Amplification

Amplification

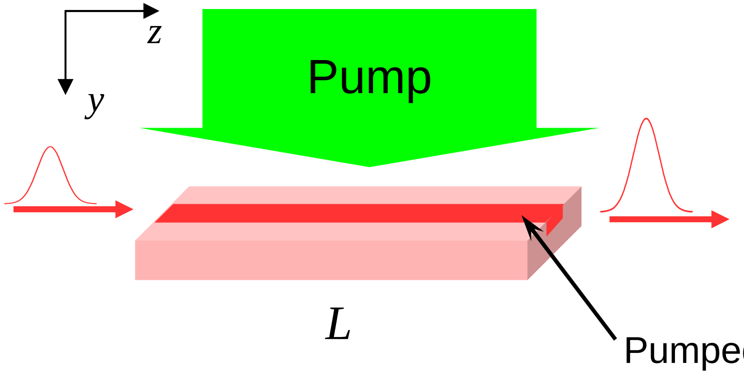
Typical output of a mode-locked oscillator:

- 1 watt at a repetition rate of 100 MHz ➔ 10 nJ per pulse
- For a 10 fs pulse ➔ ~ 1 MW peak power
- Focused to 10 micron spot ➔ 10^{14} Watts/cm^2 ➔ 2 x 10^8 V/cm
  - Not to shabby, but what if you need more? Compare: inner atomic field is about 1 GV/cm

Amplify:

- The average power that can be extracted is basically constant
  - Lower repetition rate to achieve higher pulse energy
- 1 watt at 1 kHz → 1 mJ per pulse
- 50 fs pulse ➔ 20 GW peak power
- Focused to 20 micron spot ➔ 5 x 10^{17} Watts/cm^2 ➔ 15 GV/cm
- These are “typical” numbers, higher are achievable

Gain and Saturation I

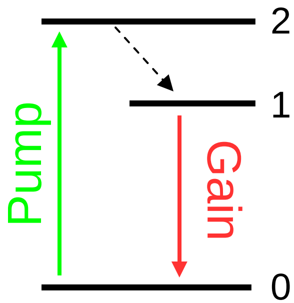


Analyze the following geometry:

Pump from the side a region of width a
How deep (b) is the pump region?
→Need to analyze saturation
Assume a 3 level system

Need to describe saturation.

The cross section of a transition is



$$\sigma(\omega)=\frac{\sigma^{(0)}}{1+T_2^2(\omega-\omega_0)^2}$$

where T_2 is the dephasing time, ω_0 is the resonance frequency and the on resonance cross-section for a transition with dipole moment μ is

$$\sigma^{(0)}=\frac{\mu T_2\omega_0}{\varepsilon_0 c n\hbar}$$

The cross-section has units of cm^2, think of it as absorption(gain) coefficient per atom (σN has units cm^{-1})

Gain and Saturation II

The energy density at position z and time t within the pulse is

$$W(z,t) = \int_{-\infty}^{t} I(z,t')\,dt'$$

The saturation energy density is

$$W_s = \hbar\omega/2\sigma$$

In a 2-level amplifying medium, an initial $W_0(t)$ becomes

$$W(z,t) = W_s \ln\left[1 - e^{a}\left(1 - e^{W_0(t)/W_s}\right)\right]$$

Where the small signal gain is

$$a = \sigma \Delta n^{(e)} z$$

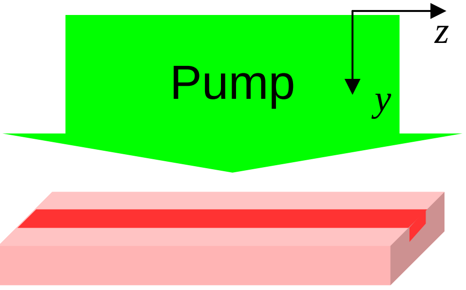
For an equilibrium population inversion $\Delta n^{(e)}$, $\Delta n = n_1 - n_0$

Define saturation ratio as

$$s = \frac{W_0}{W_s} = 2\sigma\frac{W_0}{\hbar\omega}$$

which is product of cross-section and number of photons in incident pulse

Gain and saturation III



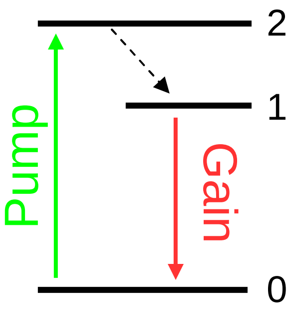
Return to the 3-level system and consider absorption of pump.

Write rate equations for number of pump photons, F_p and population of lower state, n_0 and middle state, n_1

$$\dot{n}_0(y,t) = -\sigma_{02} n_0(y,t) F_p(y,t)$$

$$\frac{\partial}{\partial y} F_p(y,t) = -\sigma_{02} n_0(y,t) F_p(y,t)$$

$$n_1(y,t) = N - n_0(y,t)$$

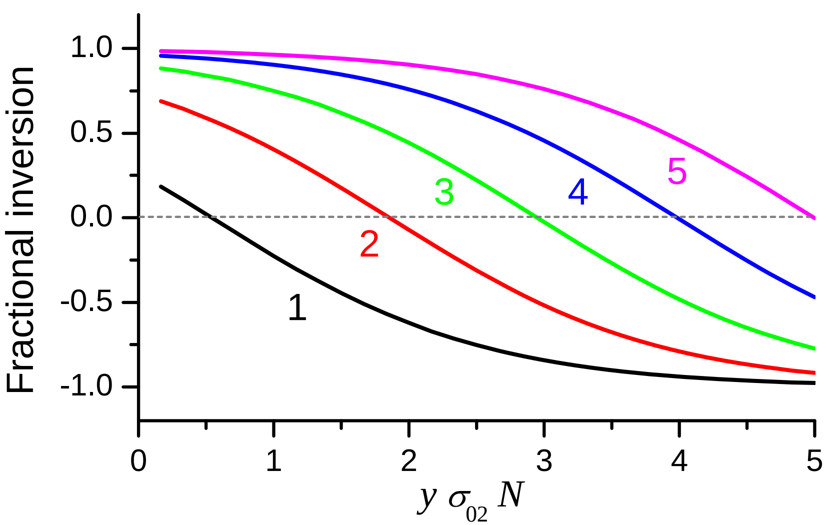


For initial condition of all atoms in ground state, $n_0(y,0) = N$

The inversion density is

$$\Delta n_{10} = n_1 - n_0 = N\left\{1 - \frac{2}{1 - e^{-\sigma_{02} N y}\left(1 - \exp\left(s_p\right)\right)}\right\}$$

From which we see it is possible to obtain a region of reasonably constant inversion(gain) if the pump saturates the gain medium (curves for varying s_p)

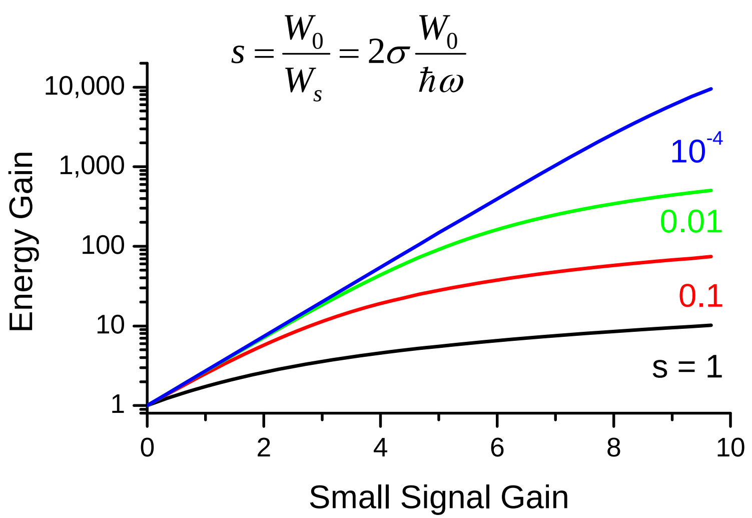


Gain and Saturation IV

The small signal intensity gain for a pass through the entire medium is

$$a = \sigma_{10}(n_1 - n_0)L$$

The energy gain, including saturation, is

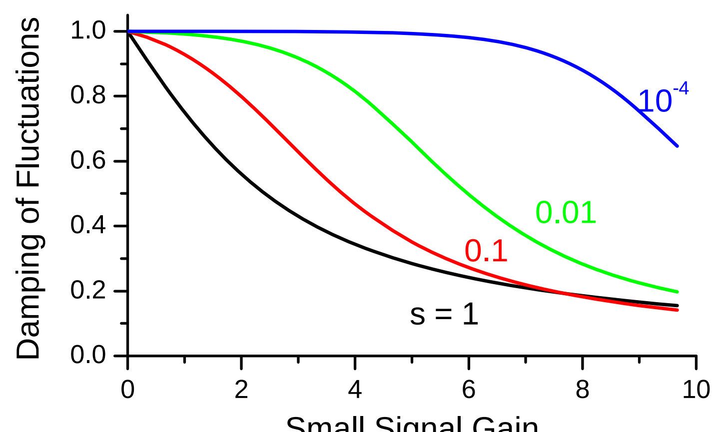


$$G = \frac{W(L)}{W_0} = \frac{\hbar\omega}{2\sigma_{10}W_0}\ln\left[1 - e^a\left(1 - e^{2\sigma_{10}W_0}\right)\right]$$

$$= \frac{1}{s}\ln\left[1 - e^a\left(1 - e^s\right)\right]$$

Working in saturation is bad for chirped pulse amplifiers (more later)
But advantageous in terms maximizing energy extraction and reduced sensitivity to pulse fluctuations

$$\frac{\Delta W(L)/W(L)}{\Delta W_0/W_0} = \frac{e^a e^s}{1 - e^a\left(1 - e^s\right)}\frac{1}{G}$$



Gain Media

Medium	λ (μm)	$\Delta\lambda$ (nm)	σ (cm^2)	Life time (s)	Typical Pump
Dyes	0.3…1.0	50	$>10^{-16}$	10^{-8}…10^{-12}	laser
Color centers	1…4	200	$>10^{-16}$	10^{-6}	laser
KrF	0.249	2	$3x10^{-16}$	10^{-8}	discharge
Alexandrite	0.75	100	$7x10^{-21}$	$3x10^{-4}$	Flashlamp
Cr:LiSAF	0.83	250	$5x10^{-20}$	$6x10^{-5}$	Flashlamp Diode
Ti:sapphire	0.78	400	$3x10^{-19}$	$3x10^{-6}$	Laser
Nd:glass	1.05	21	$3x10^{-20}$	$3x10^{-4}$	Flashlamp

Generally prefer

Smaller σ (higher saturation energy)

Longer lifetime (pump pulse can be longer, less loss to fluorescence)

Pulse Shaping in Amplifiers

Changes in the pulse shape during amplification are generally detrimental

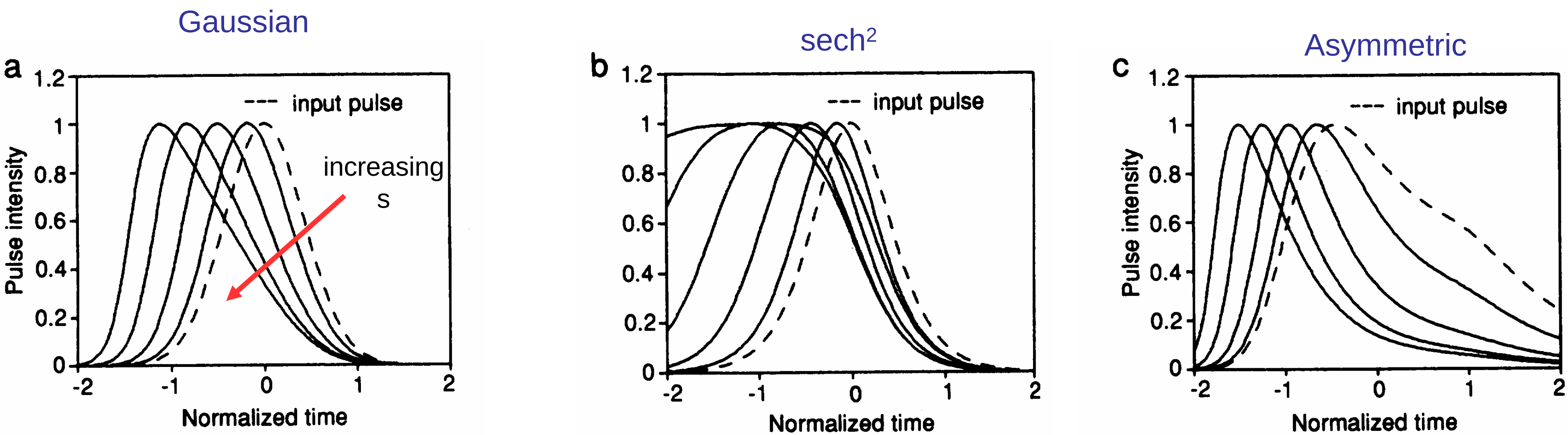
Occur due to:

- Saturation
- Dispersion
  - Discussed dispersion extensively already
  - As long as the amplifier is not strongly saturated, can be pre- or post-compensated
- Gain Narrowing
- Nonlinearity

Pulse Shaping in Amplifiers: Saturation

Saturation of the gain favors the leading edge of the pulse

- Shifts the pulse earlier in time
- Steepens the leading edge
  - The steeper the edge to start with, the less distortion
- Dramatically changes "wings" on the pulse
  - ➔insert saturable absorbers between amplification stages



Gain Narrowing

The finite bandwidth of the gain medium acts as a spectral filter

Narrows pulse spectrum ➔ lengthens duration

Same analysis as when we discussed pulse evolution inside the cavity

Assume Gaussian small signal gain spectrum

$$a = a_0 \exp\left[-\left(\Omega T_g\right)^2 / 2\right]$$

$2.36/T_g$ is spectral width

Parabolic expansion

$$a = a_0\left[1 - \left(\Omega T_g\right)^2 / 2\right]$$

Initial Gaussian pulse with temporal width $1.18\ \tau_G$ has spectrum

$$\hat{E}(\Omega) = A_0 \exp\left[-\left(\Omega \tau_G / 2\right)^2\right]$$

Which after amplification becomes

$$\hat{E}(\Omega) = A_0 \exp\left[a_0 / 2\right] \exp\left[-\Omega^2\left(\tau_G^2 + a_0 T_g^2\right) / 4\right]$$

Duration after amplification is

$$\tau'_p \cong 1.18\sqrt{\tau_G^2 + a_0 T_g^{\,2}}$$

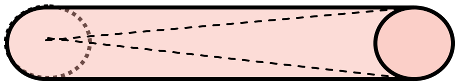
In limit of strong amplification, output pulse duration is just due to gain bandwidth, not input pulse duration

Amplified Spontaneous Emission (ASE)

Amplification of spontaneously emitted photons is a severe problem in amplification of ultrashort pulses

- Due to pump pulse duration being long compared to signal pulse
- Robs available gain from signal pulse
- Clamps inversion

Depends on geometry of gain medium



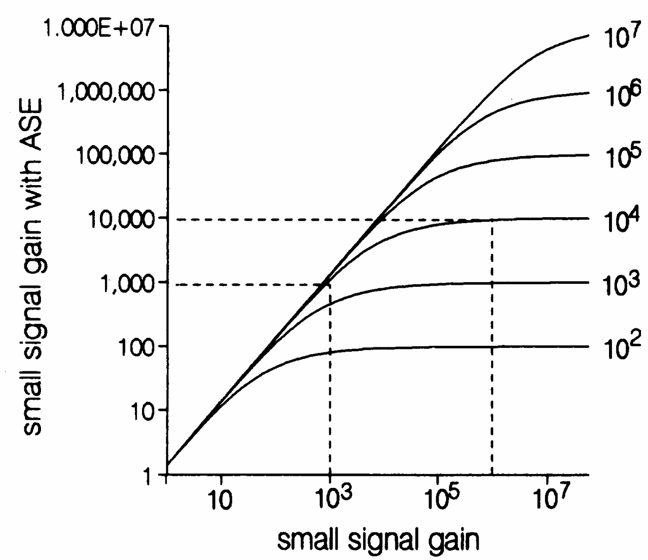
- Only spontaneous photons that travel down the gain medium matter

Plot at right shows small signal gain with ASE as a function of it without

- For varying ratio of pump photon flux to ASE photon flux, F_p/F_{ASE}

$$F_{ASE} = \frac{\eta_F \Delta\Omega \hbar \omega_{ASE}}{4\sigma_{ASE} T_{10}}$$

- Vary small signal gain without changing F_p by length or concentration



Nonlinear Index of Refraction

Nonlinear index of refraction arises due to

saturation of off-resonant amplification (or absorption)

and/or

nonlinear index of refraction of host materials

Results in

Self-phase modulation (change in spectrum)

Can be exploited to increase spectral width (shorter pulse after compression)

Self-focusing (change in spatial profile)

Generally deleterious, leads to filamentation and possibly damage

SPM due to gain saturation

The change in phase of a pulse propagating through a resonant material is

$$\frac{\partial}{\partial z}\varphi = -\frac{1}{2}\sigma\left|\frac{1}{i(\omega-\omega_0)T_2+1}\right|^2(\omega-\omega_0)T_2\Delta n$$

At position z, the time dependent change due to saturation is

$$\delta\omega(t) = \frac{\partial\varphi}{\partial t} = -\frac{1}{2}\sigma^{(0)}\left|\frac{1}{i(\omega-\omega_0)T_2+1}\right|^2(\omega-\omega_0)T_2\int_0^z\frac{\partial}{\partial t}\Delta n dz = -\frac{1}{2}(\omega-\omega_0)T_2\frac{\partial}{\partial t}\ln\frac{F(z,t)}{F_0(t)}$$

The sign depends on

Tuning above or below resonance

Absorption or Gain (F larger or smaller than F_0, F's are photon fluxes)

In limit of short signal pulse

$$\delta\omega(t) = -\frac{(\omega-\omega_0)T_2}{2}\frac{e^{-a}-1}{e^{-a}-1+e^{W(t)/W_s}}\frac{I(t)}{W_s}$$

We see that the frequency roughly tracks the intensity

Effect of SPM

Net SPM due to saturation of gain and Kerr effect

Kerr effect frequency shift is

$$\delta\omega(t) \sim -n_2 \int_0^z \frac{\partial}{\partial t} I(z',t)\,dz'$$

Tracks derivative of intensity

In saturated amplifiers, former dominates

Plot at right shows typical examples as function of saturation ratio