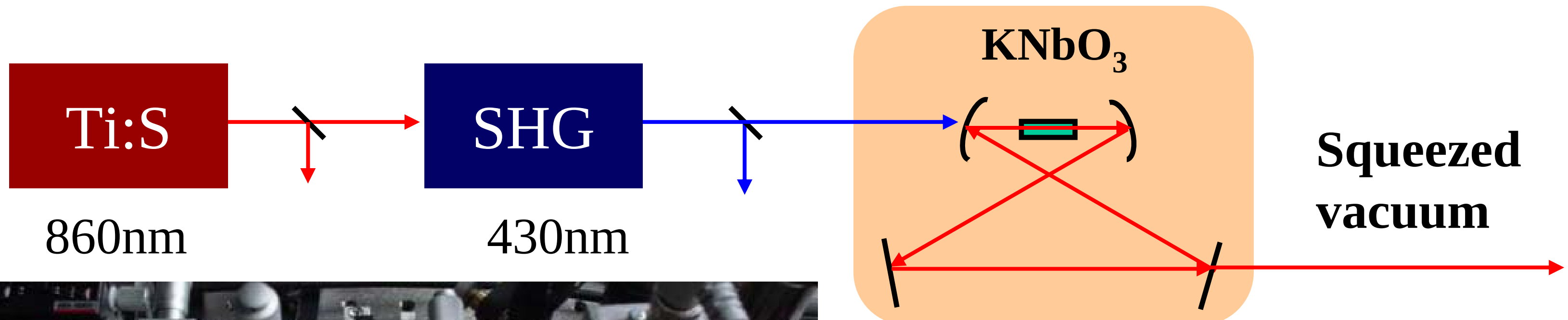


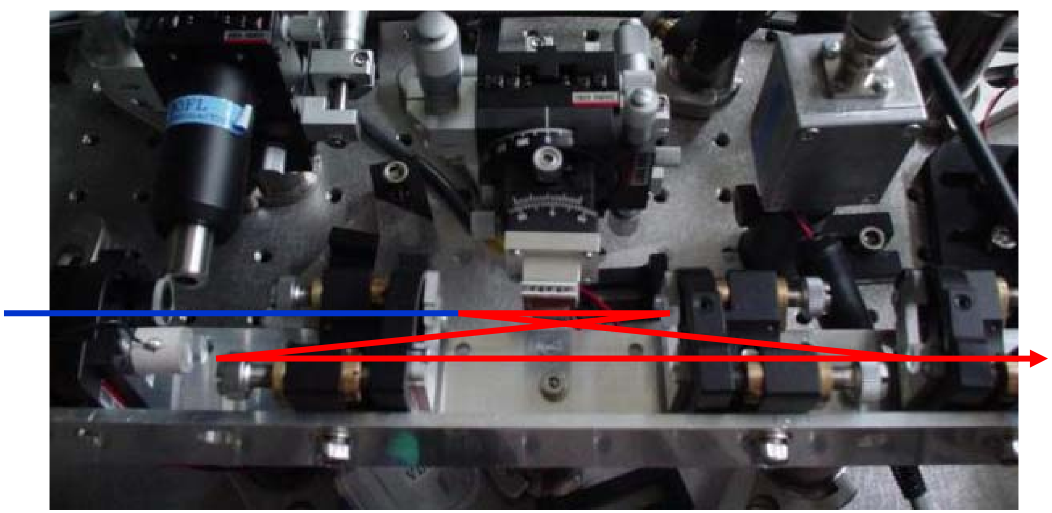
Pulsed light: A. Ourjoumtsev et al., Science **312**, 83 (2006).
CW light: J. S. Neergaard-Nielsen et al., quant-ph/0602198.

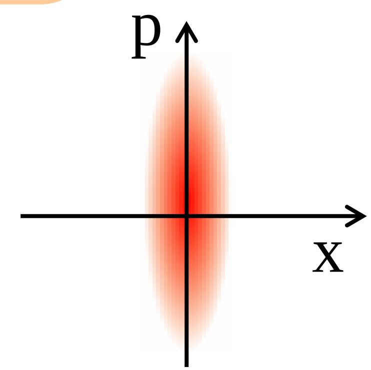


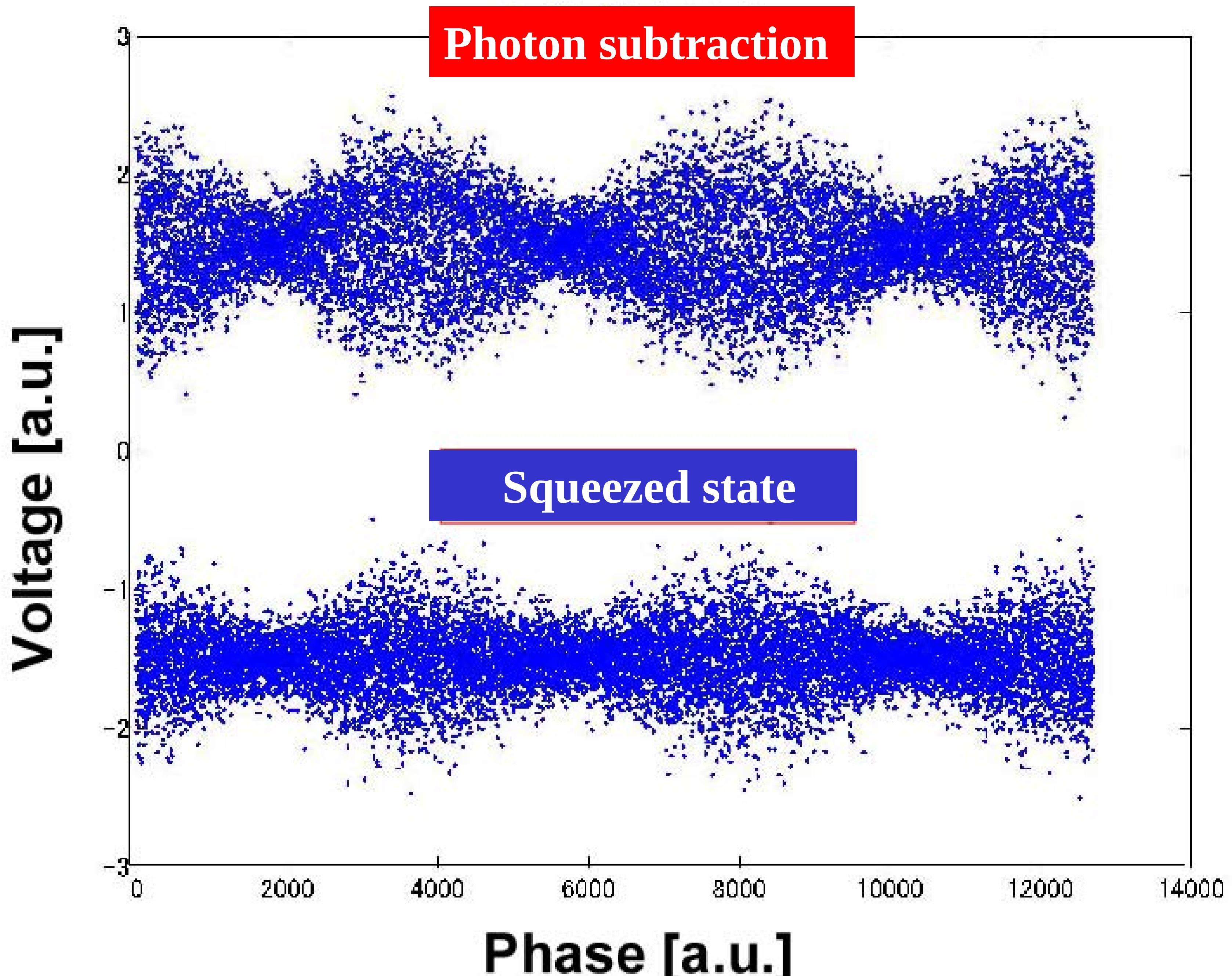
Optical Parametric Oscillator

- $KNbO_3$
- Type-I non-critical phase-matching
- Output coupler : T $\simeq$ 15%



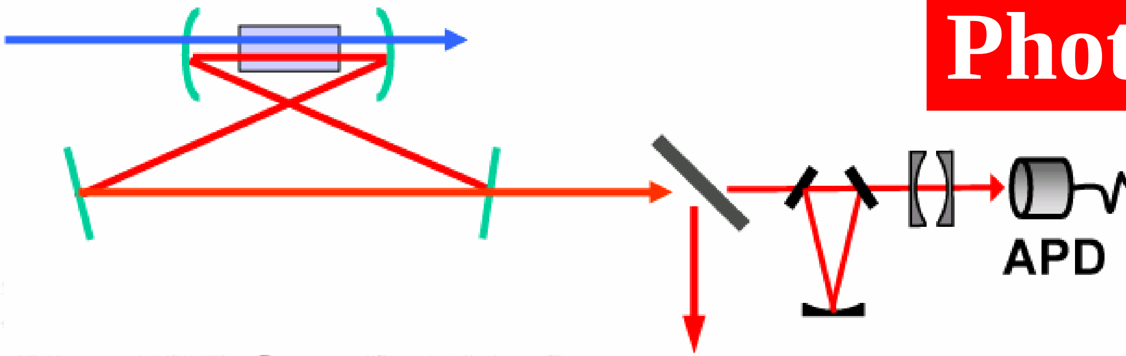


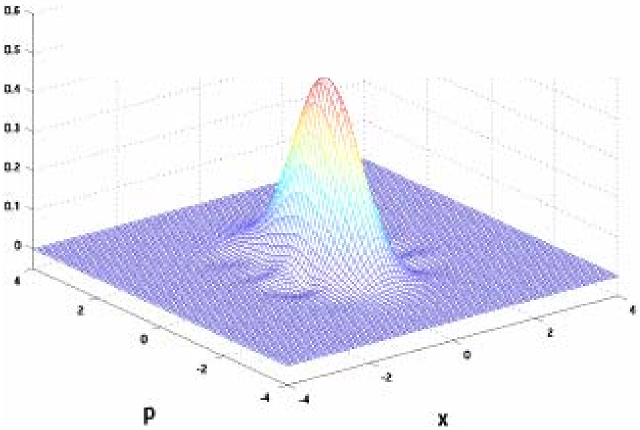




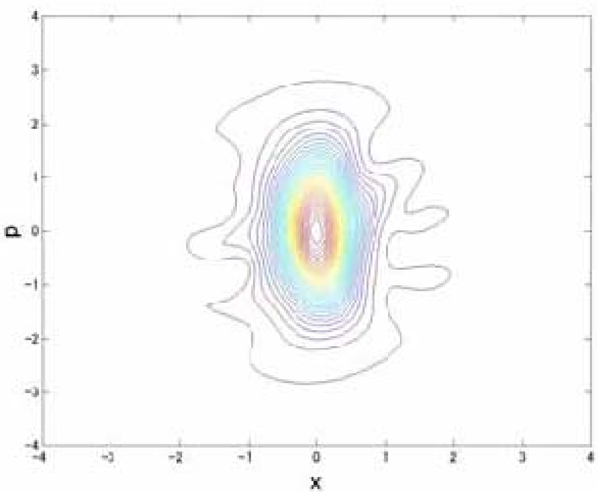
Photon subtraction
Squeezed state
Voltage [a.u.]
Phase [a.u.]
3
2
1
0
-1
-2
-3
0
2000
4000
6000
8000
10000
12000
14000

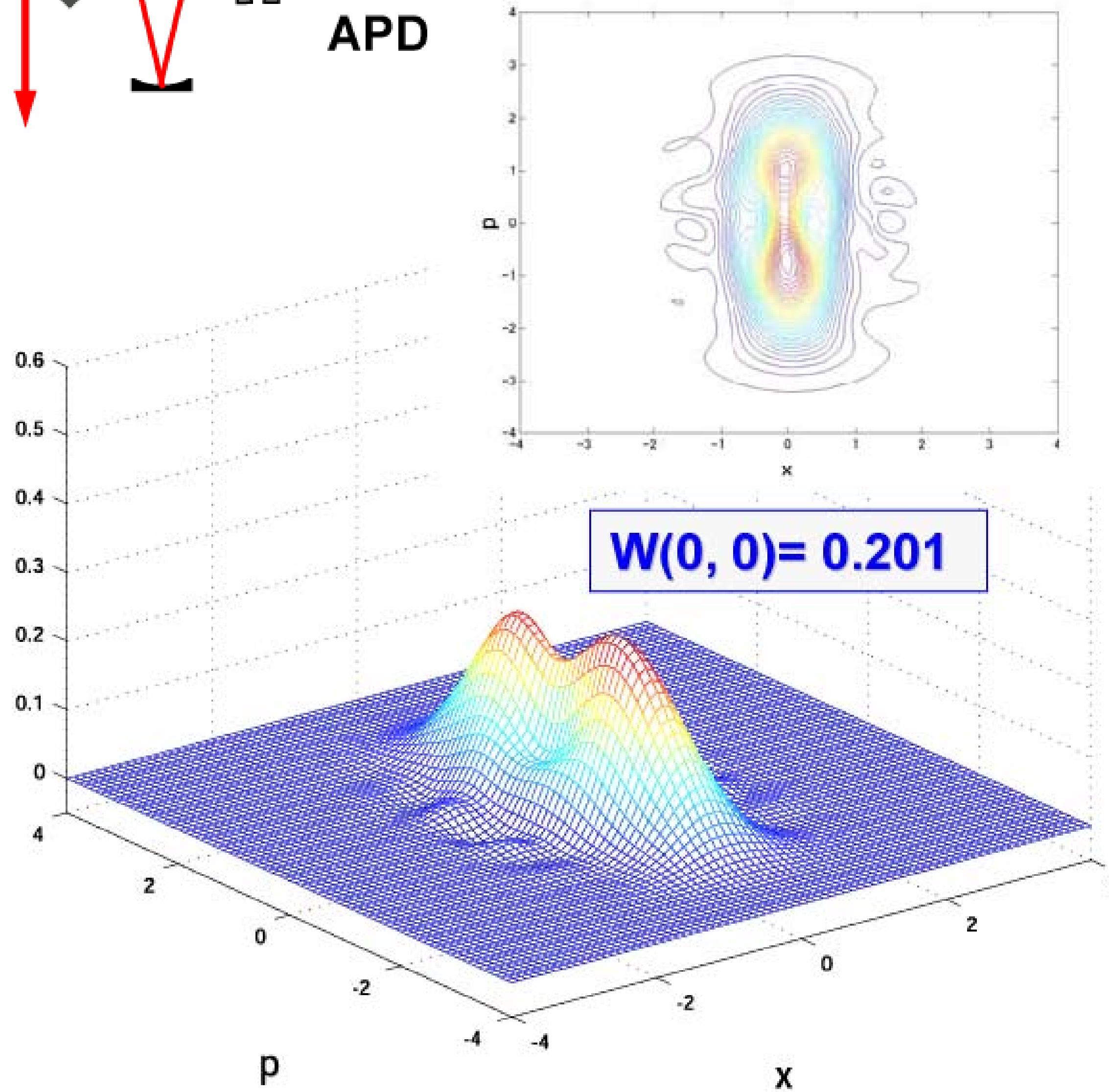
Photon subtraction



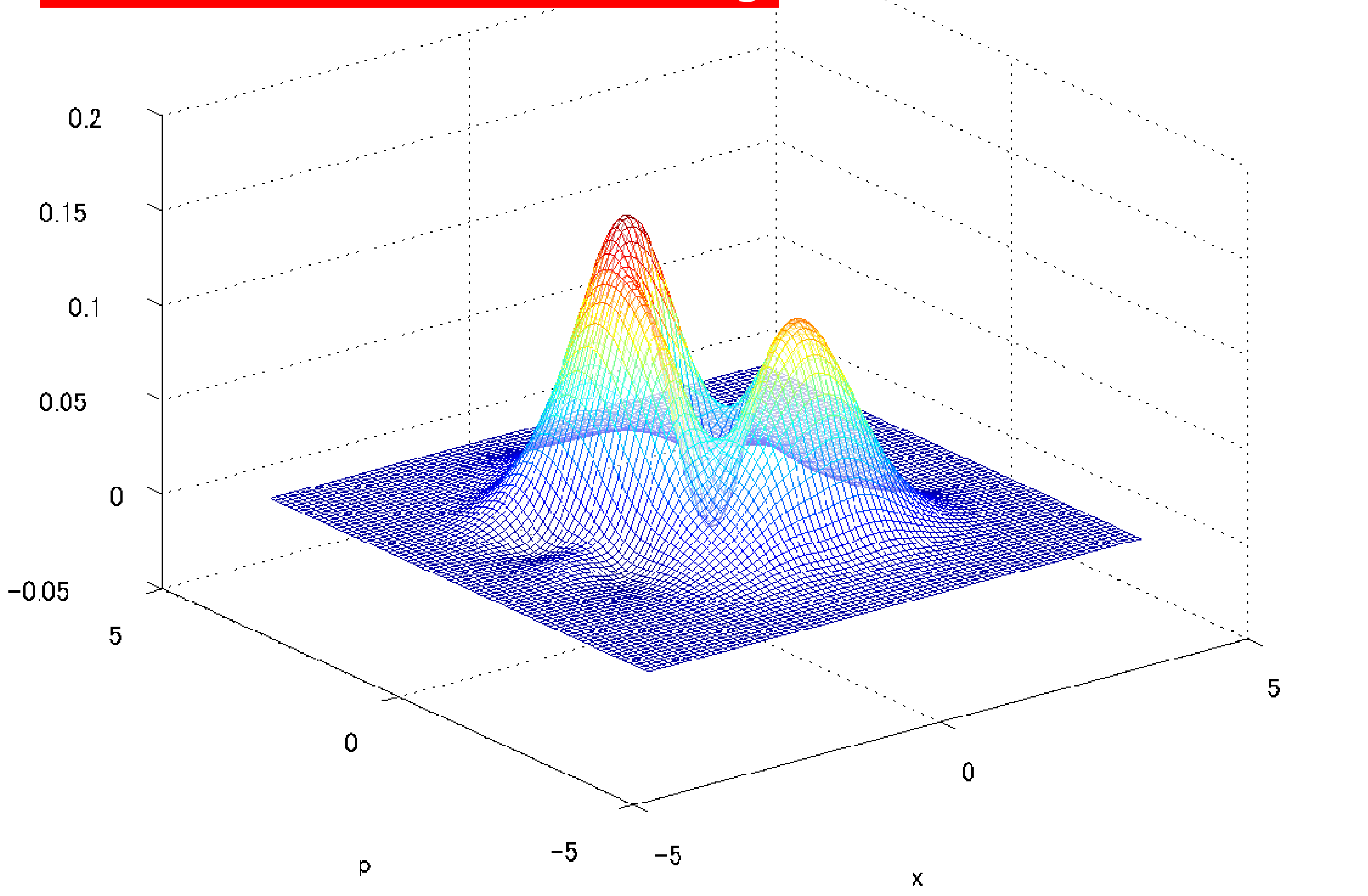


Squeezed state

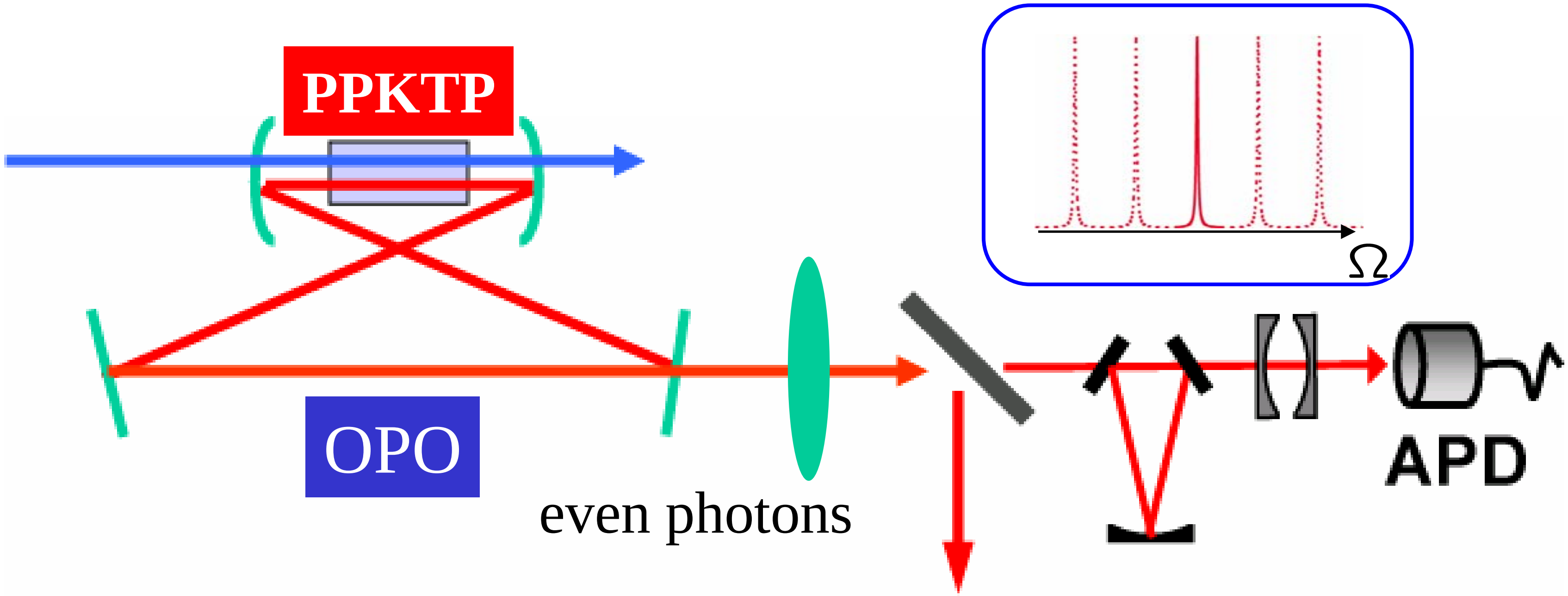


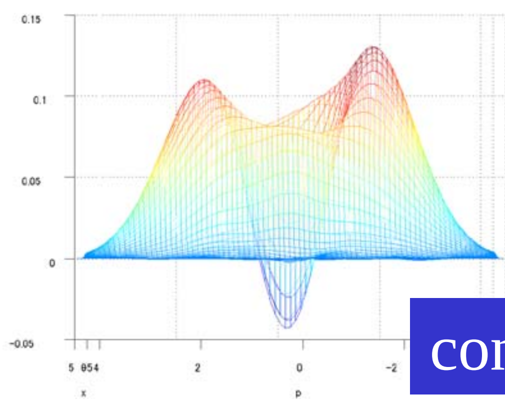


Best result with $KNbO_3$ without any correction



Photon subtraction





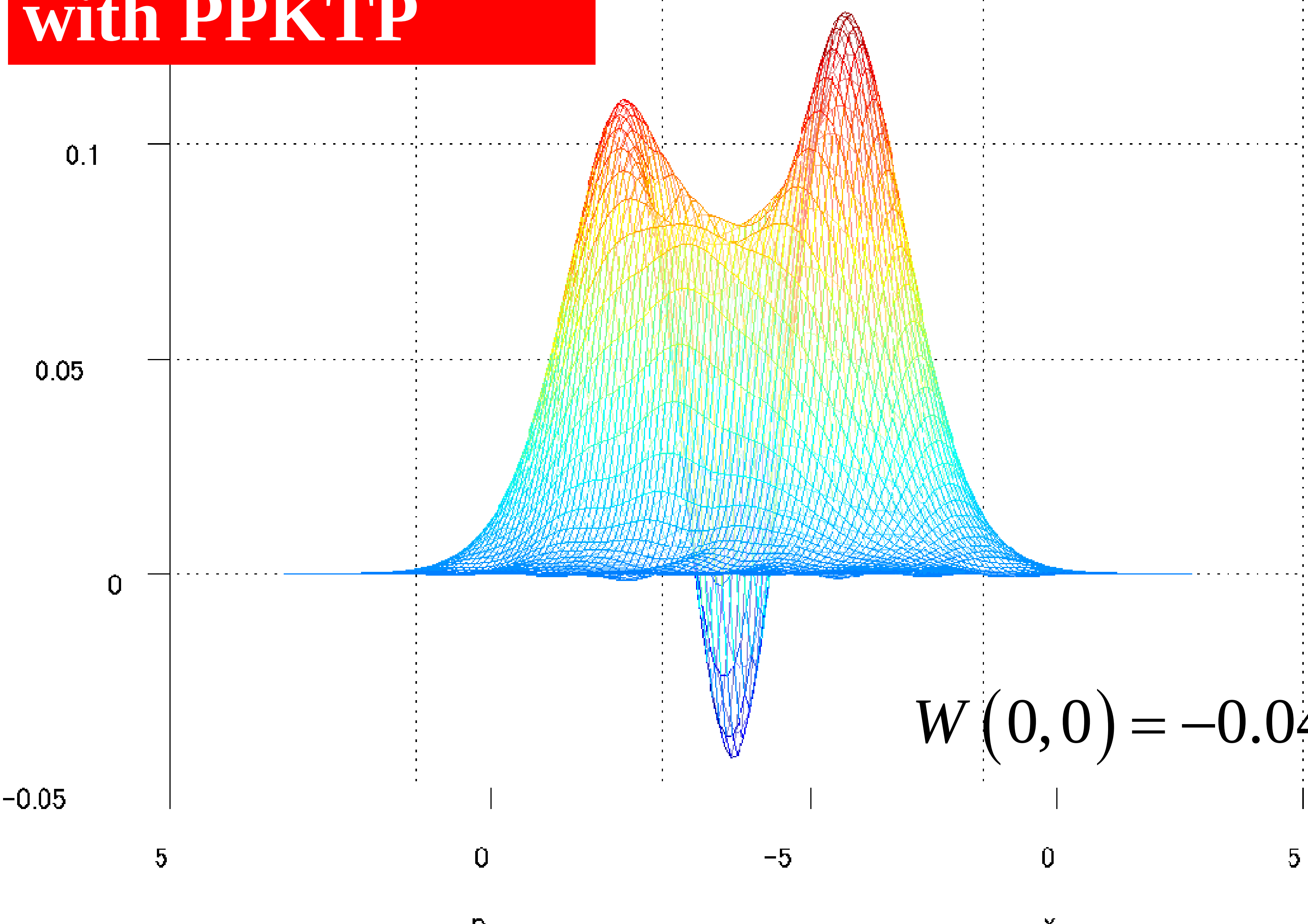
$$|\alpha\rangle - |-\alpha\rangle$$

odd photons

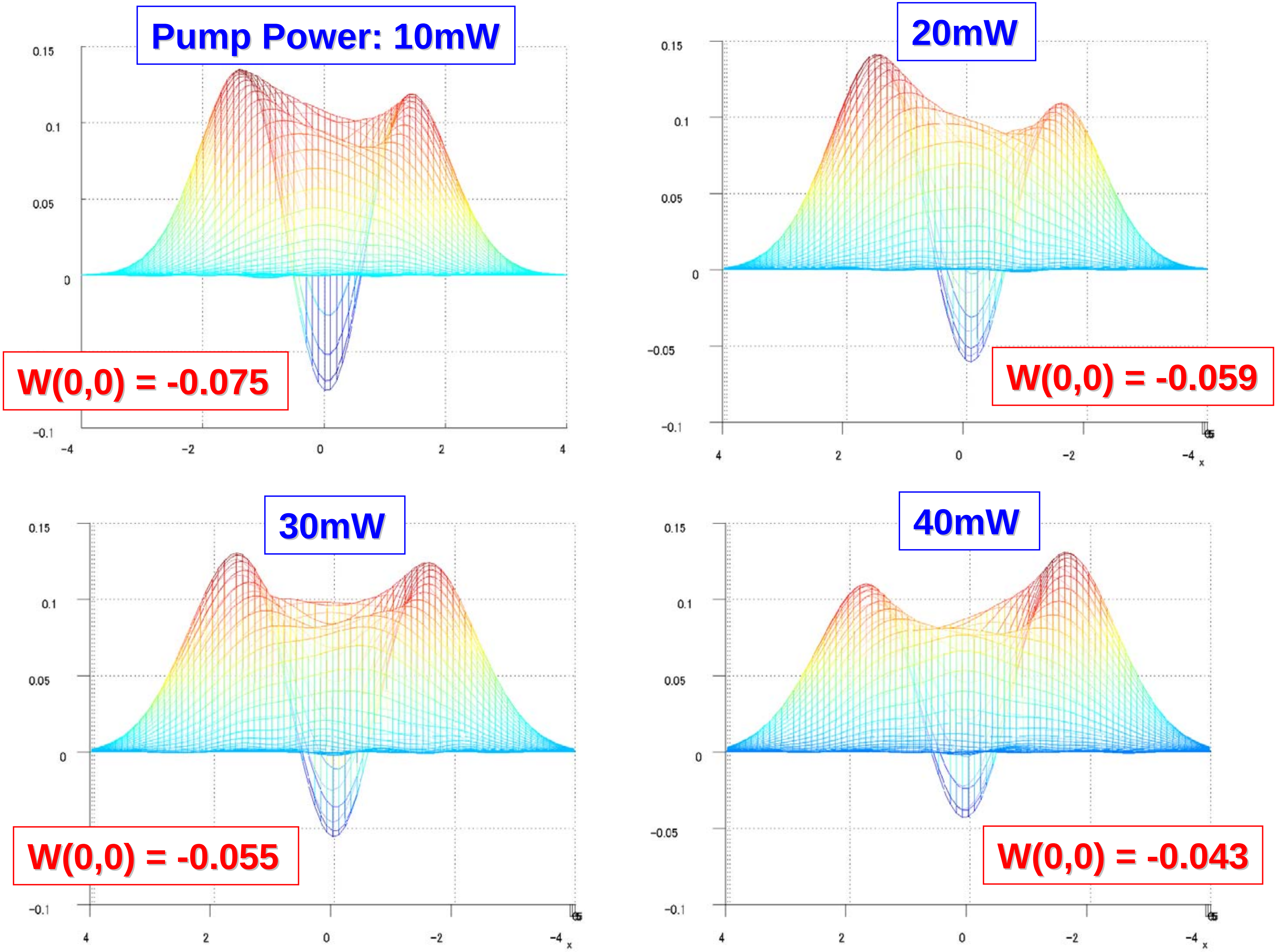
conditional homodyne tomography

One of the results with PPKTP

without any correction



$$W(0,0) = -0.043$$


Pump Power: 10mW
W(0,0) = -0.075
20mW
W(0,0) = -0.059
30mW
W(0,0) = -0.055
40mW
W(0,0) = -0.043

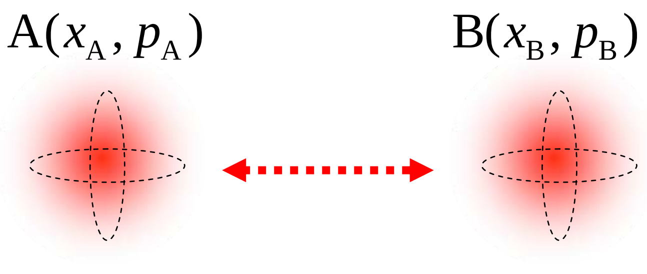
Time domain Einstein-Podolsky-Rosen (EPR) correlation

N. Takei, N. Lee, D. Moriyama, J. S. Neergaard-Nielsen, & A. Furusawa, quant-ph/0607091

Time-domain EPR correlation

$$|\mathrm{EPR}\rangle \propto \int dx\, |x\rangle_\mathrm{A} |x\rangle_\mathrm{B}$$

$$\begin{cases} x_\mathrm{A} - x_\mathrm{B} = 0 \\ p_\mathrm{A} + p_\mathrm{B} = 0 \end{cases}$$



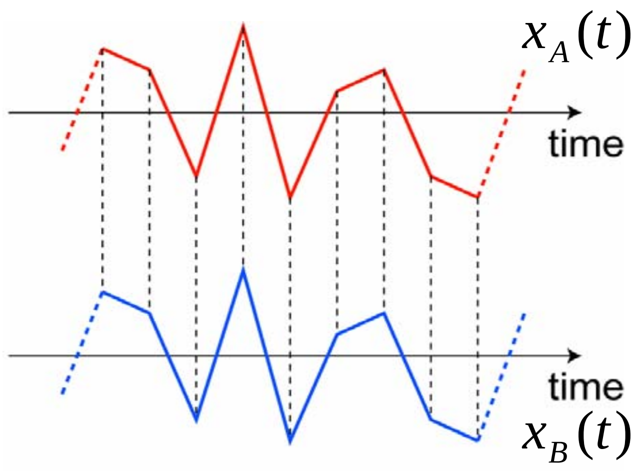
Simultaneous eigenstates of

$(\hat{x}_A - \hat{x}_B)$ & $(\hat{p}_A + \hat{p}_B)$

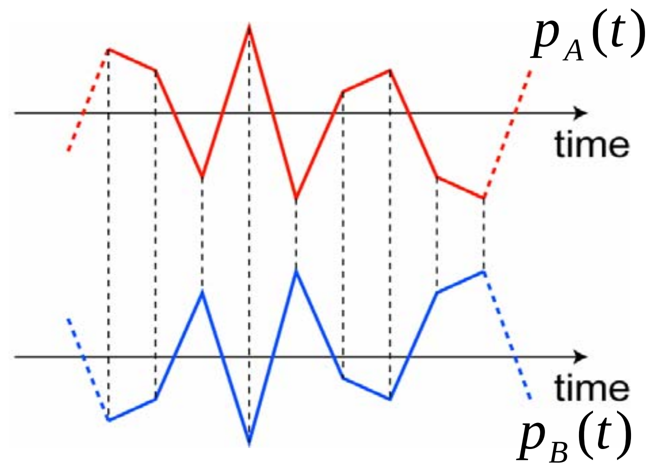
$$[\hat{x}_\mathrm{A} - \hat{x}_\mathrm{B}, \hat{p}_\mathrm{A} + \hat{p}_\mathrm{B}] = 0$$

EPR beams in quantum optics

x measurements



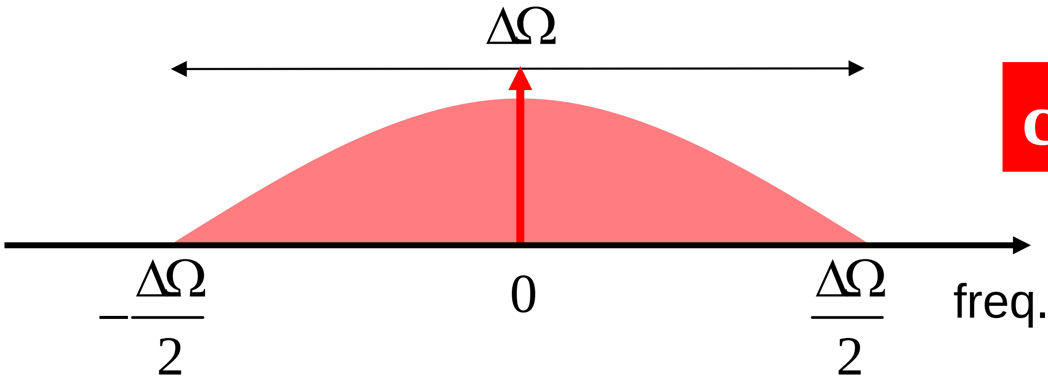
p measurements



Mode matching to photon counting

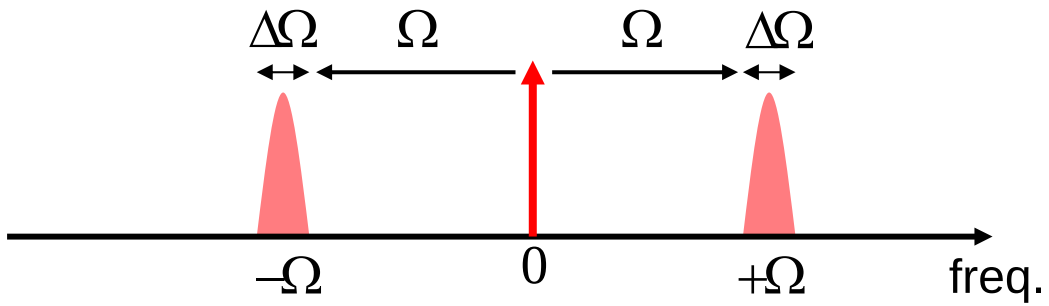
- Broad band

Time resolution $\Delta T \approx$ 1/bandwidth



cavity bandwidth

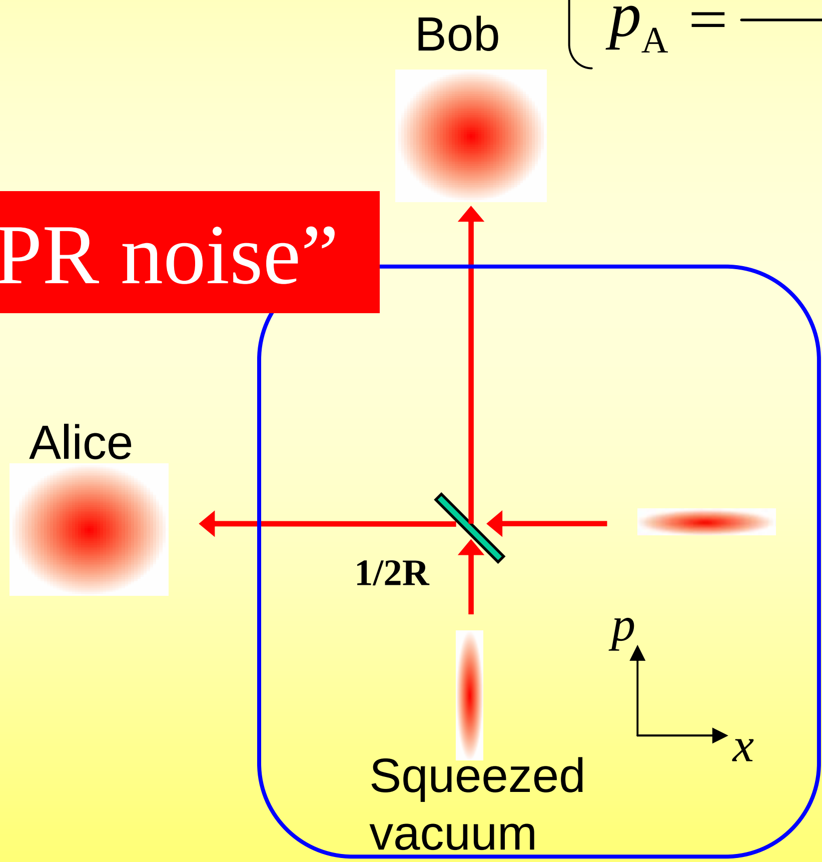
- Ordinary teleportation experiment: side band



Generation of EPR beams

$$\begin{cases} \hat{x}_A = \dfrac{e^{r}\hat{x}_1^{(0)} + e^{-r}\hat{x}_2^{(0)}}{\sqrt{2}} \\ \hat{p}_A = \dfrac{e^{-r}\hat{p}_1^{(0)} + e^{r}\hat{p}_2^{(0)}}{\sqrt{2}} \end{cases} \qquad \begin{cases} \hat{x}_B = \dfrac{e^{r}\hat{x}_1^{(0)} - e^{-r}\hat{x}_2^{(0)}}{\sqrt{2}} \\ \hat{p}_B = \dfrac{e^{-r}\hat{p}_1^{(0)} - e^{r}\hat{p}_2^{(0)}}{\sqrt{2}} \end{cases}$$

“EPR noise”



“EPR correlation”

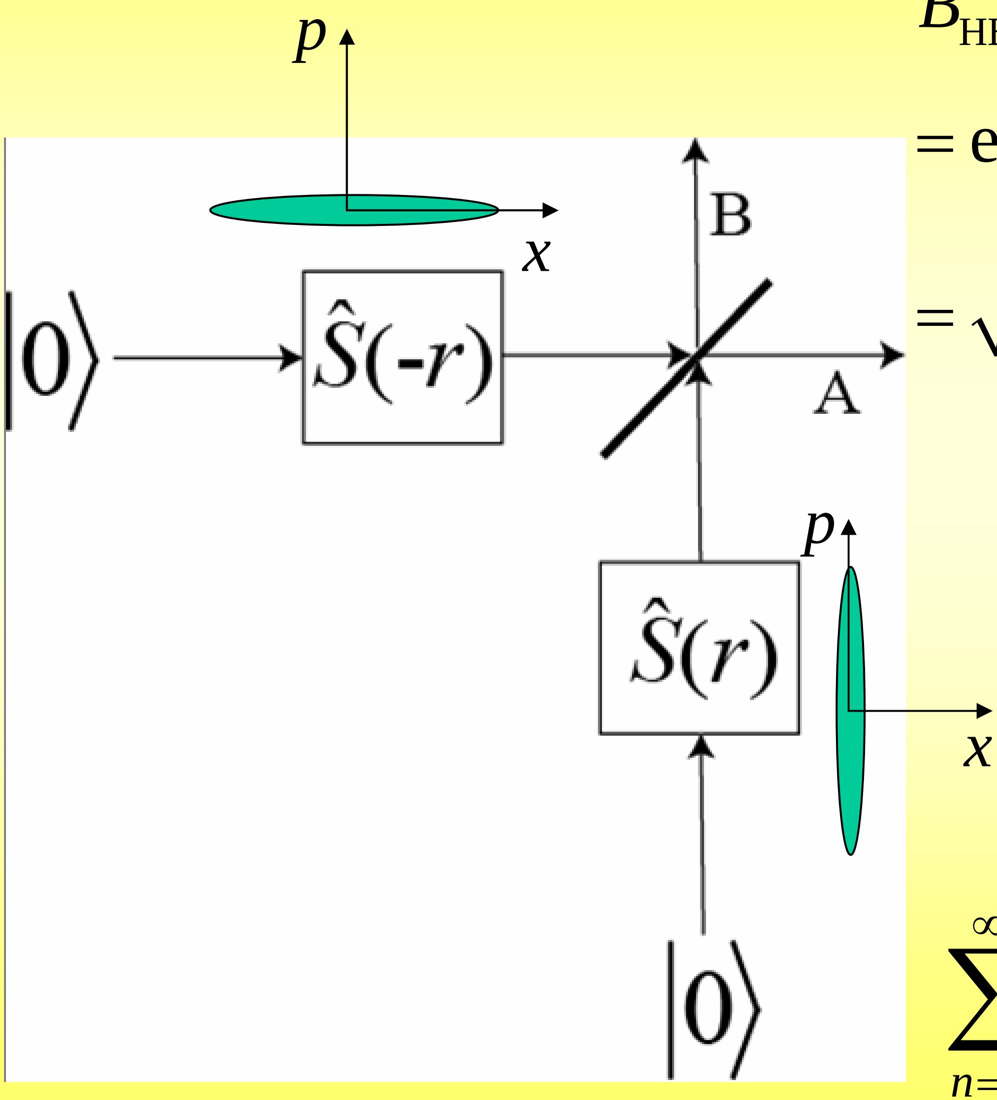
$$\hat{x}_A - \hat{x}_B = \sqrt{2}e^{-r}\hat{x}_1^{(0)}$$

$$\hat{p}_A + \hat{p}_B = \sqrt{2}e^{-r}\hat{p}_2^{(0)}$$

$r \to \infty$

0

Generation of EPR beams



$$\hat{B}_{\mathrm{HBS}}\hat{S}_{\mathrm{A}}(-r)\hat{S}_{\mathrm{B}}(r)|0\rangle_{\mathrm{A}}\otimes|0\rangle_{\mathrm{B}}$$

$$=\exp\left[r\left(\hat{a}_{\mathrm{A}}^{\dagger}\hat{a}_{\mathrm{B}}^{\dagger}-\hat{a}_{\mathrm{A}}\hat{a}_{\mathrm{B}}\right)\right]|0\rangle_{\mathrm{A}}\otimes|0\rangle_{\mathrm{B}}$$

$$=\sqrt{1-q^2}\sum_{n=0}^{\infty}q^n|n\rangle_{\mathrm{A}}\otimes|n\rangle_{\mathrm{B}}$$

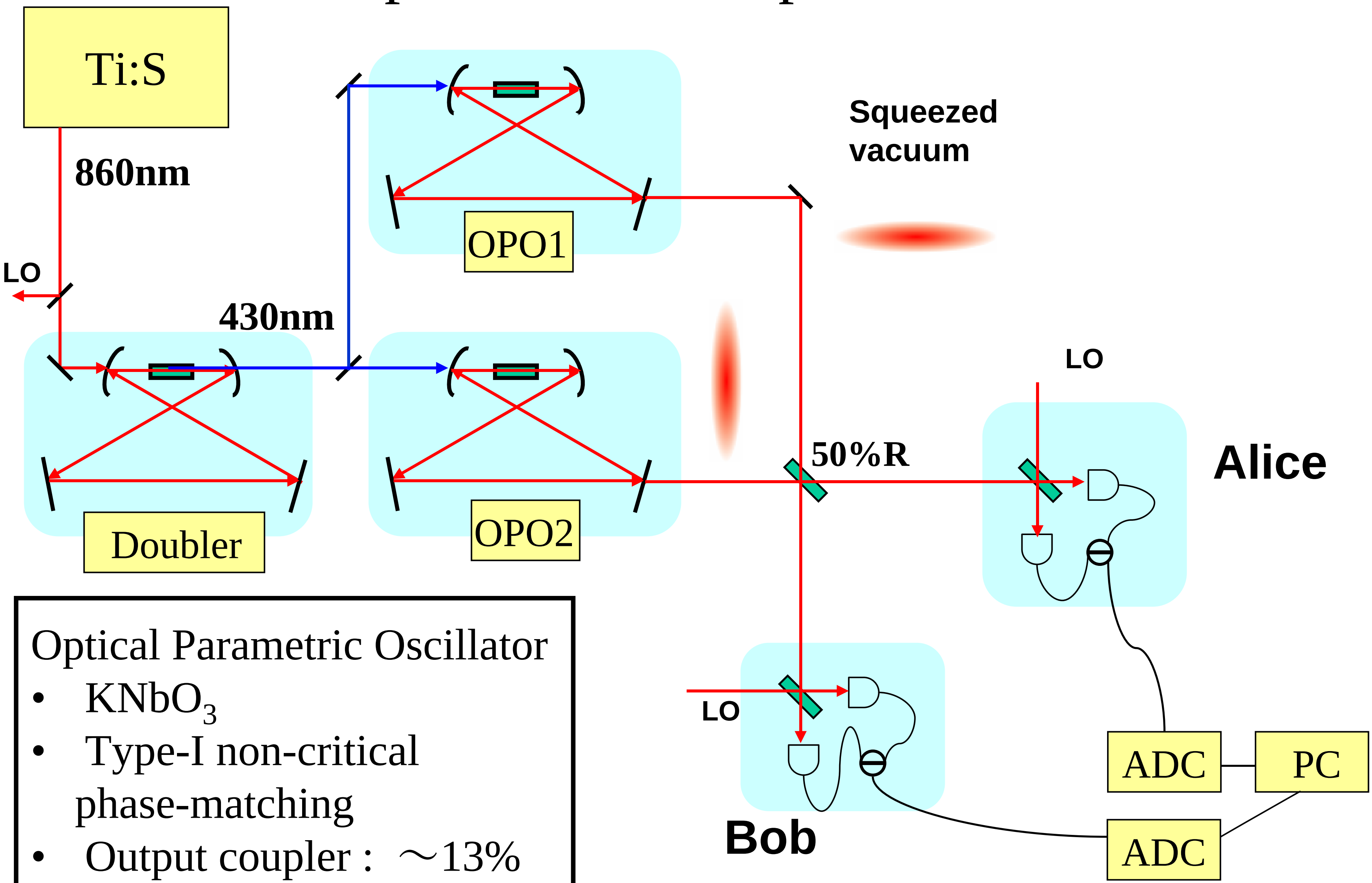
$$q=\tanh r$$

$$\sqrt{1-q^2}\sum_{n=0}^{\infty}q^n|n\rangle_{\mathrm{A}}\otimes|n\rangle_{\mathrm{B}}$$

$$r\to\infty$$

$$\sum_{n=0}^{\infty}|n\rangle_{\mathrm{A}}|n\rangle_{\mathrm{B}}=\int_{-\infty}^{\infty}dx|x\rangle_{\mathrm{A}}|x\rangle_{\mathrm{B}}$$

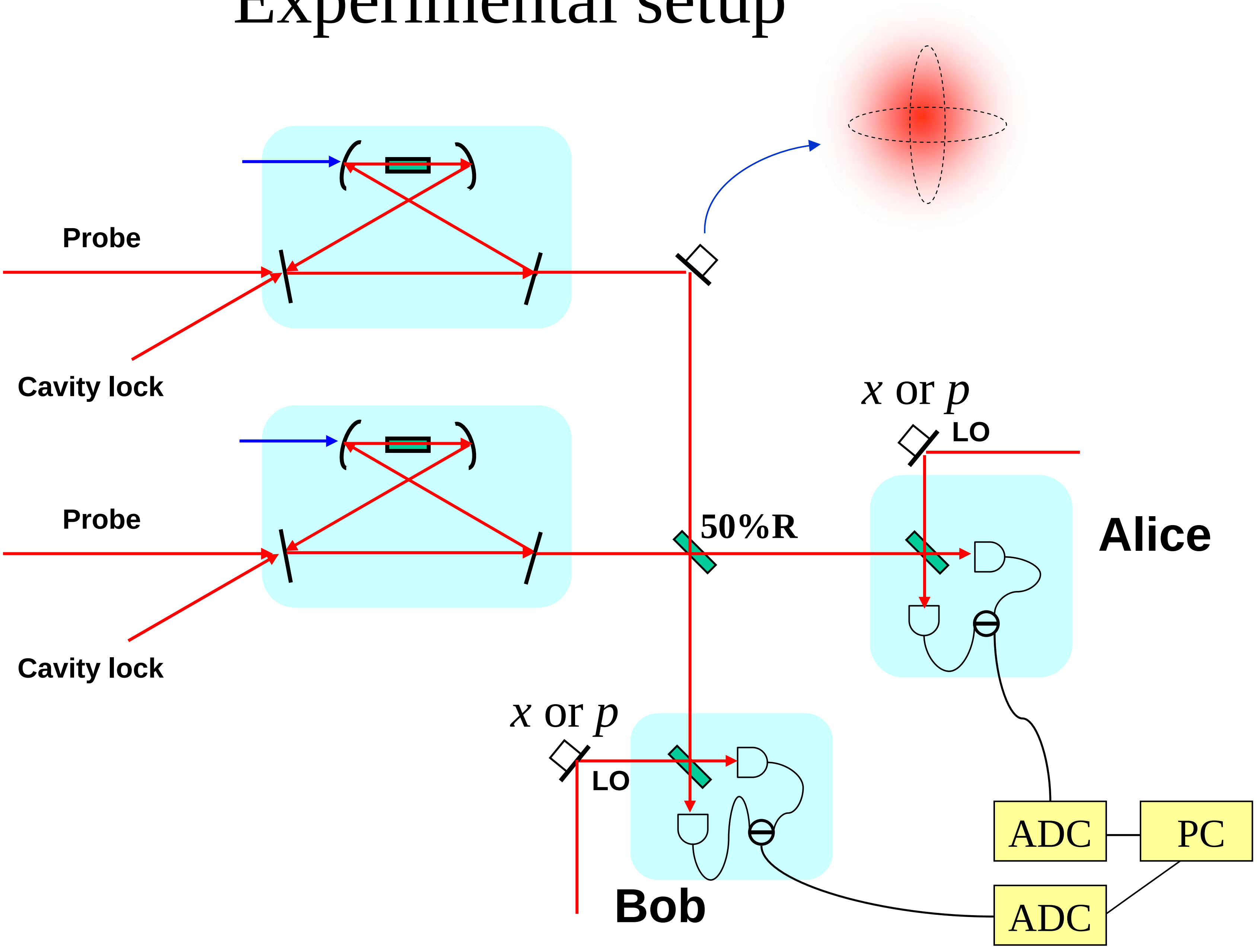
Experimental setup



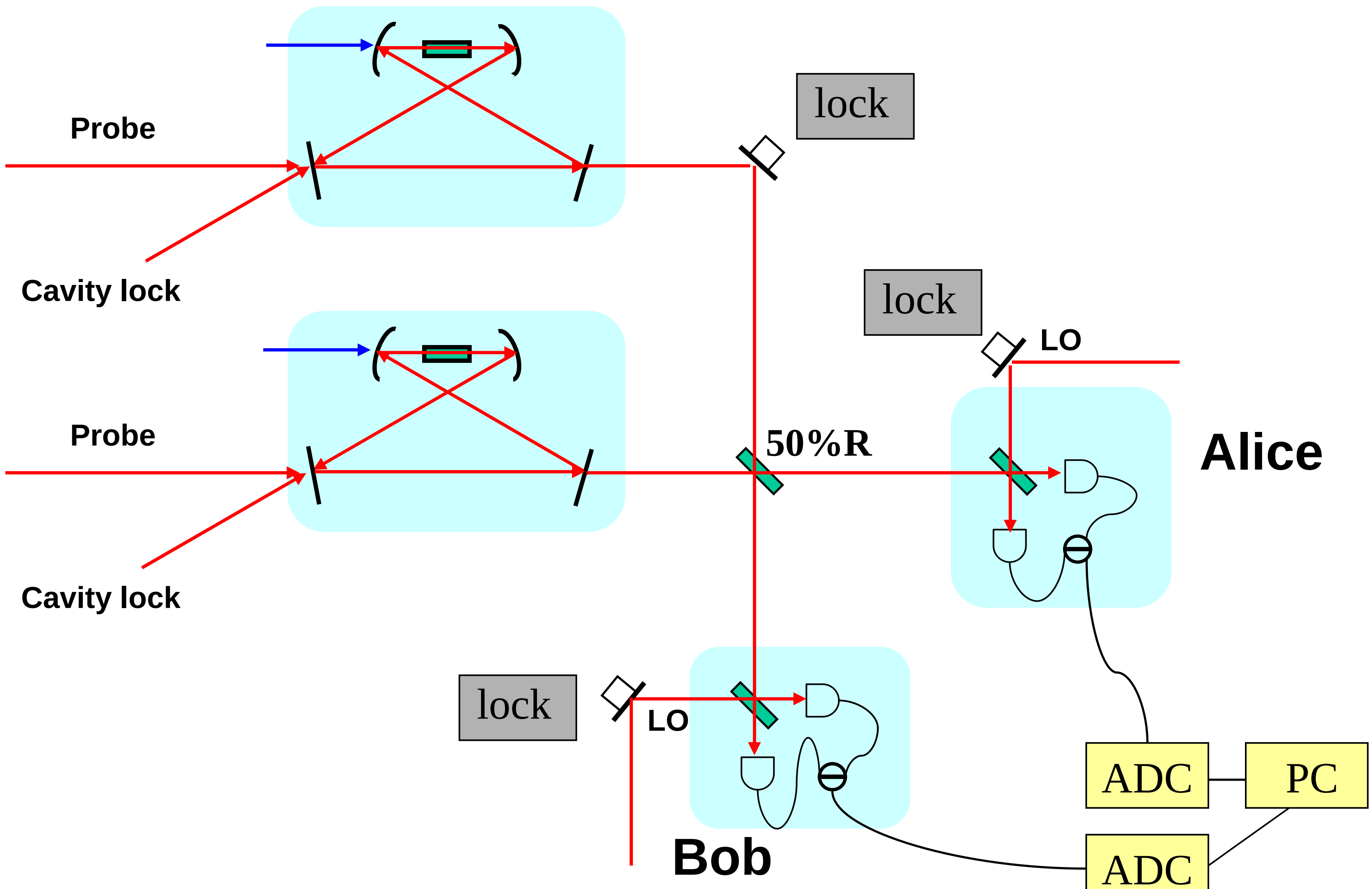
Optical Parametric Oscillator

- $KNbO_3$
- Type-I non-critical phase-matching
- Output coupler : ～13%

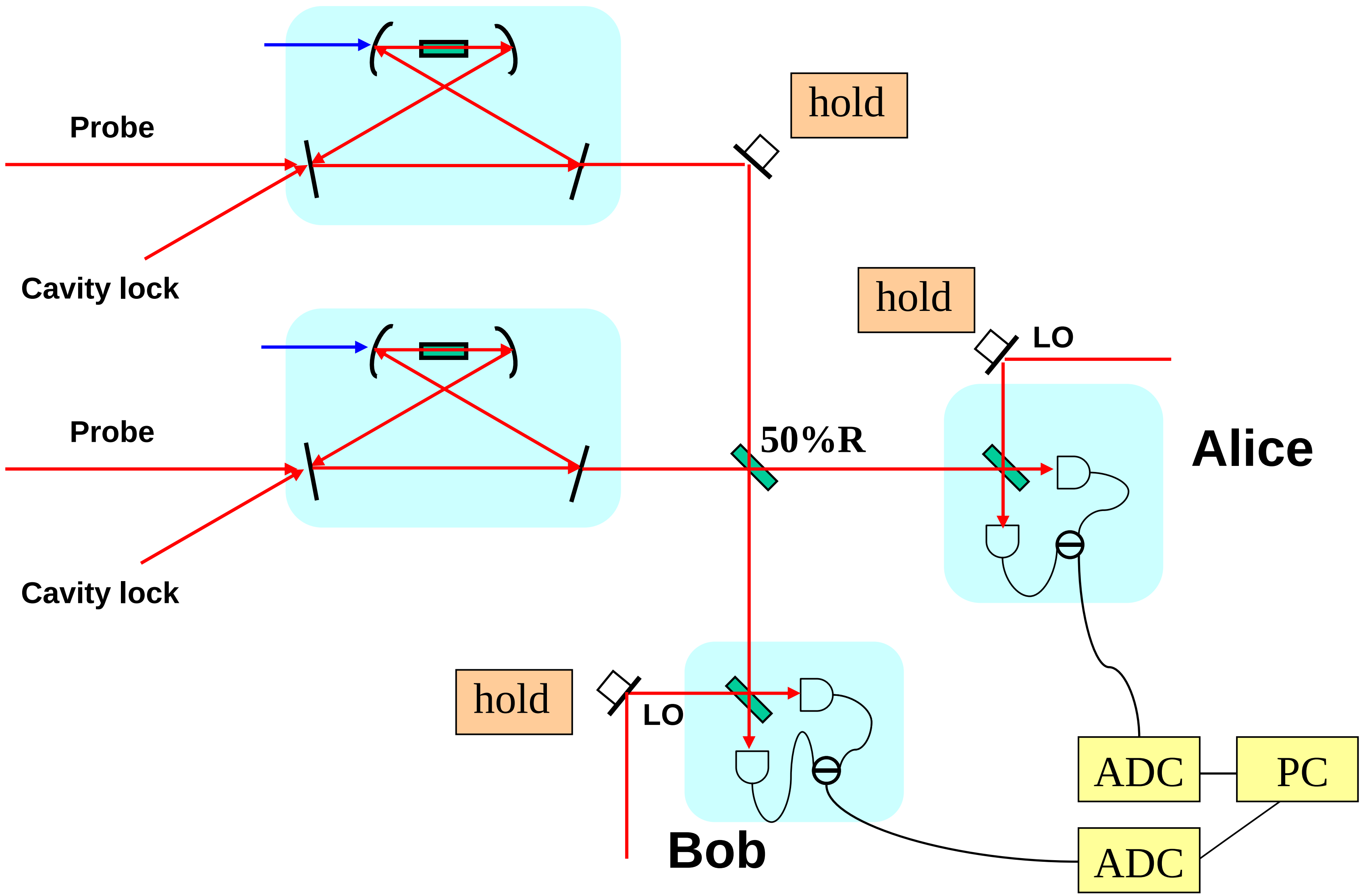
Experimental setup



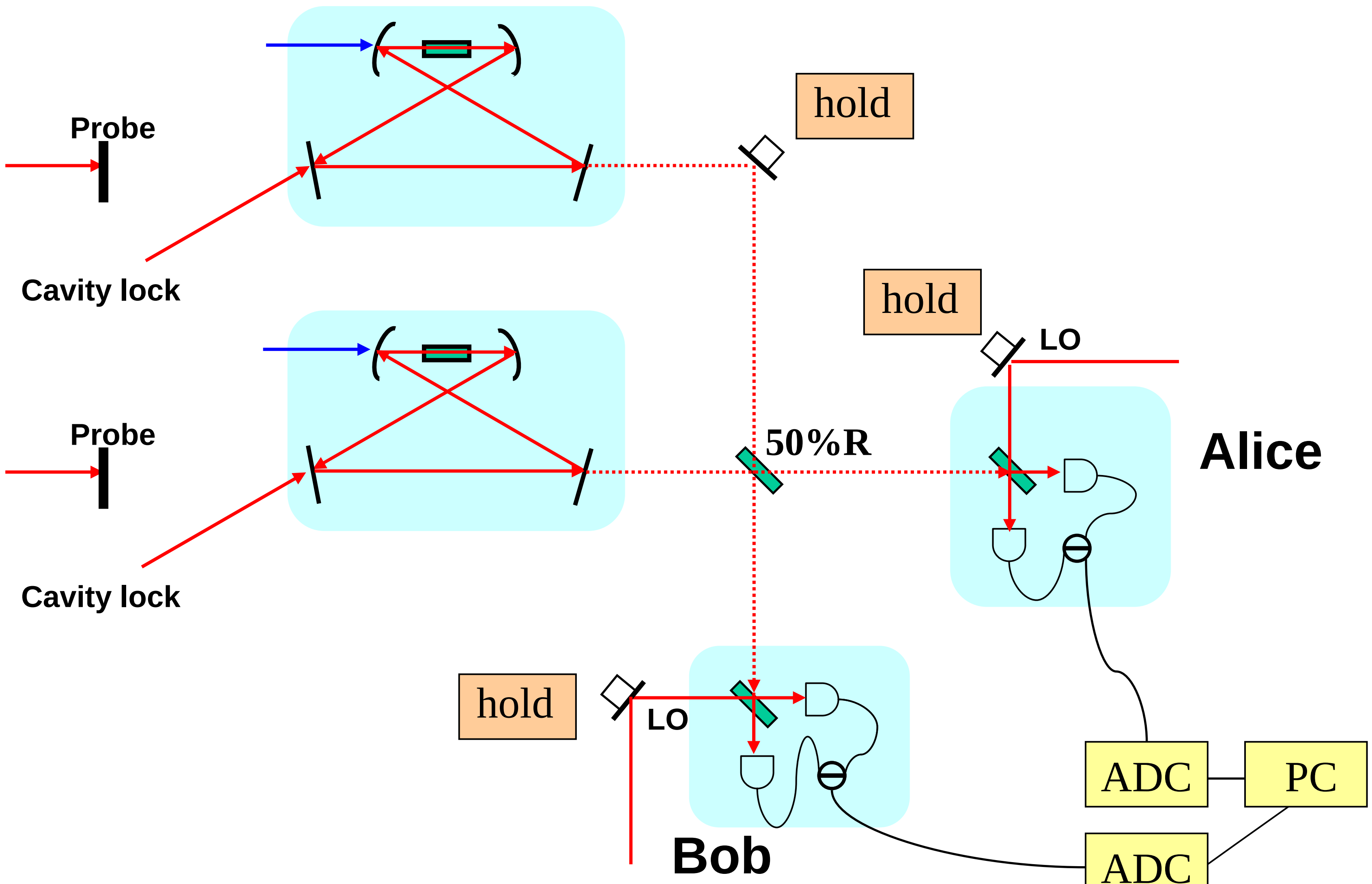
Experimental setup



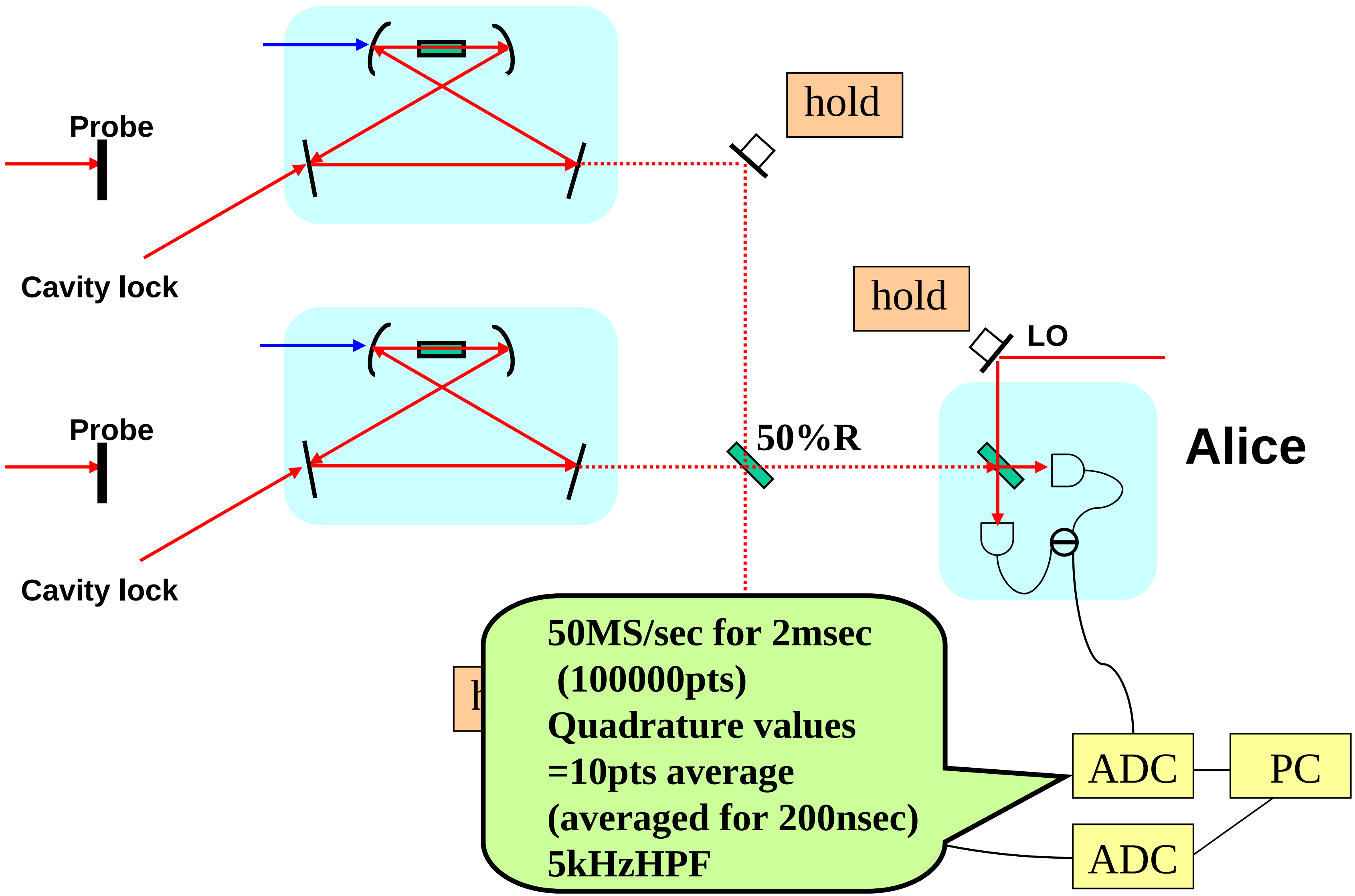
Experimental setup



Experimental setup



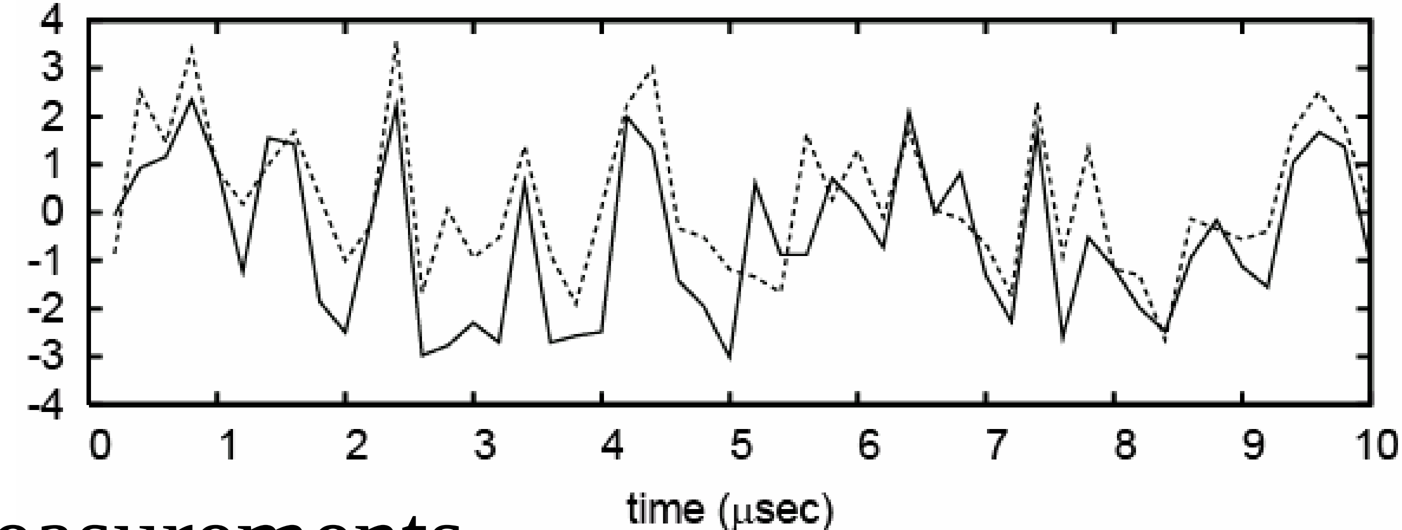
Experimental setup



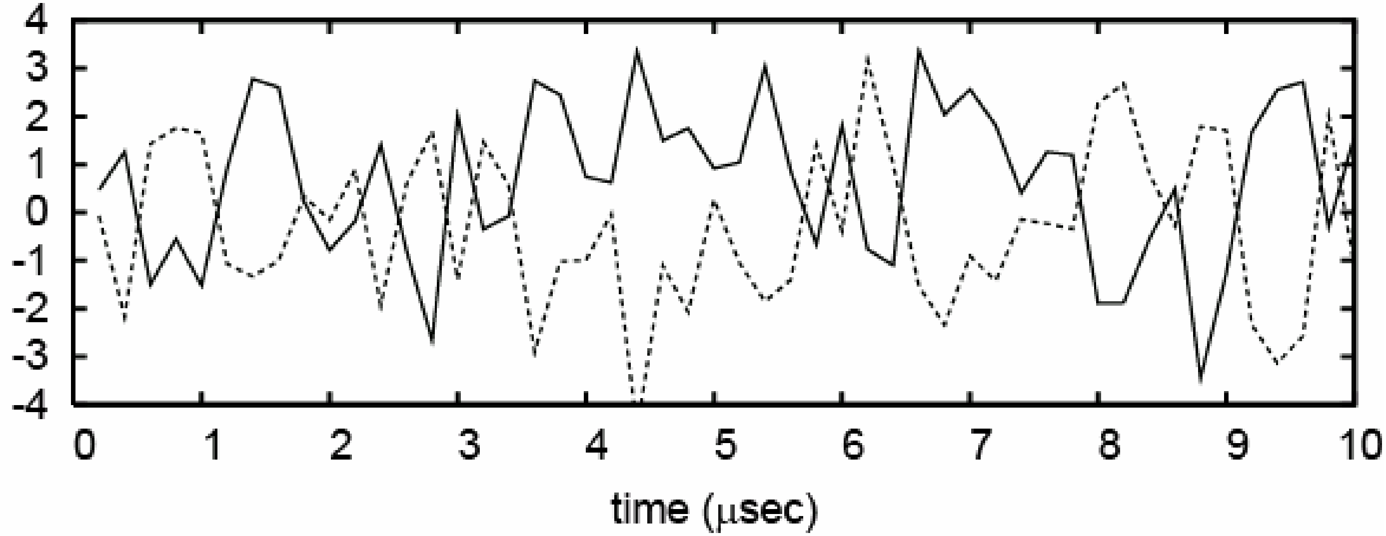
Time domain EPR correlation

Alice ———
Bob ············

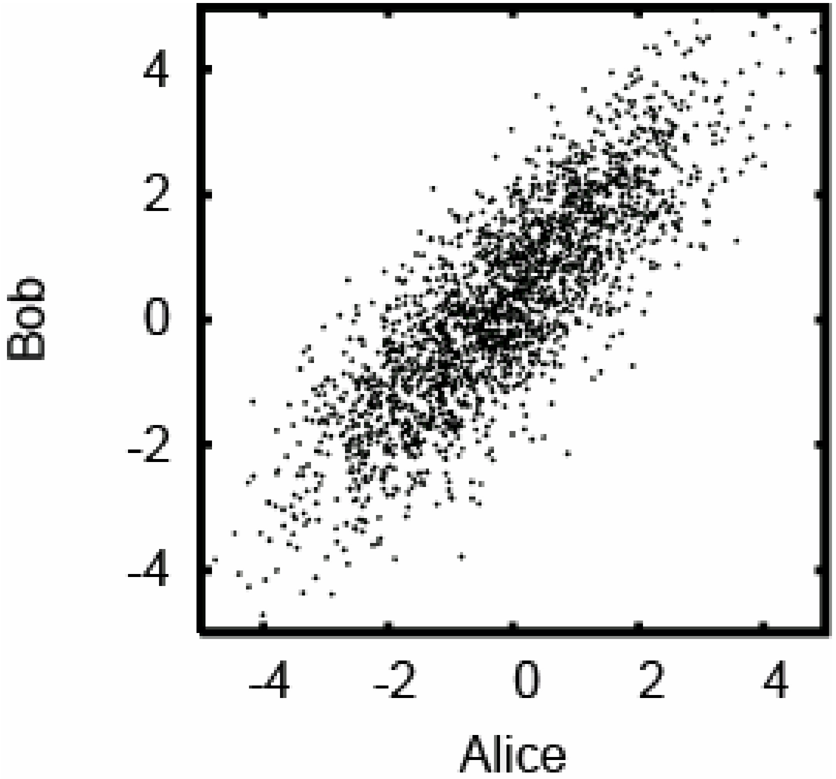
x measurements



p measurements

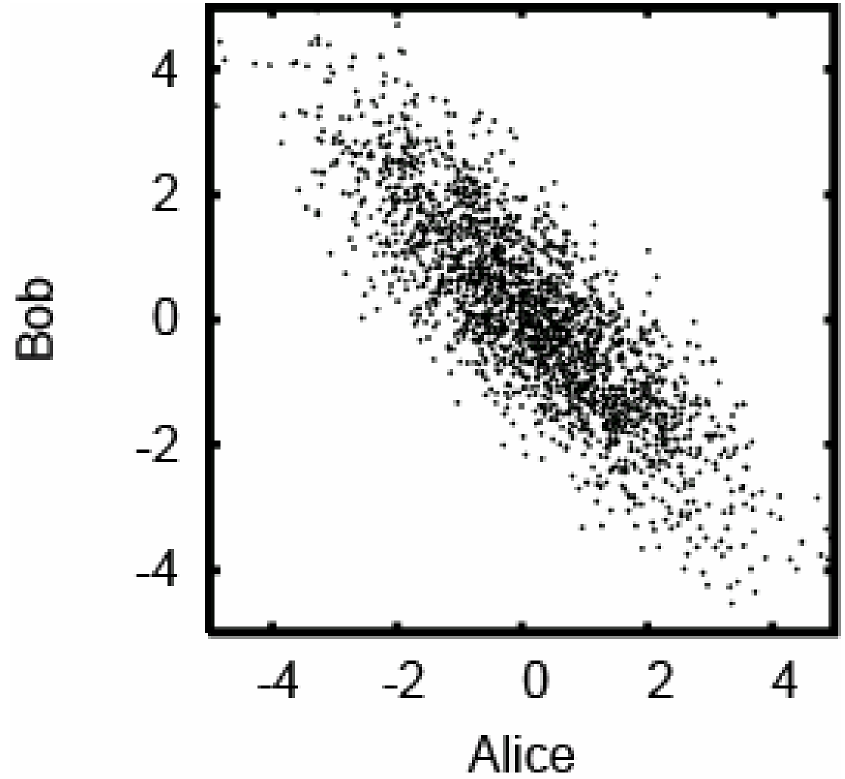


x correlation



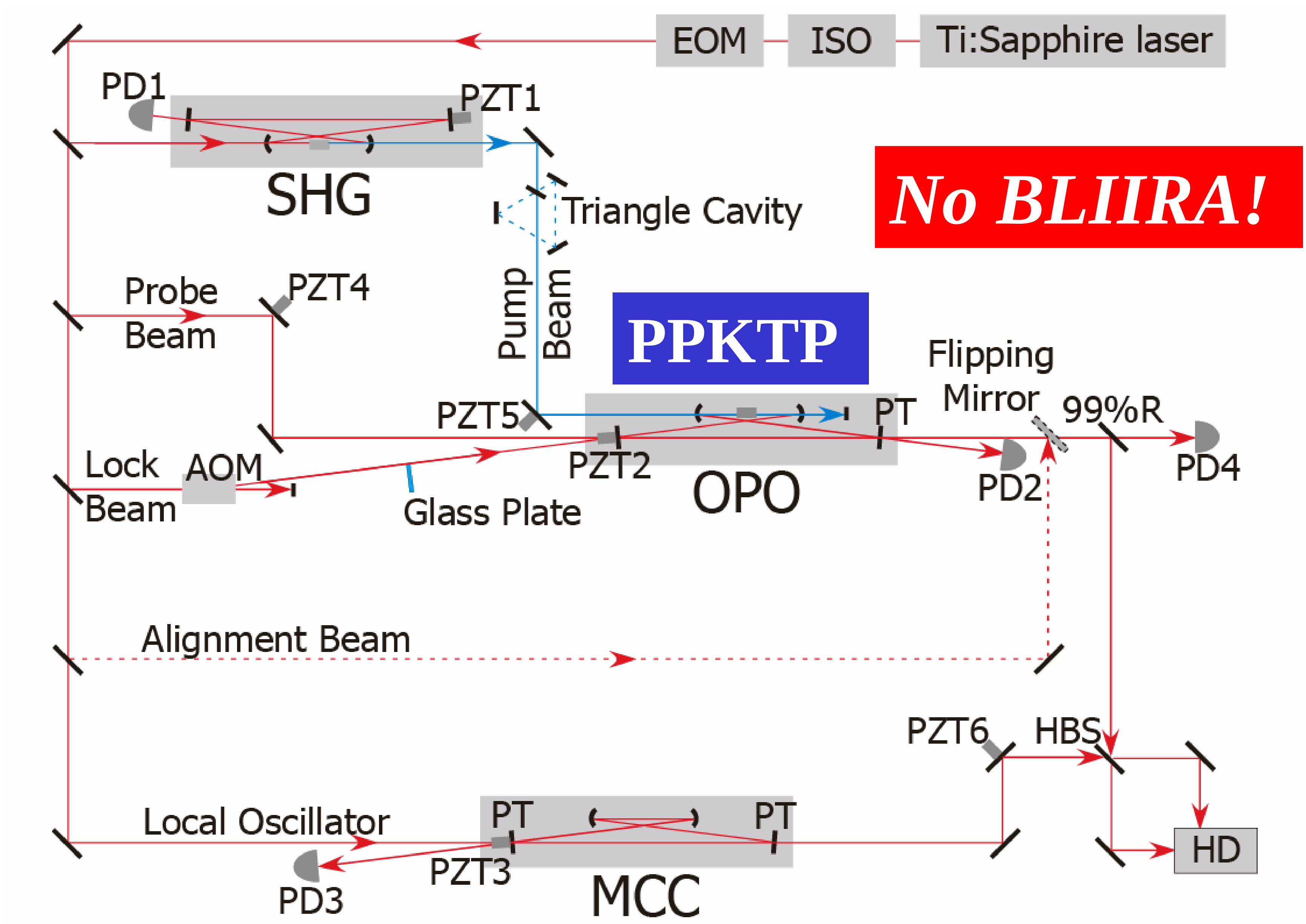
$$\left\langle [\Delta(\hat{x}_A - \hat{x}_B)^2] \right\rangle \approx -3\mathrm{dB}$$

p correlation

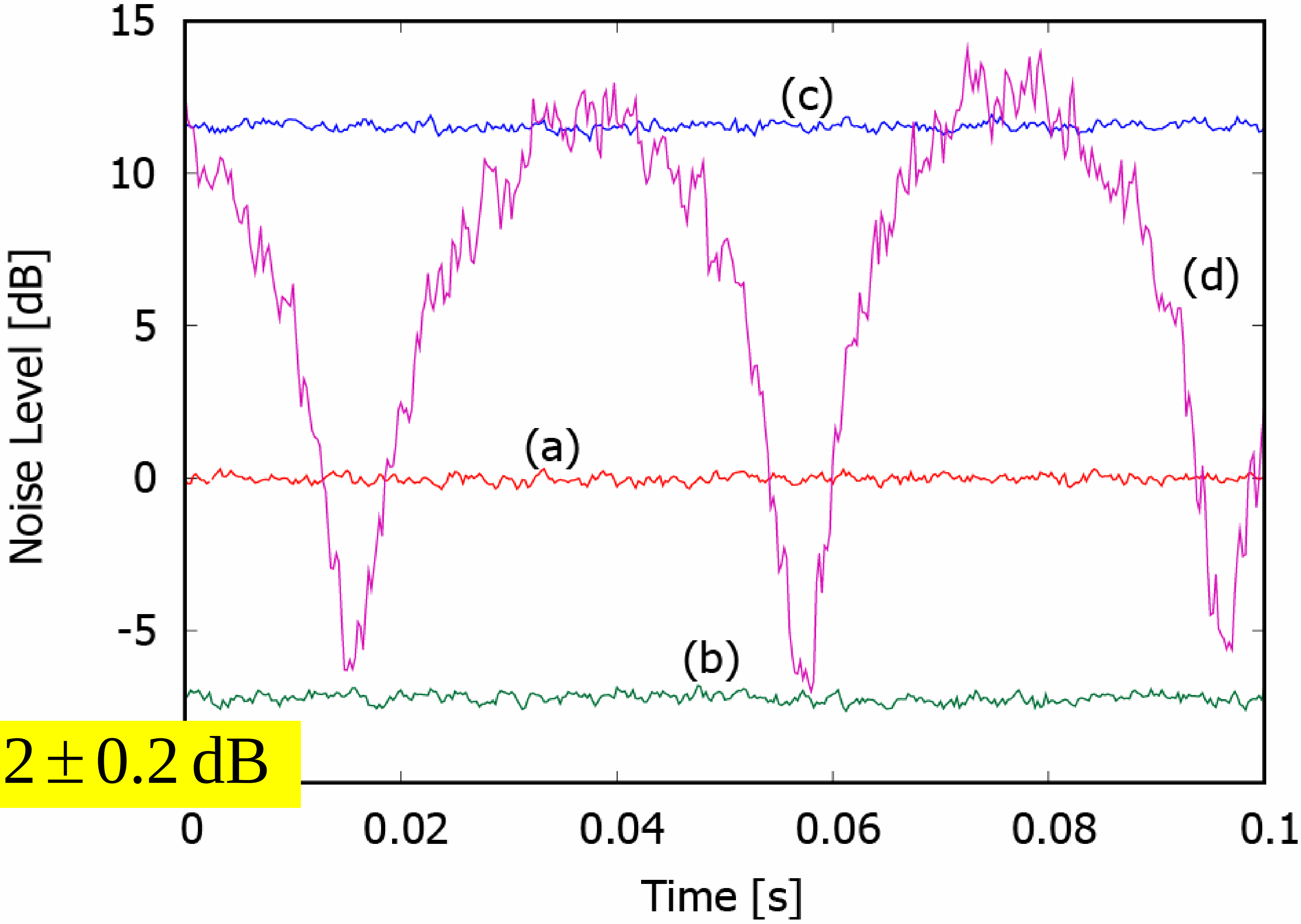


$$\left\langle [\Delta(\hat{p}_\mathrm{A} + \hat{p}_\mathrm{B})^2] \right\rangle \approx -3\mathrm{dB}$$

Trying to get more squeezing



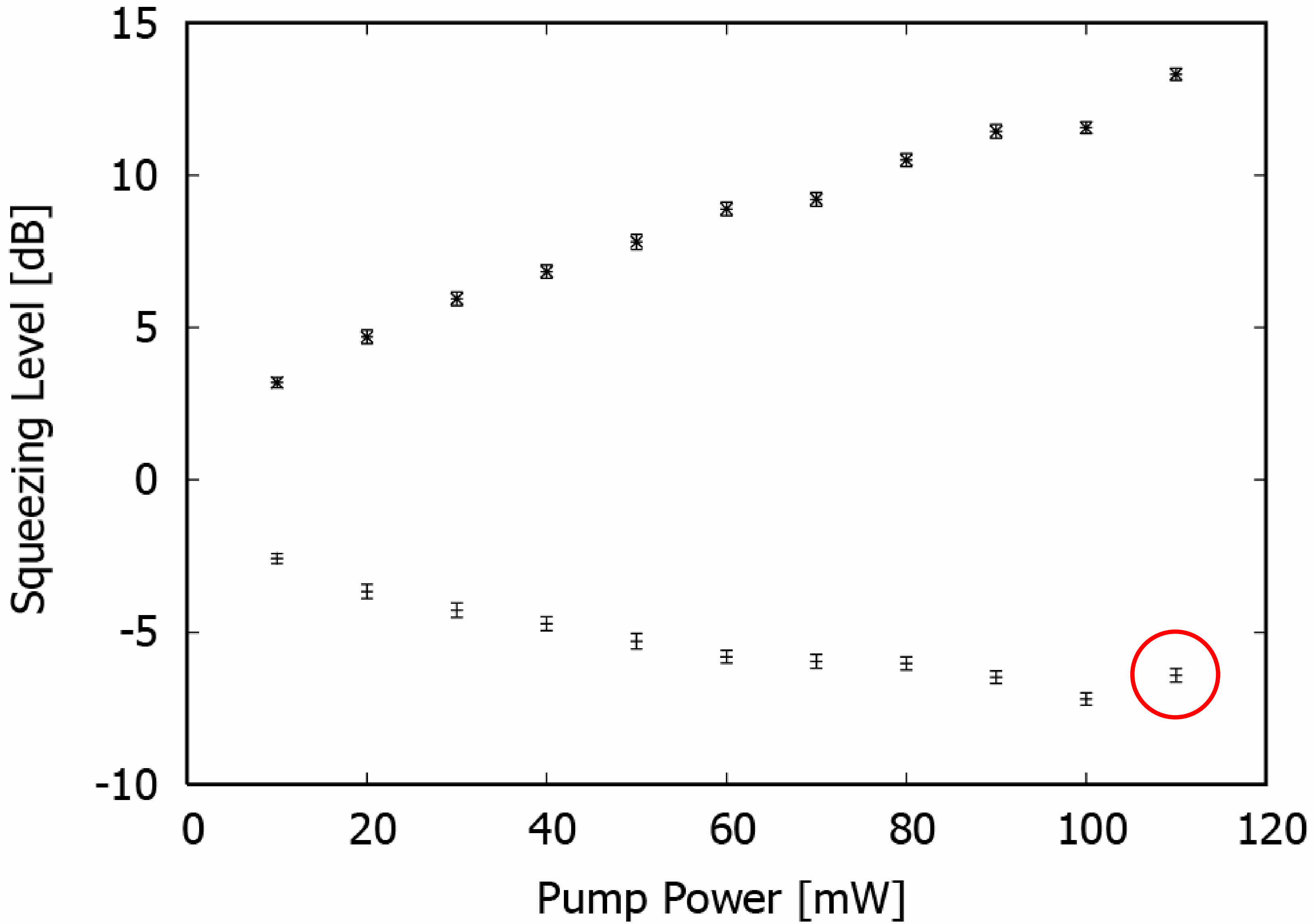
-7dB squeezing



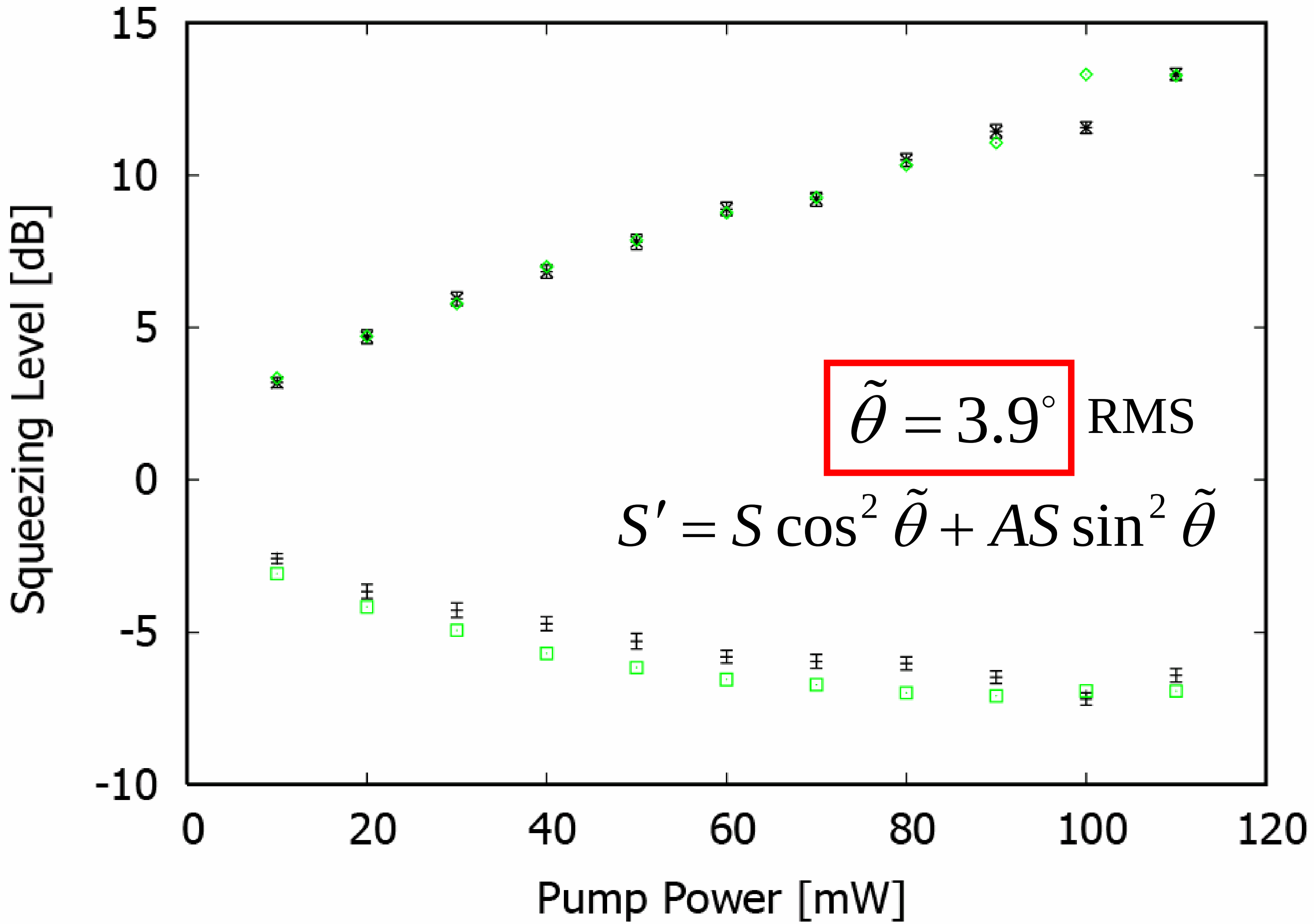
$-7.2 \pm 0.2\,\mathrm{dB}$

S. Suzuki, H. Yonezawa, F. Kannari, M. Sasaki, & A. Furusawa, APL 89, 061116 (2006).

Pump power dependence of squeezing



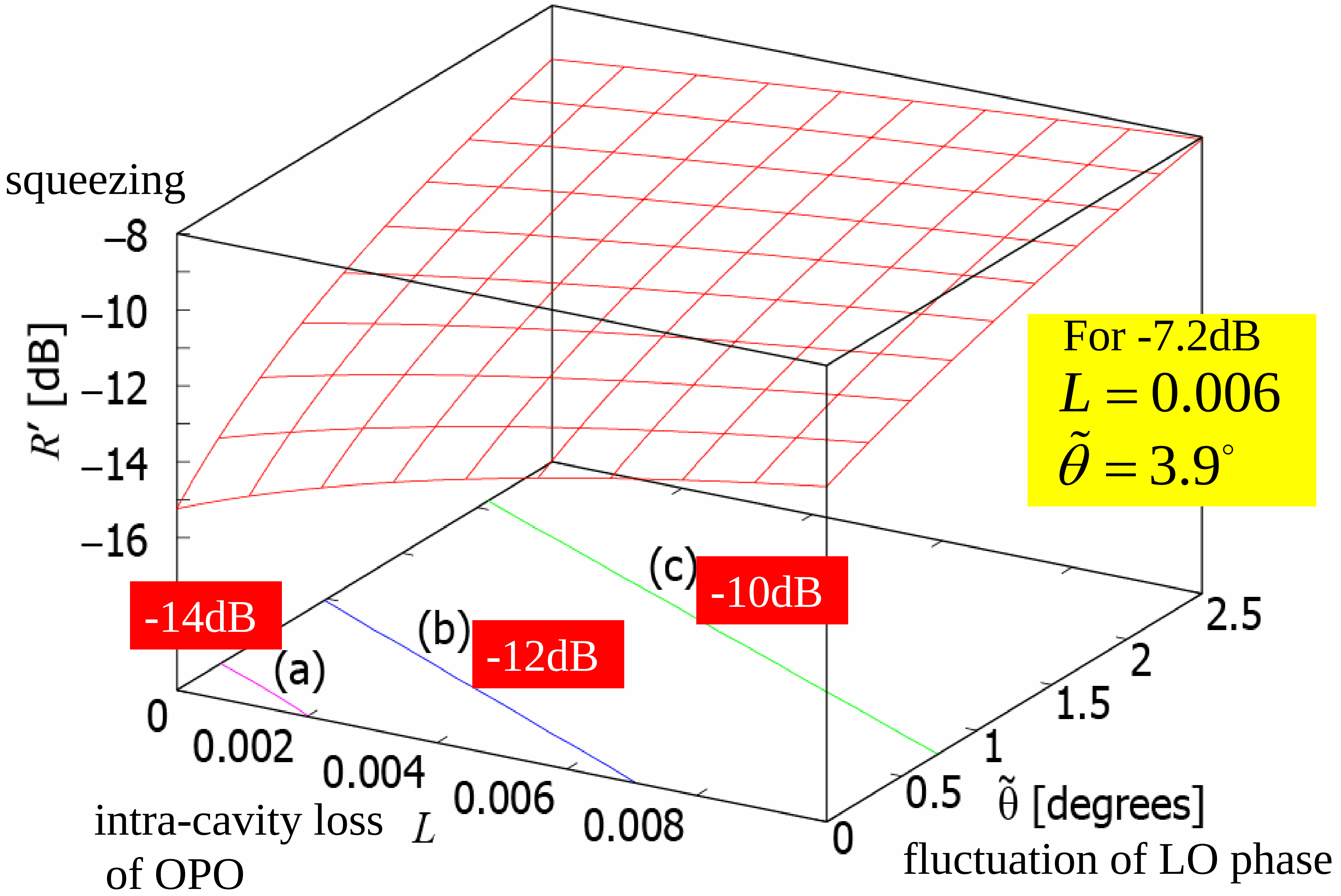
Theoretical squeezing level calculated from G+ and losses taking account of the phase fluctuation of the LO



$$\tilde{\theta} = 3.9^\circ$$ RMS

$$S' = S\cos^2\tilde{\theta} + AS\sin^2\tilde{\theta}$$

Requirements for high-level squeezing

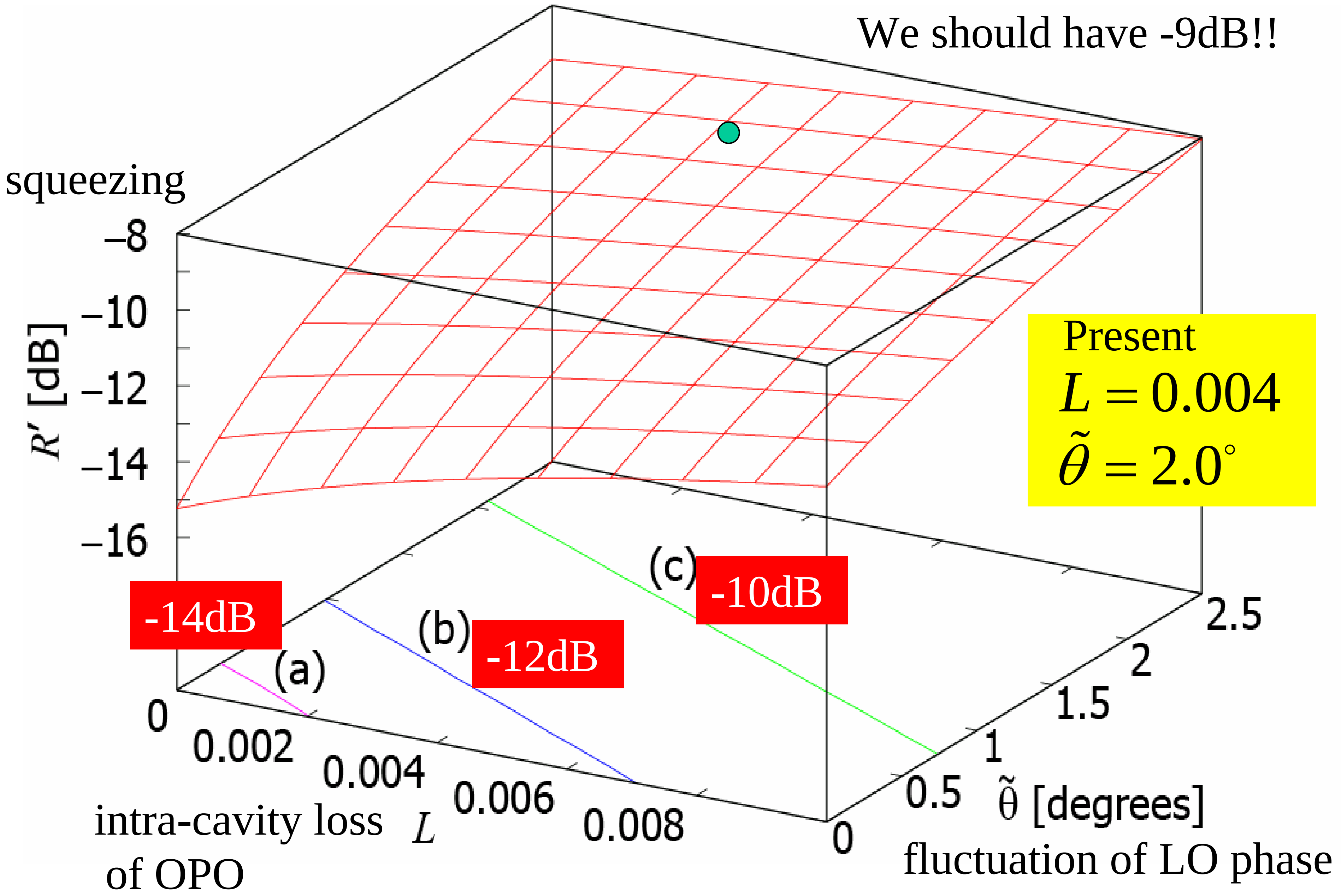


For -7.2dB

$L = 0.006$

$\tilde{\theta} = 3.9^{\circ}$

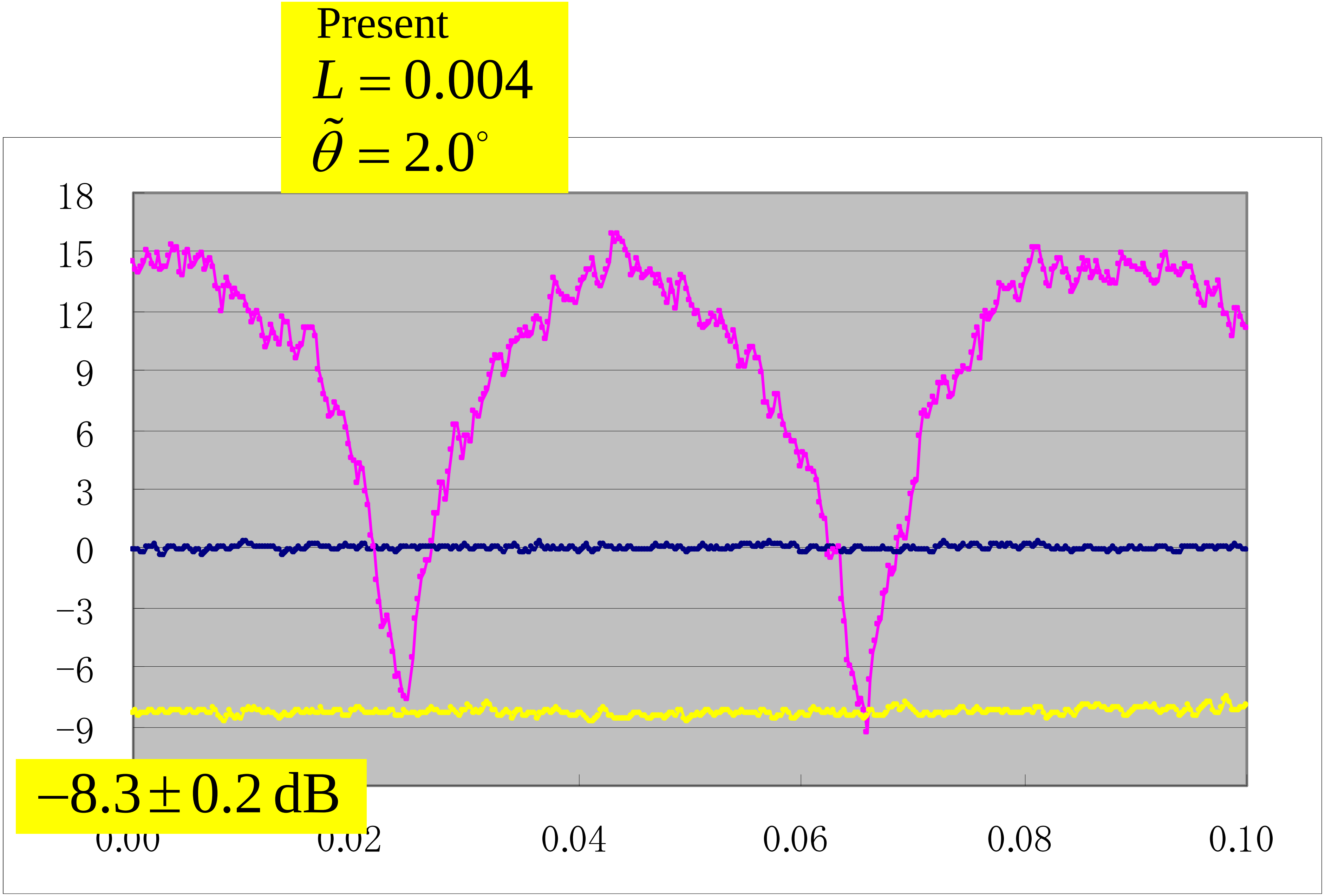
Requirements for high-level squeezing



Present

$L = 0.004$

$\tilde{\theta} = 2.0^{\circ}$


Present
$L = 0.004$
$\tilde{\theta} = 2.0^\circ$
18
15
12
9
6
3
0
−3
−6
−9
-8.3 ± 0.2 dB
0.00
0.02
0.04
0.06
0.08
0.10

Near future

Schrödinger cat state

Non-Gaussian states

+

Time domain EPR correlation

||

Quantum teleportation of non-Gaussian states