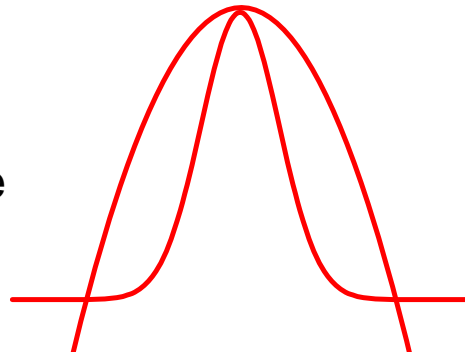


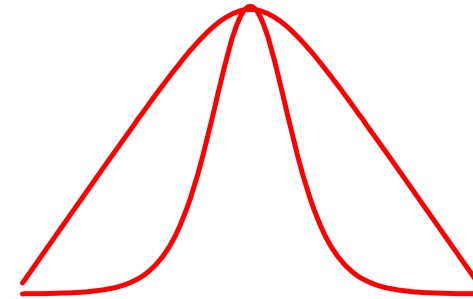
Pulse shape

What's your pulse shape



Gaussian

or



Hyperbolic secant

?

This is an interesting question

Caveat:

Must be in the regime where dominant pulse shaping mechanism is consistent with the theory

Optics Communications 97 (1993) 215-218
North-Holland

OPTICS
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Gaussian pulse wings with passive modelocking

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Roger Falcone at Berkely [1], Wayne Knox at AT&T Bell Laboratoires [2], and Bill White and Howard Nathel of Lawrence Livermore Laboratory [3] have observed SHG traces of pulses whose temporal wings are matched better by a gaussian ($\exp(-t^2/\tau^2)$) than by a secant hyperbolic (exponential) dependence ($\exp(-t/\tau)$). This is a troubling observation to those of us who have developed analytic theories of passive mode-locking. The equa-

Describing the pulse shaping dynamics

“Continuous” Approaches:

Derive a differential equation describing the evolution of the envelope

1) Nonlinear Schrödinger Equation

Includes Kerr nonlinearity (self phase modulation) and dispersion

Solution is locally invariant → pulse that does not change with propagation at all

2) “Master” Equation (Haus)

Includes saturable absorber & gain, spectral filtering (extensions include dispersion and nonlinearity)

Solution that is self-consistent after one round trip

Ordering of elements does not matter

Both of these give $\text{sech}^2(t)$ solutions

Both consider only the net dispersion in cavity, ignore that it is due to compensation between regions of opposite sign

“Dispersion managed” approaches:

Explicitly include variation regions of opposite dispersion.

Gives **Gaussian** solution in limit that changes in dispersion are large compared to net

Nonlinear Schrödinger Equation

Recall propagation equation from 2nd lecture:

$$\frac{\partial}{\partial z'} \hat{E} - \frac{i}{2} k_0'' \frac{\partial^2}{\partial t'^2} \hat{E} = 0$$

We need to add nonlinearity, given by $i\gamma |\hat{E}|^2 \hat{E}$ where $\gamma = \frac{n_2 \omega_0}{c A_{eff}}$

for a beam with an “effective” cross-section area of A_{eff} yielding

$$i \frac{\partial}{\partial z'} \hat{E} = \frac{1}{2} k_0'' \frac{\partial^2}{\partial t'^2} \hat{E} + \gamma |\hat{E}|^2 \hat{E} \quad \leftarrow \text{NLSE}$$

Assume a solution

$$\hat{E}(z', t') = \sqrt{P_0} \operatorname{sech}\left(\frac{t'}{t_0}\right) e^{i\gamma P_0 z'}$$

“soliton” with peak power P_0
and FWHM $1.763 t_0$

Substitution into NLSE yields relationship

$$\frac{\gamma P_0 t_0}{\beta_2} = 1$$

This describes a family of solutions, typically this determines t_0 as other parameters are fixed by operating conditions

Solitons I

Where'd the $\text{sech}(t)$ come from? Just a lucky guess?

Nope, it can be derived from “inverse scattering theory”

Developed by Zakharov and Shabat

Classic text: Ablowitz and Segur “Solitons and the inverse scattering transform” (1981)

Inverse scattering theory yields

A set of solutions, corresponding to “eigenvalues” N

The $N=1$ solution is $\text{sech}(t)$

Higher order solutions are more complicated and periodic

The pulse velocity depends on N

Plus a “radiation” field

(analogy to quantum mechanics: think of eigenstates and radiation field like the bound and continuum wavefunctions of a finite potential well respectively)

An arbitrary pulse is decomposed into eigenstates + radiation

Eigenstates propagate unchanged

Solitons II

Decomposition has an interesting result: an arbitrary initial condition evolves into a soliton:

Decompose into eigenstate + radiation \rightarrow radiation spreads and dissipates, leaving soliton

Implies stability/robustness against perturbation

Soliton theory seems like a pretty major simplification

Ignored “dissipative” terms

Gain & Loss

Saturation of Gain & Loss (remember gain is always saturated in a laser!)

Higher order dispersion

Ignored “lumped” nature of elements in a bulk optic laser

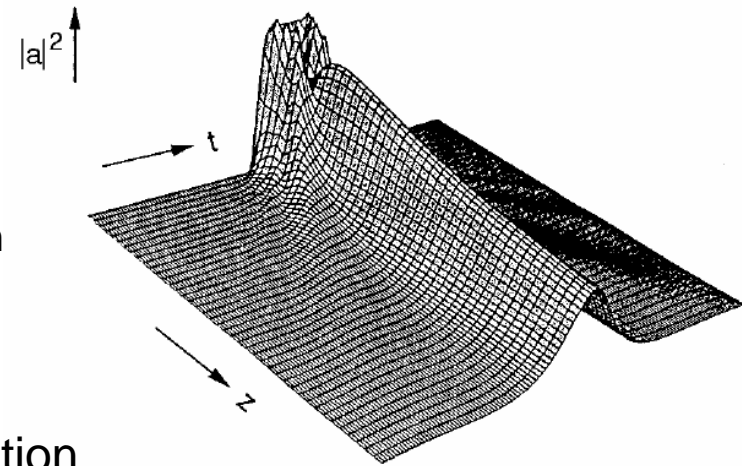
Soliton theory does a pretty good job when these are reasonable, e.g., for a fiber laser where

Pulse shaping is dominated by GVD and nonlinearity

Which are distributed throughout the cavity

Tends to underestimate stability regimes (due to no saturation)

Nevertheless, it does give insight into one phenomena that occurs in bulk optic (KLM Ti:sapphire) lasers...



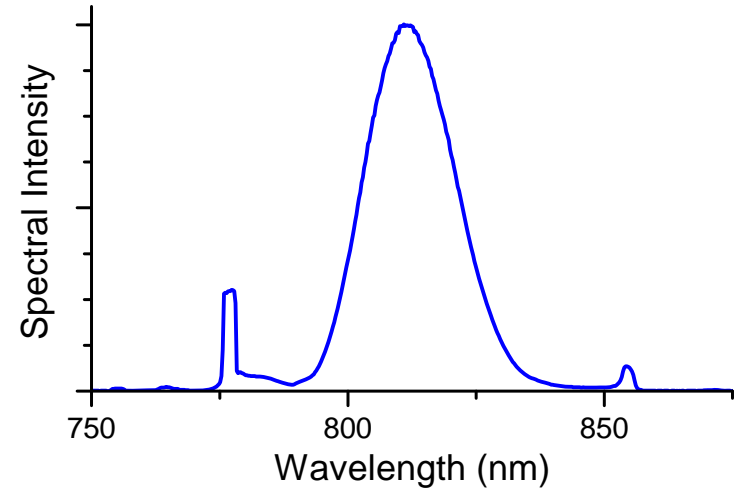
Solitons: Resonant (“Kelley”) Sidebands

Look closely at ti:sapph spectrum:

Often notice a pair of “sidebands”

Usually requires log scale

Data at right for laser where they are particularly strong



Due to “phase matching” of radiation field arising from perturbations by gain/loss

- 1) Assume you have a proper soliton
- 2) It hits output coupler, pulse energy decreases
- 3) It has to get longer in time/narrower in spectrum to still be soliton
- 4) Excess spectrum is “shed” – travels at different speed
- 5) Due to periodicity of cavity, radiation of certain wavelengths add constructively → phase matching of side bands

$$\frac{\gamma P_0 t_0}{\beta_2} = 1$$

Strength of radiation field \leftrightarrow magnitude of perturbation

Spacing of sidebands \leftrightarrow inverse cavity length

Note that the sidebands can “clamp” the spectral width, limits minimum duration

Master Equation Approach

Write an “operator” for action of each element in the cavity on the pulse envelope

In steady state the sum has to have no effect

$$[g - l + S + D + i\psi + T_D + \gamma]E(t) = 0$$

Gain } May be time dependent
 Loss } Frequency dependence included in S
 Spectral filtering } Can be combined
 Dispersion } (dispersion = imaginary spectral filter)
 Phase shift }
 Time delay } Needed for self-consistency because other
 Nonlinearity } elements can cause delay or phase shifts

What are each of these operators?

Easy ones:

ψ – just a number

$$T_D = t_D \frac{d}{dt}$$

Spectral filter, gain, loss, and dispersion

Gain g with a bandwidth ω_g , in frequency domain transforms $E(\omega)$ to

$$\hat{E}'(\omega) = \left[1 + g \left(1 - \frac{(\omega - \omega_0)^2}{\omega_g^2} \right) \right] \hat{E}(\omega)$$

Where we have assumed the filter function is parabolic with frequency

Fourier transform in to time domain

$$\hat{E}'(t) = \left[1 + g \left(1 + \frac{1}{\omega_g^2} \frac{d^2}{dt^2} \right) \right] \hat{E}(t)$$

Loss is same, with $g \rightarrow -l$, typically lump together into single spectral filter

Non saturable loss and gain are easy, just numbers g_0, l_0

Group velocity dispersion is similar, just a complex spectral filter

$$\hat{E}'(t) = \left[1 + ik_0'' \frac{d^2}{dt^2} \right] \hat{E}(t)$$

Active modelocker

$$\left[g \left(1 + \frac{1}{\omega_g^2} \frac{d^2}{dt^2} \right) - l - m(1 - \cos \omega_m t) \right] \hat{E}(t) = 0$$

modulation function

Parabolic approximation for modulation function

$$T = 1 - m(1 - \cos \omega_m t) \cong 1 - \frac{1}{2} m (\omega_m t)^2$$

The solution is then

$$\hat{E}(t) = E_0 \exp\left(-\frac{t^2}{\tau^2}\right)$$

with

$$\tau = \sqrt{\frac{1}{\omega_g \omega_m} \left(\frac{8g}{m}\right)^{1/4}}$$

Parabolic approximation for pulse peak

$$\hat{E}(t) = E_0 \left(1 - \frac{t^2}{\tau^2} \right)$$

Passing through the modulator yields

$$\frac{1}{\tau^2} \rightarrow \frac{1}{\tau^2} + \frac{m\omega_m^2}{2}$$

Giving a pulse shortening rate

$$\frac{\Delta\tau}{\tau} = \frac{m\omega_m^2 \tau^2}{4}$$

Slow saturable absorber I

$$\left[\underbrace{g(t)}_{\text{saturable gain}} - \underbrace{l_a(t)}_{\text{saturable loss}} - \underbrace{l_0}_{\text{nonsaturable loss}} + i\psi + \frac{l_0}{\omega_g^2} \frac{d^2}{dt^2} + t_D \frac{d}{dt} \right] \hat{E}(t) = 0$$

Where

$$l_a(t) = l_i \exp\left(-\sigma_a \int_{-\infty}^t |\hat{E}(t')|^2 dt'\right) \quad g(t) = g_i \exp\left(-\sigma_g \int_{-\infty}^t |\hat{E}(t')|^2 dt'\right)$$

The initial loss and gain are l_i and g_i , the effective cross-sections are σ_a and σ_g

Expand these to second order in $\int_{-\infty}^t |\hat{E}(t')|^2 dt'$

The solution is

$$\hat{E}(t) = E_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$$

with

$$\tau = \frac{1}{\omega_g} \frac{4}{\sigma_a W} \sqrt{\frac{l_0}{l_i}} \quad \text{where} \quad W = \int_{-\infty}^{\infty} |\hat{E}(t')|^2 dt'$$

Increasing

Gain bandwidth

Pulse energy

Saturable absorption

Ease of saturation

Decreases pulse duration

Increase non-saturable loss increases duration

Slow saturable absorber II

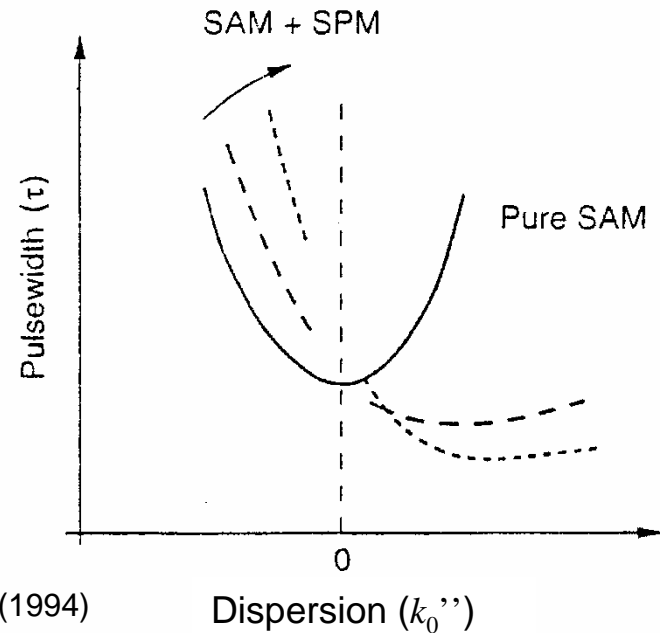
The pulse shortening rate $l_i(\sigma_a W)^2$ due to competition between absorber and finite gain bandwidth \rightarrow independent of pulsewidth

If we include GVD and self-phase modulation, the solution is a chirped pulse

$$\hat{E}(t) = E_0 \operatorname{sech}\left(\frac{t}{\tau}\right) \exp[i\beta \ln \operatorname{sech}(t/\tau)]$$

Where we assume the SPM is resonant due to the absorber \rightarrow opposite sign from non-resonant (Kerr) SPM

Resulting relation between dispersion and pulsewidth



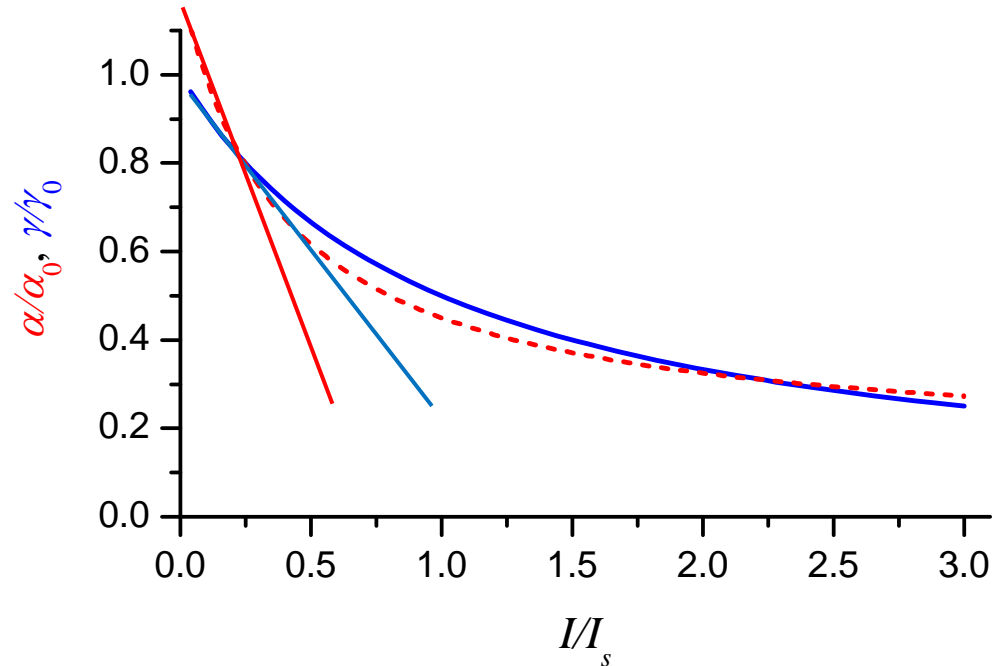
Fast Saturation -- Linearization

Absorption or gain saturation follows the standard saturation curves

$$l_a = \frac{l_a^0}{1 + I/I_s} \quad g = \frac{g_0}{1 + I/I_s}$$

where

$$I = \left| \hat{E}(t) \right|^2$$



Using this form, the master equation cannot be solved analytically

It can be if saturation is linearized

$$l_a = l_a^0(1 - I/I_s) \quad g = g_0(1 - I/I_s)$$

This form can produce solutions of the master equation, but does not provide insight about stability

To determine if the solution is stable against perturbations, numerics can be used, or more sophisticated theory

Fast saturable absorber I

$$\left[g - l + i\psi + \frac{g}{\omega_g^2} \frac{d^2}{dt^2} + iD \frac{d^2}{dt^2} + t_D \frac{d}{dt} + (\gamma - i\delta) |\hat{E}(t)|^2 \right] \hat{E}(t) = 0$$

self phase modulation
self amplitude modulation
dispersion

The solution to this is also

$$\hat{E}(t) = E_0 \operatorname{sech}\left(\frac{t}{\tau}\right) \exp\left[i\beta \ln \operatorname{sech}(t/\tau)\right]$$

Note SPM due to Kerr effect

Plugging this into the master equation yields

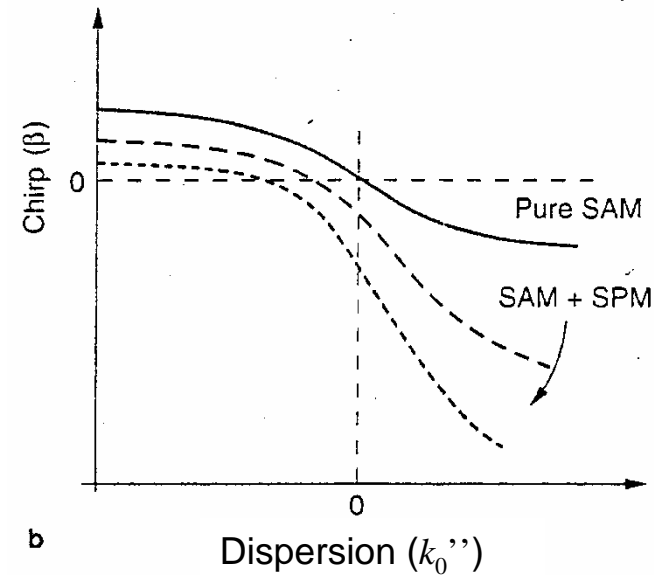
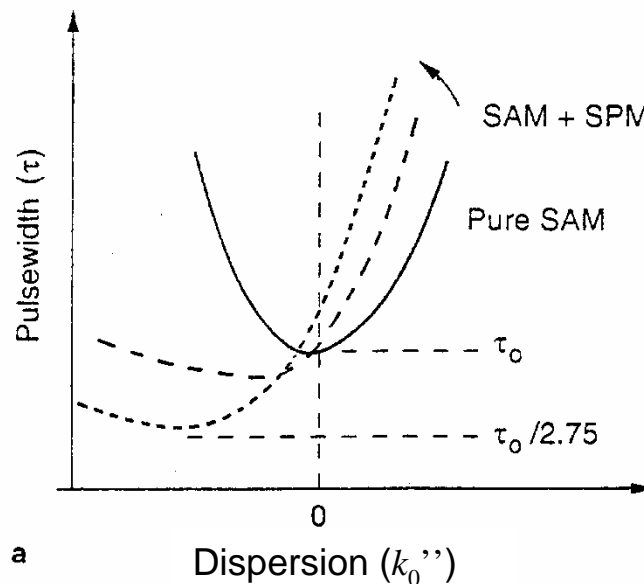
No SPM, minimum width at 0 GVD is

$$\tau_0 = \frac{4g}{\gamma W \omega_g^2}$$

With SPM, GVD for no chirp is

$$D = \frac{\delta}{\gamma} \frac{g}{\omega_g^2}$$

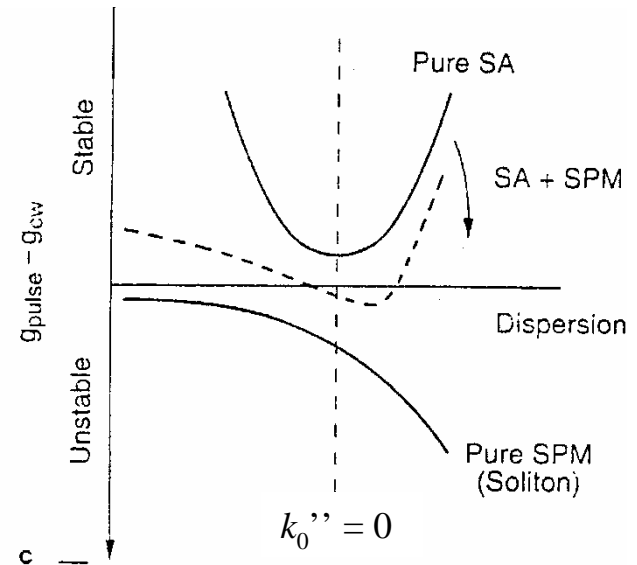
$$\tau = \frac{4|D|}{\delta W}$$



Fast saturable absorber II

A stable pulse requires pulse gain greater than CW gain

Instability in pure SPM case due to limited gain bandwidth



The pulse shortening rate is

$$\frac{\Delta\tau}{\tau} = \frac{\gamma W}{2\tau}$$

Dispersion Management I

The fact that the dispersion is not constant turns out to be quite important

Alternating regions of positive and negative dispersion

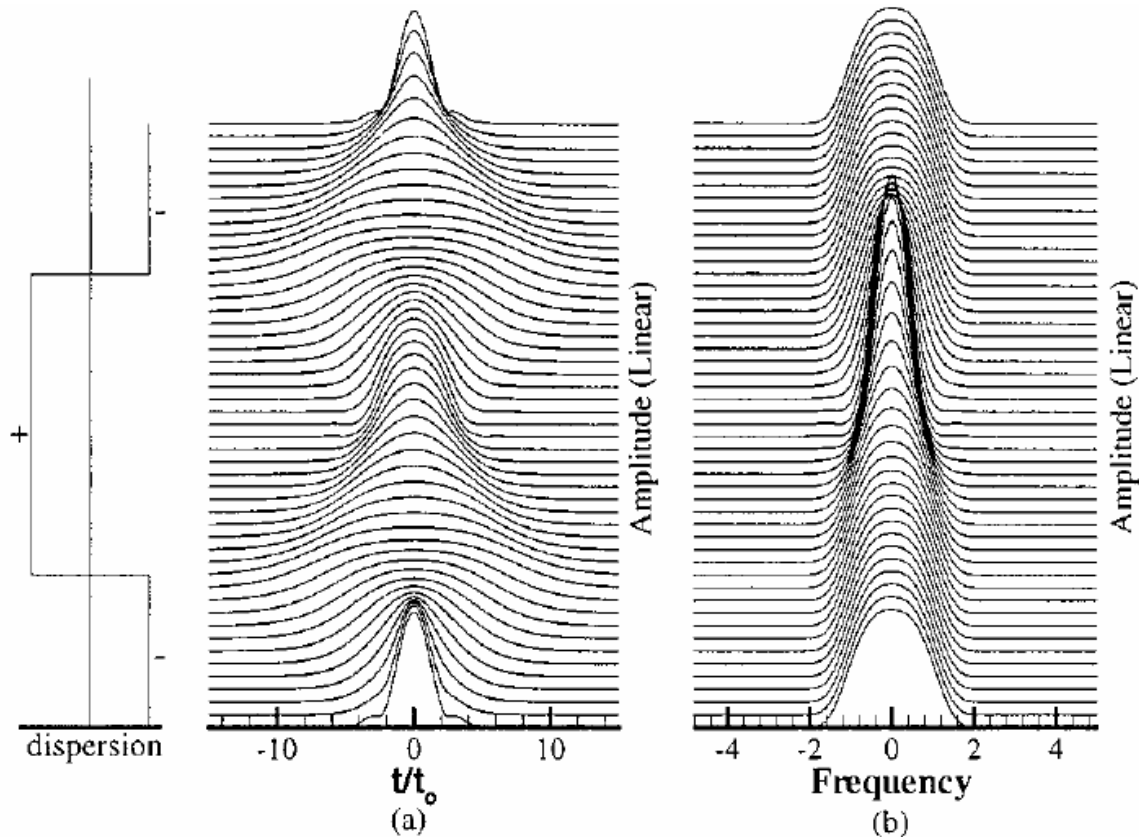
→ Causes the pulse to “breath”, stretching and recompressing twice per “period”

The pattern of dispersion variation is called the “dispersion map”

In a bulk optic laser, the nonlinear coefficient also varies

Non zero in gain medium

Zero elsewhere



Dispersion Management II

$$\frac{\gamma P_0 t_0}{\beta_2} = 1$$

Dispersion management

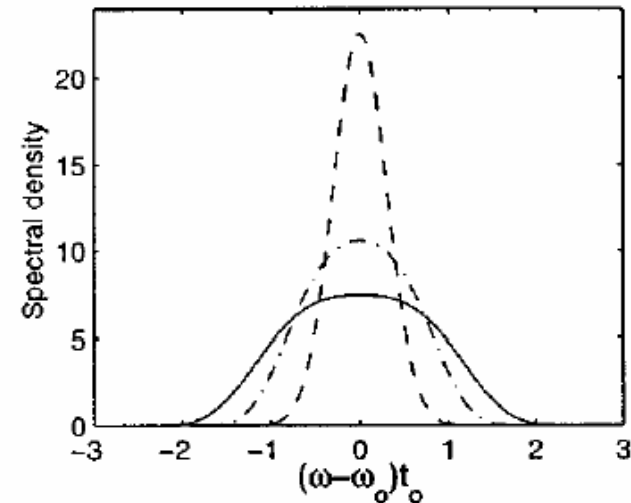
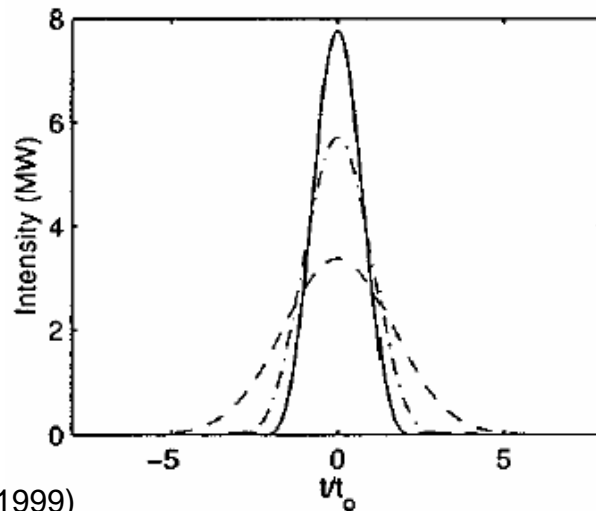
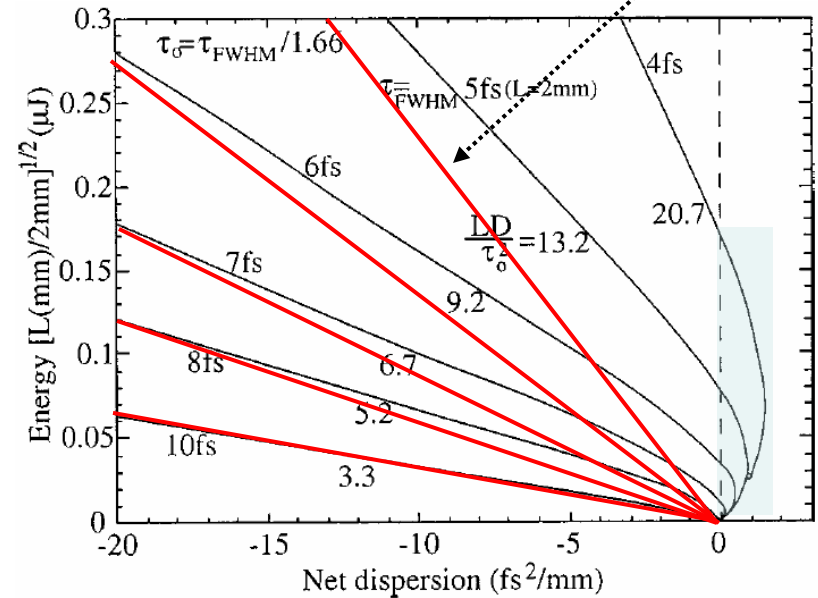
alters the relationship between power, dispersion and width

Generally need higher power since pulse is stretched much of the time

Allows stable solution in region of normal net dispersion

Changes pulse shape (semi-continuously)

Gaussian in certain cases!



Pulse shape: limits in the real world

This analysis has been useful for understanding the physical process of pulse formation, but

in the ultrabroad band/short pulse limit

Higher order effects become very important – particularly dispersion

Spectral filtering becomes strong (parabolic approximation invalid)

Self-amplitude modulation saturates

