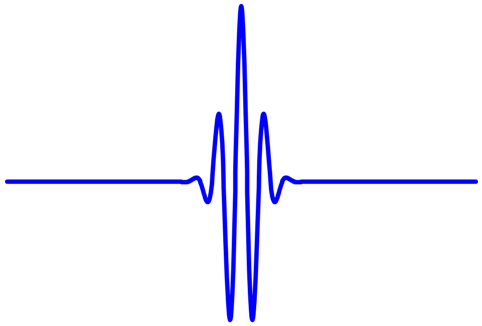
Typical Material Dispersion Values

At 800 nm

Material	k'' fs^2/mm	k''' fs^3/mm	L_D (10 fs pulse)
Air	~0.03	~0.03	2 m
Fused Silica	35	28	1.8 mm
BK7	42	32	1.5 mm
SF10	144	111	0.44 mm

Measurement of Dispersion

White light vs. femtosecond pulses



Similarity between white light and fs pulse:

Broad spectrum → short coherence time

Indistinguishable based on spectrum or field correlation
(both waveforms above have similar spectra)

Must look at intensity

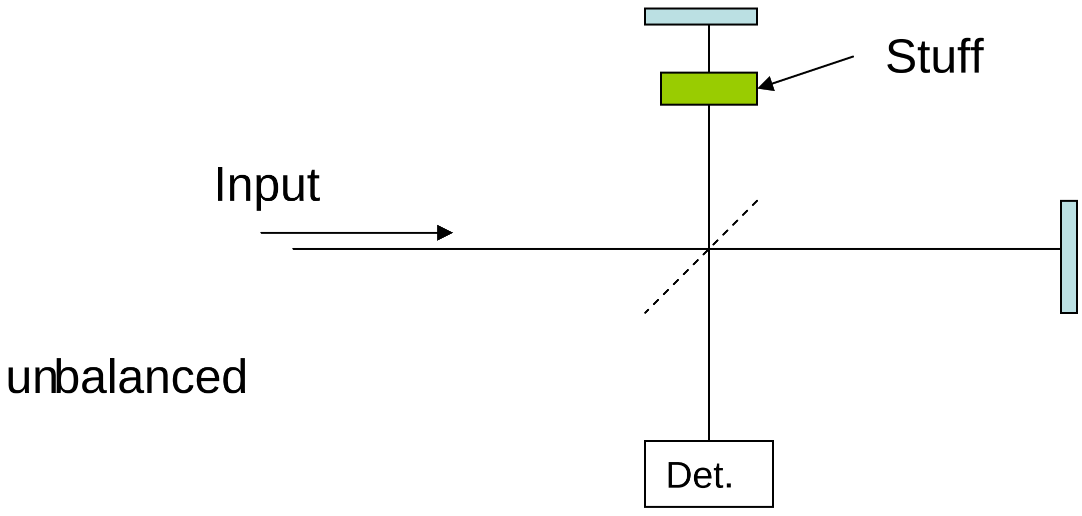
white light – constant average intensity

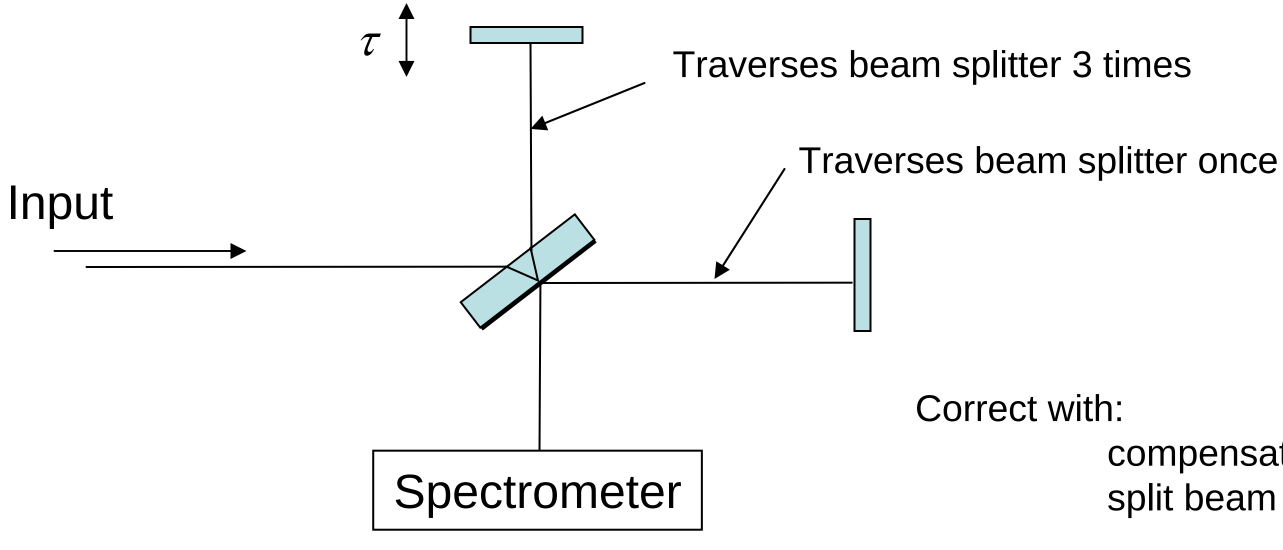
fs pulse – intensity follows envelope

→ need to consider intensity correlation function

However: unbalanced cross-correlations are similarly sensitive to dispersion

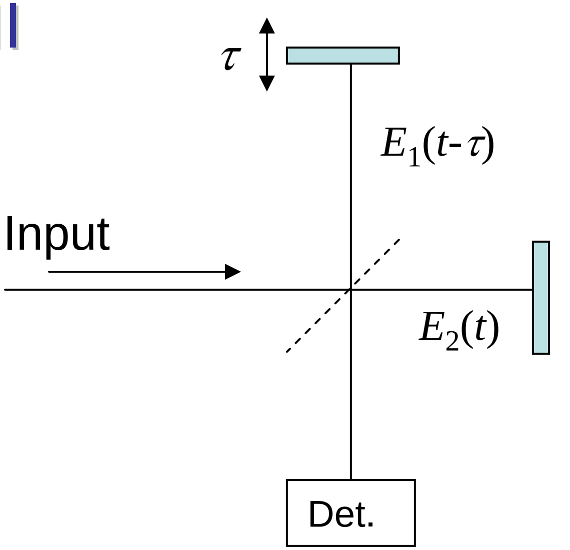
Michelson Interferometer I





Correct with:
compensating plate
split beam splitter

Michelson Interferometer II



Detected intensity is mag-squared e-field averaged over one cycle (T)

$$I(t,\tau)=\varepsilon_0 cn\frac{1}{T}\int_{t-T/2}^{t+T/2}\left|E_1(t'-\tau)+E_2(t')\right|^2 dt'$$

Where in the second line, we've decomposed the field into envelope and carrier

Assume detector response slow → integration limits go to infinity, signal time independent

$$I(\tau)\propto\left\langle \hat{E}_1^2\right\rangle+\left\langle \hat{E}_2^2\right\rangle+\underbrace{\left\langle \hat{E}_1^*(t-\tau)\hat{E}_2(t)e^{i\omega_c\tau}\right\rangle}+\underbrace{\left\langle \hat{E}_1(t-\tau)\hat{E}_2^*(t)e^{-i\omega_c\tau}\right\rangle}$$

$\langle f \rangle$ Denote averaging

Real and imaginary parts of a correlation function

Michelson Interferometer III

Correlation functions are most easily expressed in the frequency domain

$$A_{12}^{+}(\tau) = \left\langle \hat{E}_1^{*}(t-\tau)\hat{E}_2(t)e^{i\omega_c\tau} \right\rangle$$

$$\tilde{A}_{12}^{+}(\Omega) = \int_{-\infty}^{\infty} \hat{A}_{12}^{+}(\tau)e^{-i\Omega\tau}\,d\tau$$

$$= \tilde{E}_1^{*}(\Omega)\tilde{E}_2(\Omega)$$

Now consider an unbalanced interferometer. E_2 picks up a phase factor due, for example, to propagation through dispersive media

Subtract out free space

$$\tilde{E}_2(\Omega) = \tilde{E}_1(\Omega)\exp\left\{-iL\left[k(\Omega) - \frac{\Omega}{c}\right]\right\}$$

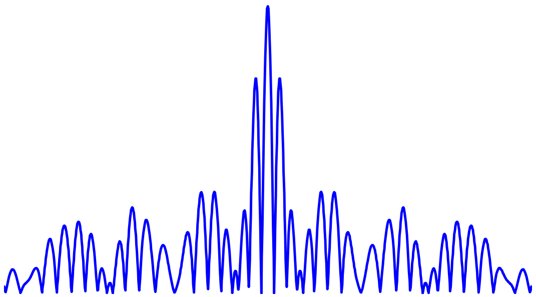
$$\approx \tilde{E}_1(\Omega)\exp\left\{-i\left[\left(k_0 - \frac{\Omega}{c}\right)L + k_0' L(\Omega-\omega_0) + \frac{k_0'' L}{2}(\Omega-\omega_0)^2\right]\right\}$$

Where we have made the standard truncated Taylor expansion for $k(\Omega)$.

Michelson Interferometer IV

Time domain interferograms

Works equally well for white light or fs pulses



Note: white light interference fringes are hard to observe due to stringent pathlength matching requirements

1) Take interferogram for balanced interferometer

2) Fourier transform to get $|E_1(\Omega)|^2$

3) Take interferogram with dispersive material

4) Take Fourier transform and divide by $|E_1(\Omega)|^2$, yielding

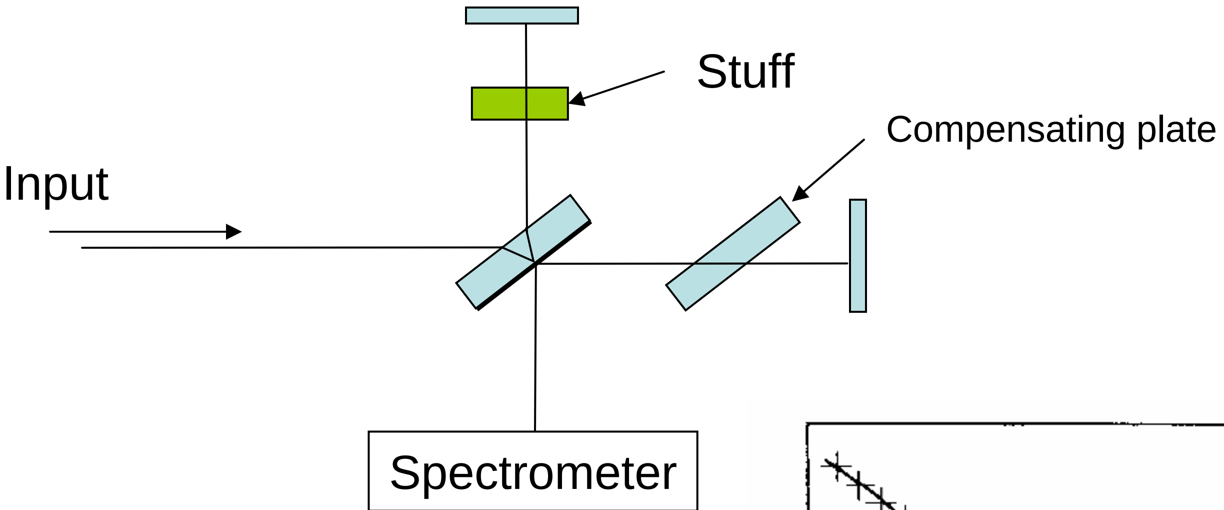
GVD

$$\exp\left\{-i\left[\left(k_0 - \frac{\Omega}{c}\right)L + k_0' L(\Omega - \omega_0) + \frac{k_0'' L}{2}(\Omega - \omega_0)^2\right]\right\}$$

These two terms require knowledge of L to fraction of λ, generally not meaningful (phase shift, group delay) or interesting

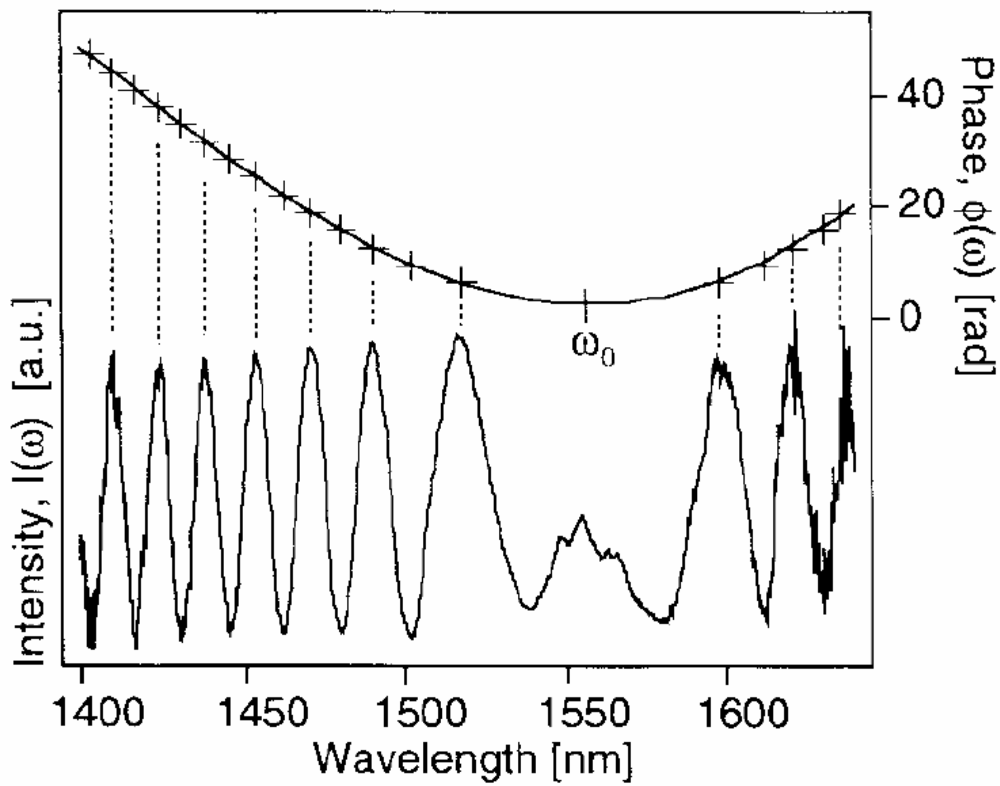
Michelson Interferometer V

Spectral interferometry



Adjust arm lengths so they match at zero GVD wavelength

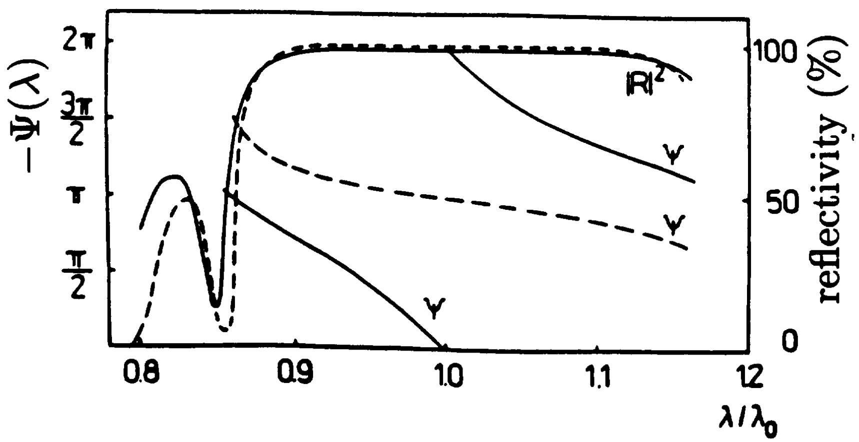
Fringes → pathlength difference depends on wavelength, directly read dispersion



Dispersion of Optical Elements

Mirror dispersion

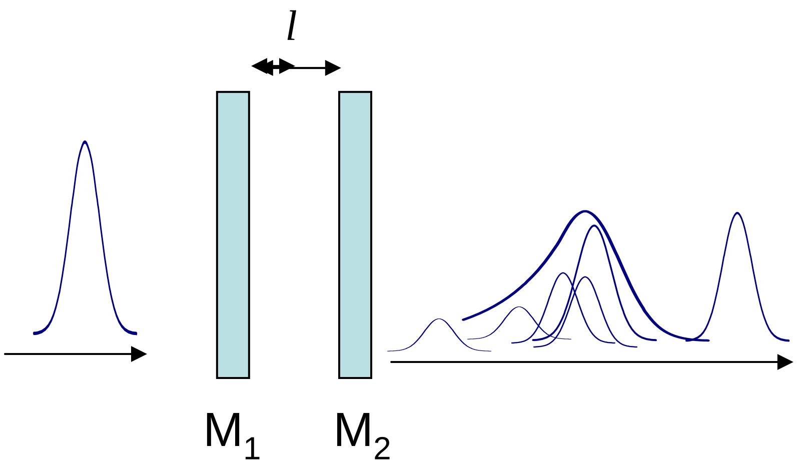
Mirrors generally have some dispersion, particularly dielectric stacks



Although the reflectivity spectrum may be flat in the region of interest, Kramers-Kronig makes the phase sensitive to spectrally remote resonant structures

Measure mirror dispersion using Michelson interferometer – replace one mirror with unknown mirror.

Fabry-Perot interferometer



Complex transfer function:

$$\tilde{H}(\Omega) = \frac{t_1 t_2 e^{i\delta/2}}{1 - r_1 r_2 e^{i\delta}}$$

$$\delta = -2\frac{\Omega n(\Omega) l}{c}\cos(\theta)$$

t_i and r_i are the field transmission and reflection coefficients of mirror M_i, respectively

θ is the angle of incidence

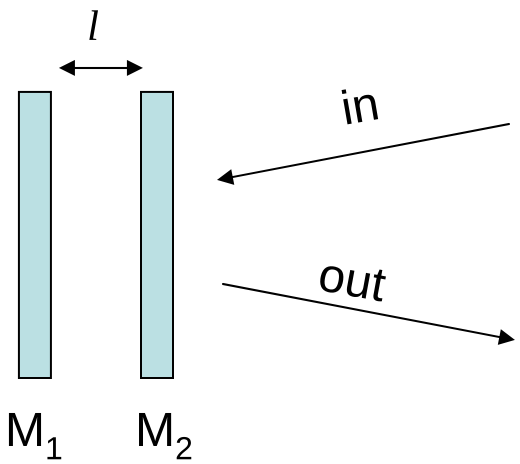
Case 1: $l > ct$

single pulse turns into train of (non-interfering) pulses
free spectral range < spectral width
spectrum is comb spaced by free spectral range
under envelope of single pulse

Case 2: $l < ct$

interference, single distorted pulse is transmitted
free spectral range > spectral width
spectrum is incident times shape of single resonance

Gires-Tournois interferometer



Special case of Fabry Perot for dispersion compensation:

$r_1 \sim 1$

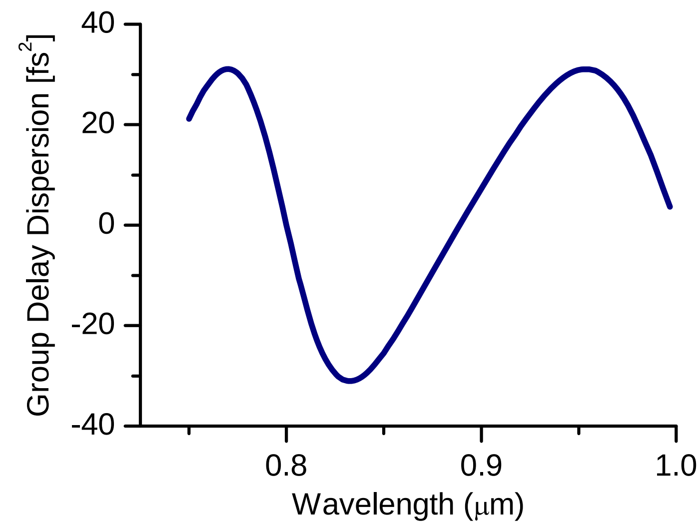
r_2 small

$l \sim \lambda$

Used in reflection, total reflection coefficient ~ 1
Dispersion due to interference with partial wave from M_2

$$\left.\frac{d^2}{d\Omega^2}\phi(\Omega)\right|_{\omega_0} = \frac{2r\left(r^2-1\right)\sin\delta}{\left(1+r-2r\cos\delta\right)}\left(\left.\frac{d\delta}{d\Omega}\right|_{\omega_0}\right)^2$$

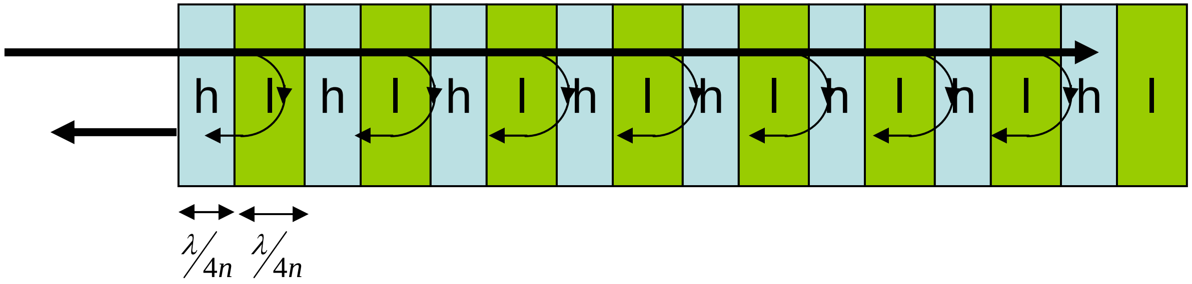
It is possible to achieve anomalous GVD
limited wavelength region
depends on l



J. Desbois, F. Gires and P. Tournois, IEEE J. Quantum Electr. **9**, 213 (1973).

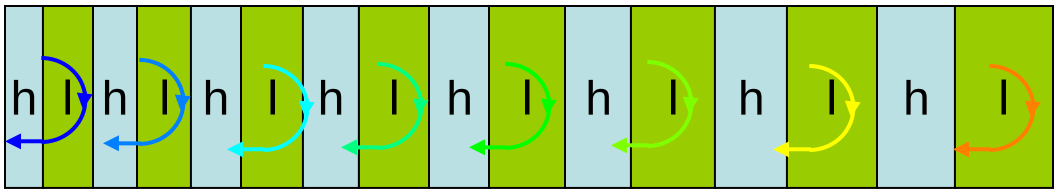
Dispersion compensating dielectric mirrors

Engineer the dispersion of dielectric mirrors.
A simple dielectric mirror consists of alternating layers of high and low index, each layer is ¼ λ thick



Constructive interference of partial wave from each h-l pair results in high total reflection

Anomalous dispersion: slow down red light
➔why not make red light reflect deeper in stack?



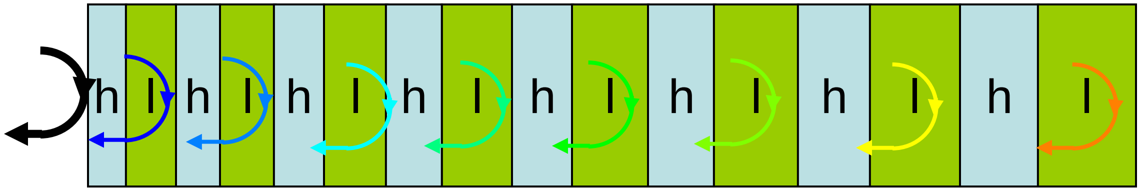
"Chirped mirror"
additional advantage: much broader reflection bandwidth

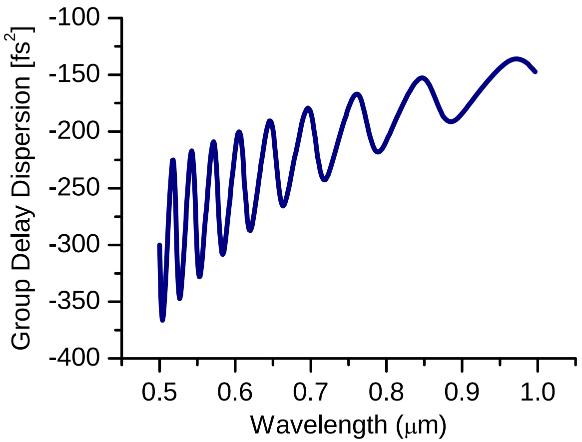
Chirped mirror

Problem with straight forward chirped mirror:

broadband strong reflection off of first interface

→ effective Gires-Tournois interferometer





Get overall anomalous GVD, but GTI effect results in undesirable "resonances"

Solutions:

1) Computer optimization of individual layers to minimize resonances
(difficult to make due to required precision of layers)

2) Reduce first surface reflection:

a) enter through wedged substrate

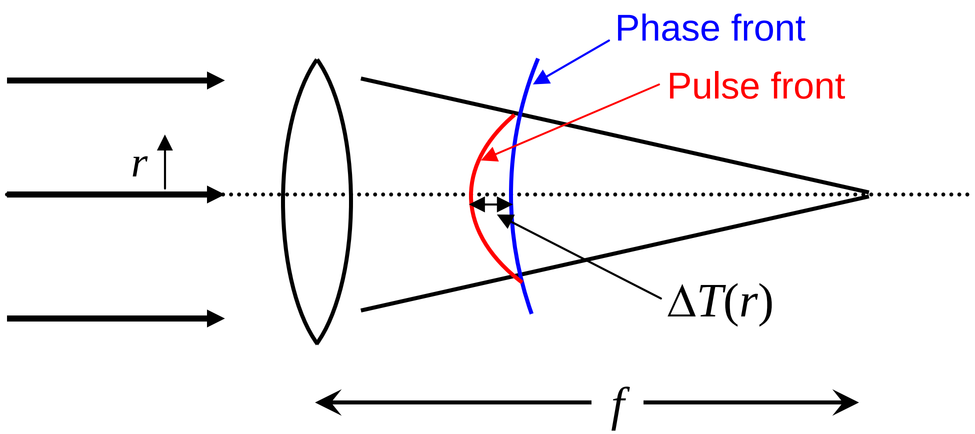
b) effective anti-reflection coating

3) Double chirped structure (thickness and period varies)

usually combined with 2a

Lenses I

Difference between phase and group velocity → pulse front does not match phase front



The delay between phase and pulse fronts depends on radius:

$$\Delta T(r) = \frac{r_0^2 - r^2}{2c} \lambda \frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

Estimate smearing in time for beam with radius r_b as difference between $\Delta T(0)$ and $\Delta T(r_b)$:

$$\Delta T(r_b) = \frac{r_b^2}{2c} \lambda \frac{d}{d\lambda}\left(\frac{1}{f}\right) = \frac{w^2}{2cf(n-1)} \lambda \frac{dn}{d\lambda}$$

Gaussian beam diameter w

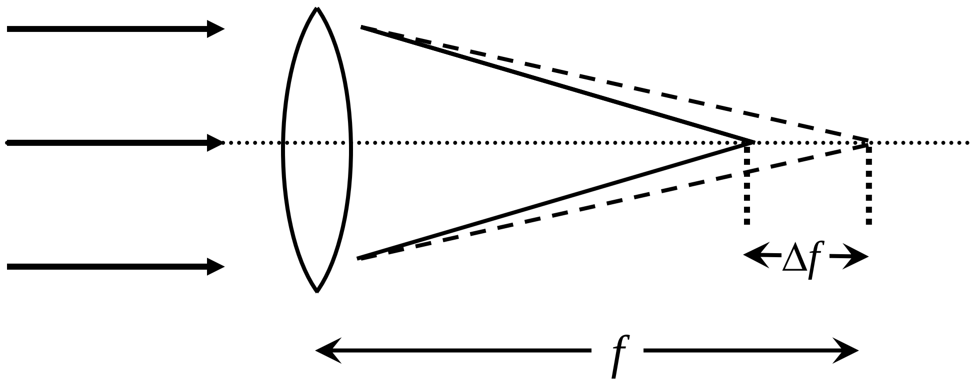
Numbers:

$\Delta T \sim 50$ fs for

$w = 1$ mm, $f = 5$ mm, $\lambda = 800$nm

Lenses II

Chromaticity of lens also affects spatial distribution



Deviation in focal length depends on λ, for Gaussian with duration τ

$$\Delta f = -\frac{f\,\lambda^2}{c(n-1)}\frac{0.441}{\tau}\frac{dn}{d\lambda}$$

Numbers:

$\Delta f \sim 100\ \mu\text{m}$ for

$\tau = 10$ fs, $f = 5$ mm, $\lambda = 800$nm

Lenses III

With Chromatic aberration

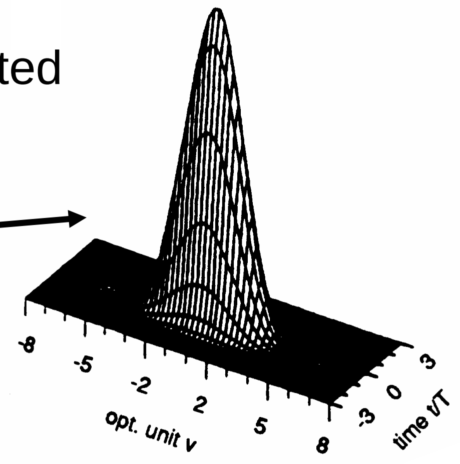
Previous expressions are all approximations

ray optics

ignoring spherical aberation

Including these, life is complicated

No aberration

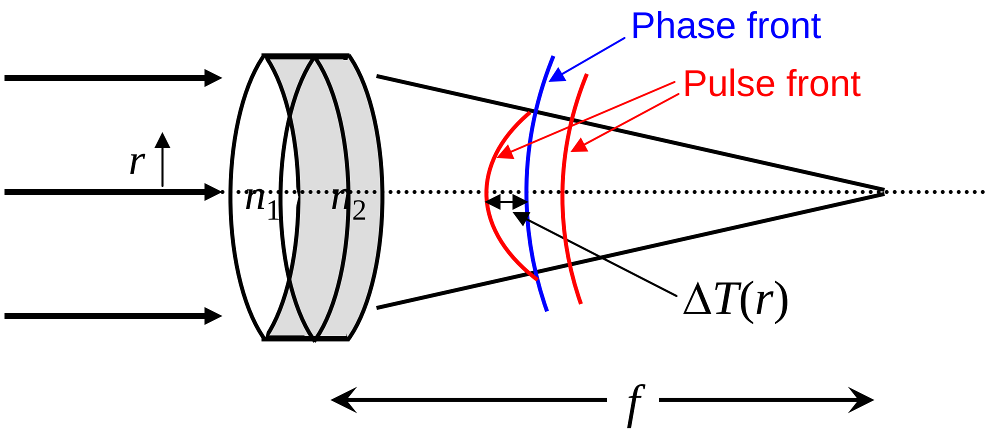


With both Chromatic and spherical aberration

Intensity distribution at focal plane

Doublet Lens

A properly designed achromatic doublet can eliminate temporal spread due to difference between phase and group velocities



$$T(r) = \frac{d_1}{c}\left\{n_1 - \lambda \frac{dn_1}{d\lambda}\right\} + \frac{d_2}{c}\left\{n_2 - \lambda \frac{dn_2}{d\lambda}\right\} + \frac{\lambda r^2}{2c}\frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

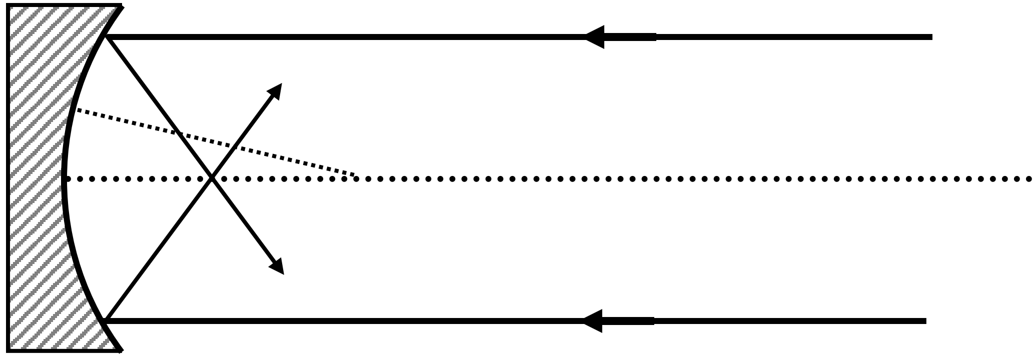
Design for right side equal to zero

then GVD is only source of broadening

But: thicker lens → GVD is worse

Focusing with Mirrors

Spherical mirror, perfectly achromatic



Temporal spreading only due to spherical aberration

$$\Delta T(r) = \frac{3}{4}\frac{R}{c}\left(\frac{r}{R}\right)^4$$

Numbers

f = 25 mm, beam diameter = 3 mm, $\Delta T = 1.6$ fs

f = 25 mm, beam diameter = 1 cm, $\Delta T = 200$ fs

Disadvantages:

focus inside incoming beam – off axis induces astimatism
first order sensitive to tip & tilt

Angular Dispersion

Angular dispersion and tilted pulses

Angular dispersion is useful:

- Generate tilted pulses
  - Tilted pulse: propagation direction not perpendicular to pulse front
  - Due to (1) difference between phase and group velocities or (2) Grating

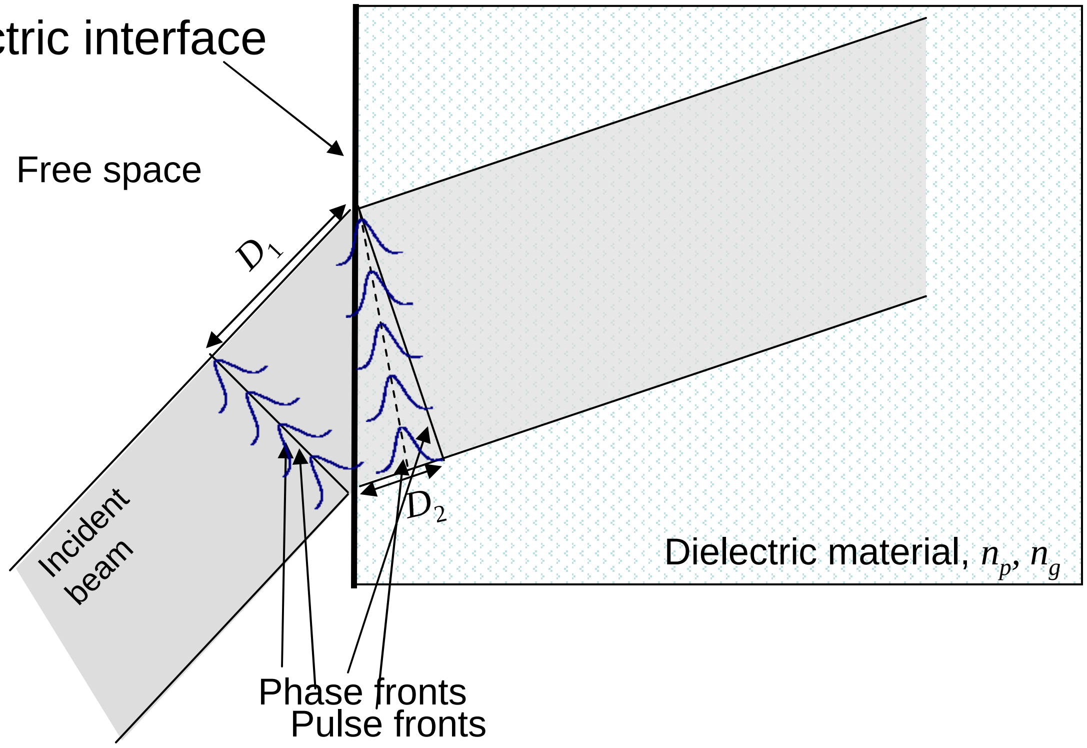
- "Decompose" pulse into frequency components
  - Allows individual frequency components to be manipulated
    - Generate anomalous GVD
    - Pulse shaping (later)
  - Due to phase index dispersion

Optical elements with angular dispersion include

- Simple dielectric interface
- Prism (e.g. two interfaces)
- Gratings

Tilted pulses due to a dielectric interface

Consider a simple dielectric interface



Phase fronts determined by time to propagate D_1 = time to propagate D_2

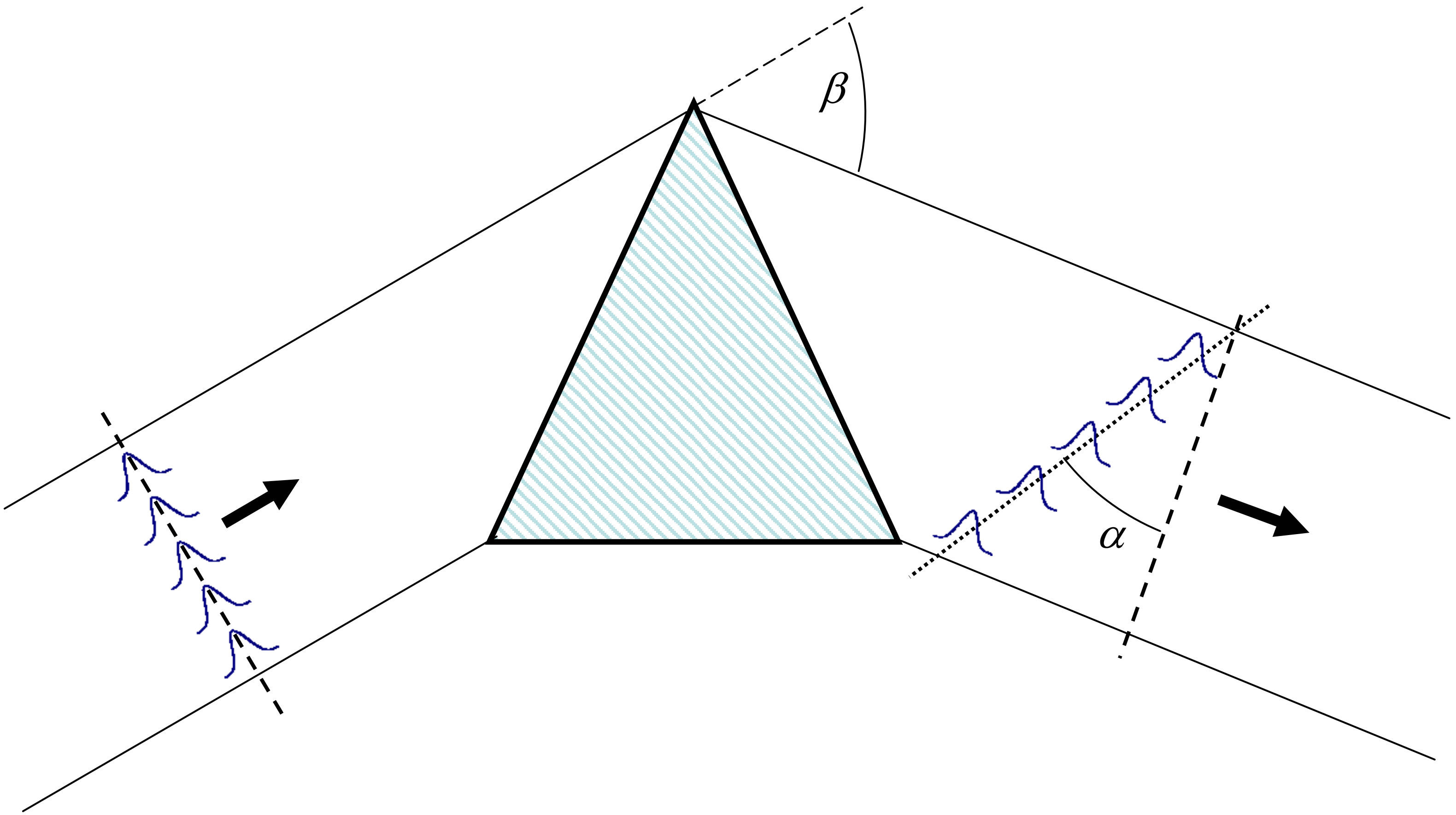
$$D_1 = n_p D_2$$

This determines the refraction angle.

However, the angle of the Pulse front is determined by the n_g (in a similar way)

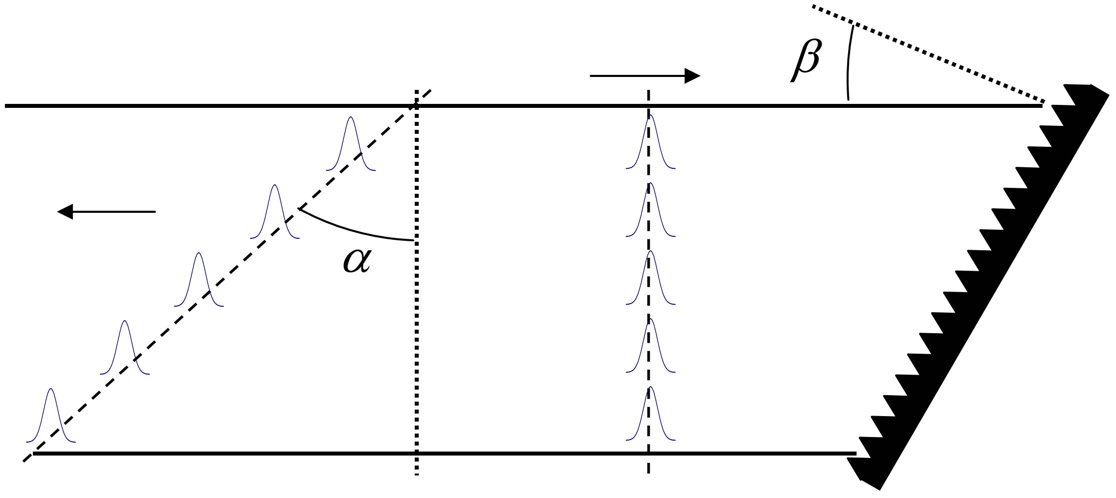
Tilted pulses due to a prism

Just two dielectric interfaces



Tilted pulses due to a grating

Simplest case to consider: -1 order diffraction counter propagates relative to incident beam

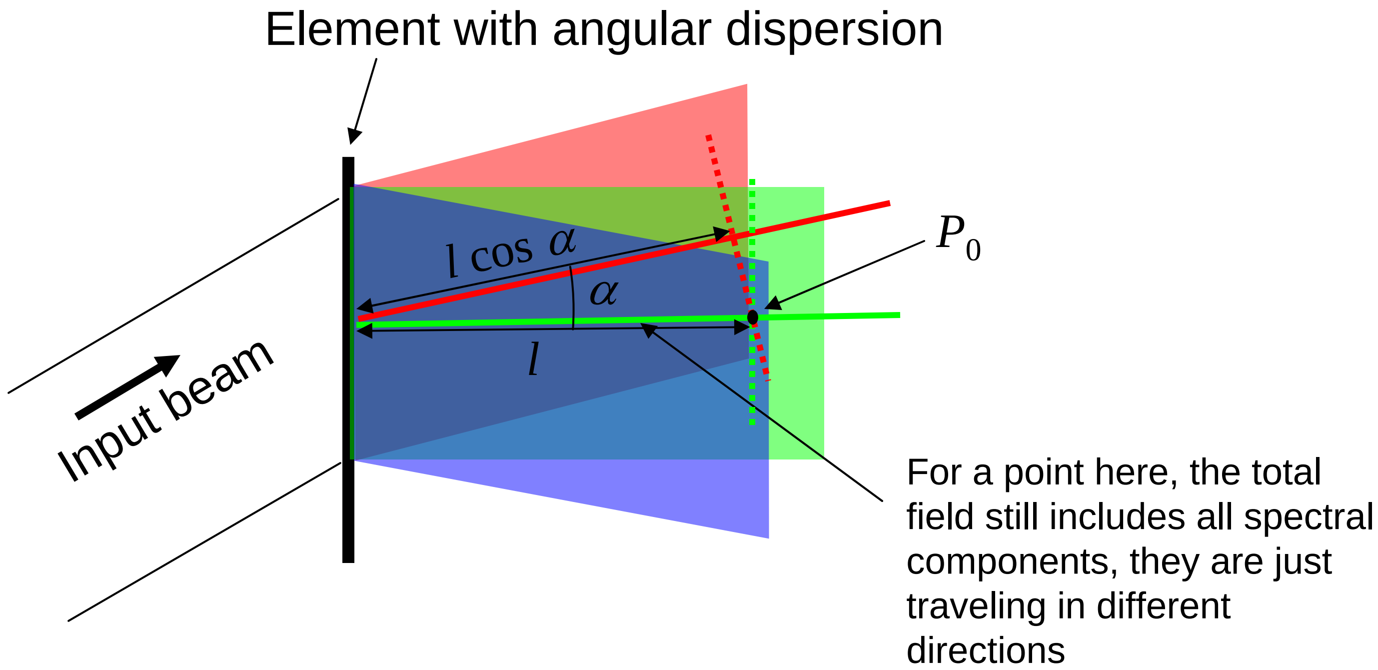


General relationship between angular dispersion and pulse front tilt:

$$|\tan\alpha| = \lambda \left|\frac{d\beta}{d\lambda}\right|$$

Geometric Anomalous Dispersion

Angular dispersion and GVD



Consider a point P_0 a distance l from the element
The phase delay of a frequency Ω that makes angle α is

$$\Psi(\Omega) = \frac{\Omega}{c} l \cos\alpha$$

GVD is the second derivative of phase:

$$\left.\frac{d^2\Psi}{d\Omega^2}\right|_{\omega_0} = -\frac{l}{c}\left\{\sin\alpha\left[2\frac{d\alpha}{d\Omega} + \Omega\frac{d^2\alpha}{d\Omega^2}\right] + \Omega\cos\alpha\left(\frac{d\alpha}{d\Omega}\right)^2\right\}\Bigg|_{\omega_0} \approx -\frac{l\omega_0}{c}\left(\left.\frac{d\alpha}{d\Omega}\right|_{\omega_0}\right)^2$$

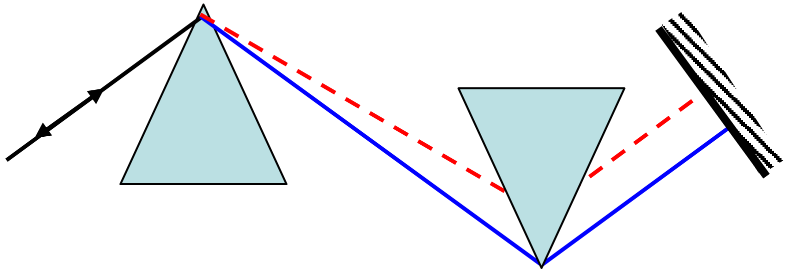
Note that the GVD is always negative, independent of the sign of $d\alpha/d\Omega$

GVD Compensation with Prisms I

Anomalous dispersion due to angular dispersion alone is not so useful as the pulse eventually “falls apart” in the transverse spatial dimension

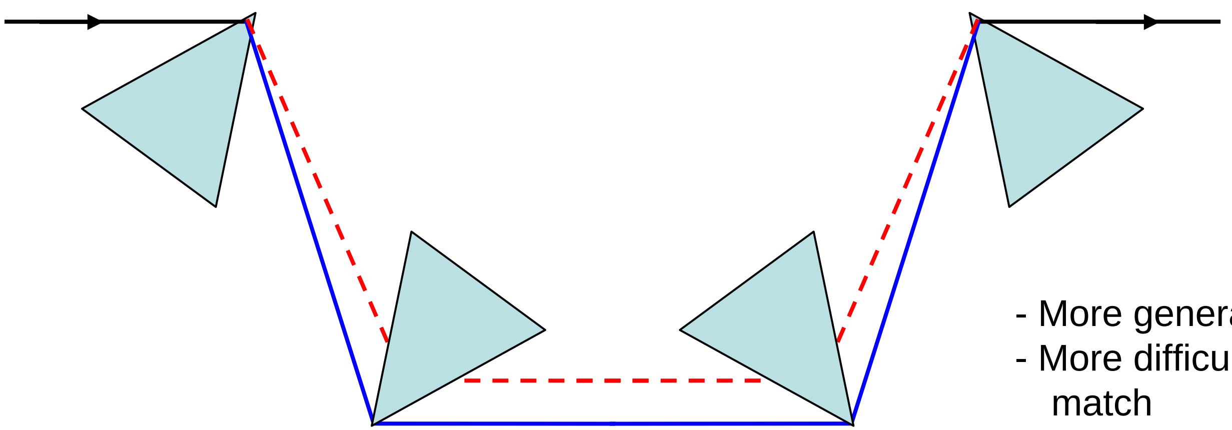
Fix this with multiple prisms:

1) Double pass a pair



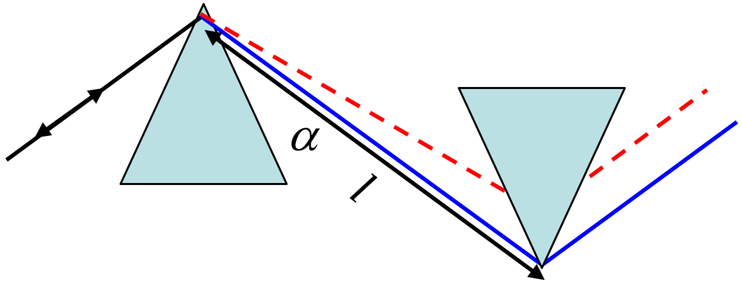
- Use directly at one end of a standing wave laser
- Offset beam vertically

2) Two pairs



- More general, usable in ring laser
- More difficult to align, prism pairs must match

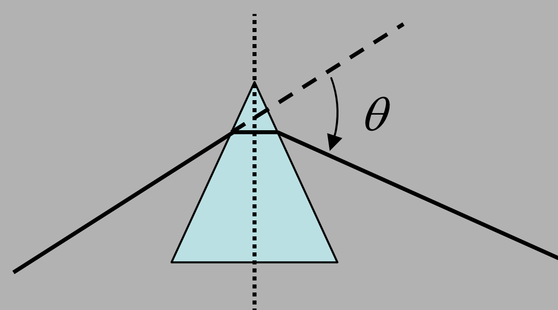
GVD Compensation with Prisms II



From ou

$$\frac{d\alpha}{d\Omega} =$$

Minimum Deviation



Orientation of prism that minimizes θ

- ray inside prism is perpendicular to bisector of apex angle
- choose apex angle such that incident ray is at Brewsters angle (depends on λ)
- Find by simply rotating prism, transmitted spot "turns around"

At minin

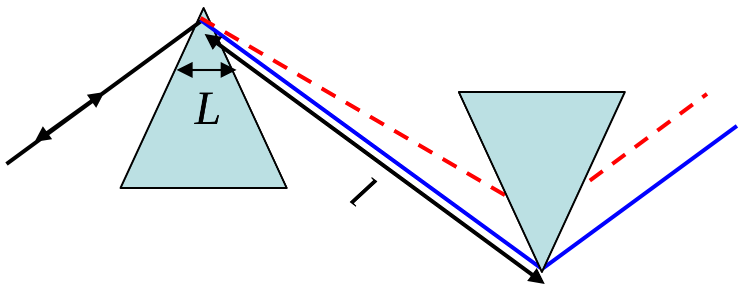
$$\frac{d\alpha}{dn} =$$

Thus

$$\left.\frac{d^2\Psi}{d\Omega^2}\right|_{\omega_0} \approx -4\frac{\omega_0}{c} l \left(\left.\frac{dn}{d\Omega}\right|_{\omega_0}\right)^2 = -4\frac{\lambda_0^3}{2\pi c^2} l \left(\left.\frac{dn}{d\lambda}\right|_{\lambda_0}\right)^2$$

→ Generates anomalous dispersion

GVD Compensation with Prisms III



In addition to angular dispersion, there is ordinary dispersion due to the glass, denote path length in glass L

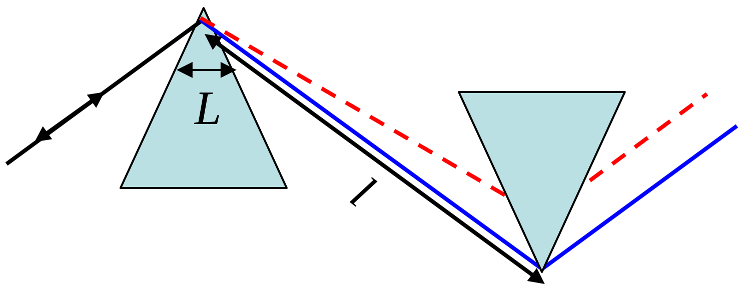
$$\Psi''_L(\omega_0) = \frac{\lambda_0^3}{2\pi c^2} L \frac{d^2 n}{d\lambda}\bigg|_{\lambda_0}$$

Thus the net GVD is

$$\Psi''_{tot}(\omega_0) = \frac{\lambda_0^3}{2\pi c^2}\left[Ln'' - 4l n'^2\right]$$

Material	For GVD = 0 at 800 nm, 1 mm beam diameter	
	l/L	l_0 (mm)
Fused Silica	16.7	136
BK7	15.6	128
SF10	8.4	70

GVD Compensation with Prisms IV



Third order angular dispersion:

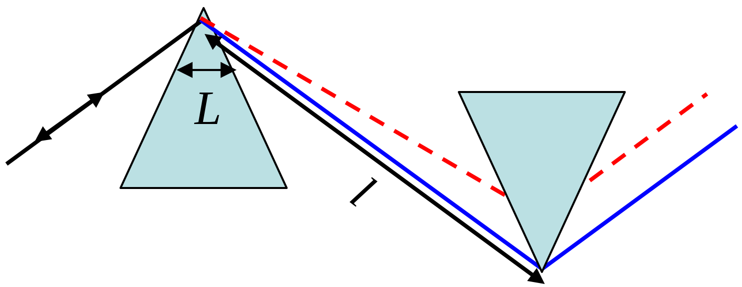
$$\Psi_L'''(\omega_0) \approx \frac{12 l \lambda_0^4}{(2\pi)^2 c^3} \left\{ \left(\frac{dn}{d\lambda} \right)^2 \left[1 - \lambda_0 \frac{dn}{d\lambda} \left(n^{-3} - 2n \right) \right] + \lambda_0 \left(\frac{dn}{d\lambda} \frac{d^2 n}{d\lambda^2} \right) \right\} \Bigg|_{\lambda_0}$$

Including material third order dispersion, net TOD is

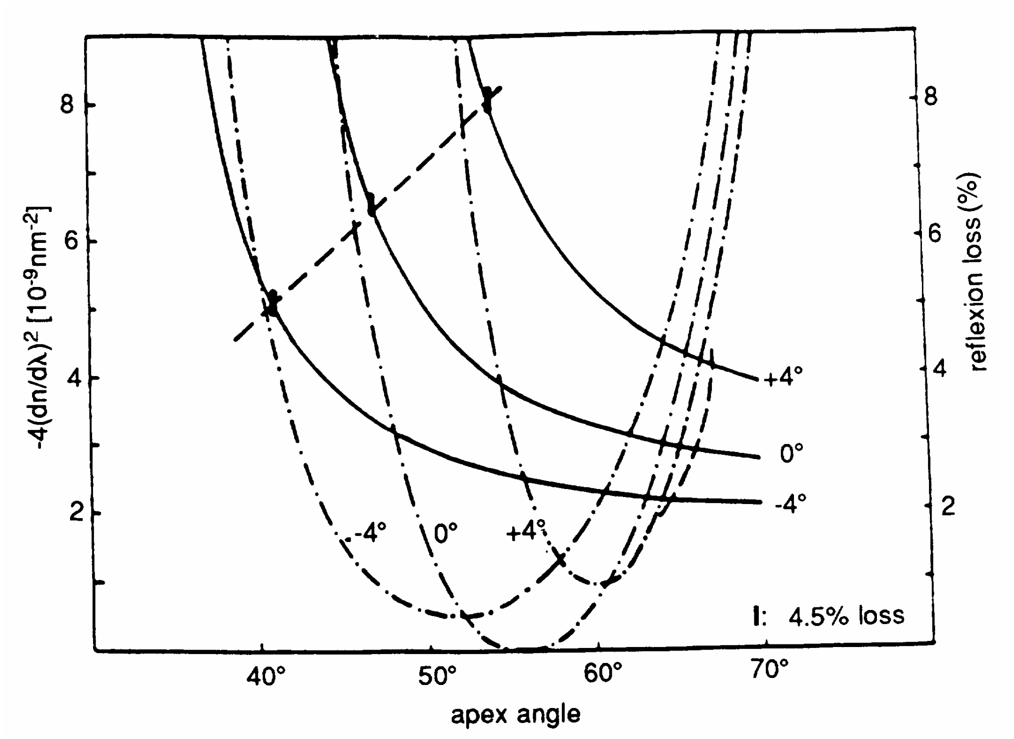
$$\Psi_{tot}'''(\omega_0) \approx \frac{\lambda_0^4}{(2\pi)^2 c^3} \left[12 l \left\{ n'^2 \left[1 - \lambda_0 n' \left(n^{-3} - 2n \right) \right] + \lambda_0 n' n'' \right\} - L \left(3n'' + \lambda_0 n''' \right) \right]$$

By appropriate choice of material and separation, it is possible to minimize (even eliminate) TOD, fused silica works around 800 nm

GVD Compensation with Prisms V

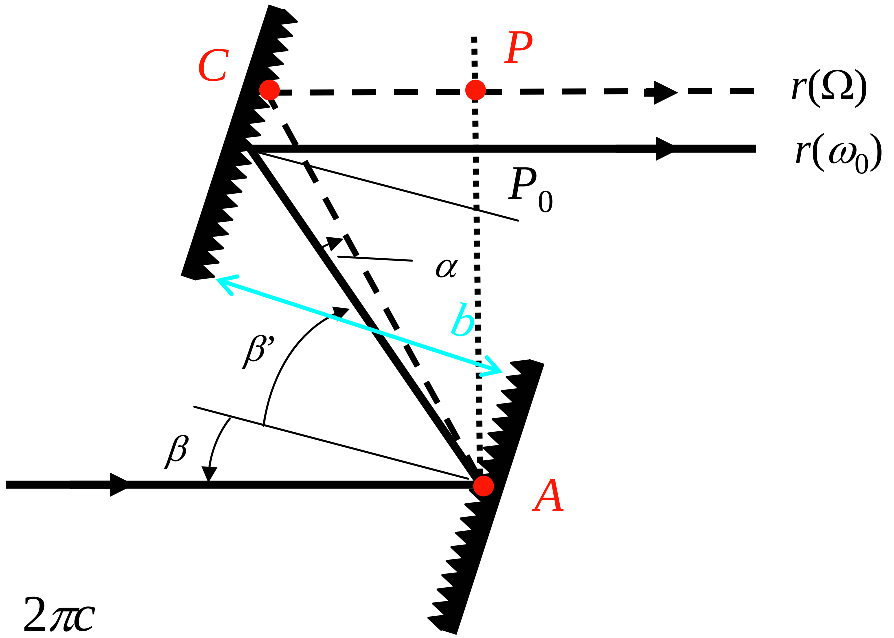


Tilting the prisms changes the GVD, however increasing anomalous GVD is accompanied by increasing TOD:



Ray tracing calculations

GVD Compensation with Gratings I



Optical path length (b - grating separation)

$$\overline{ACP} = \frac{b}{\cos(\beta' + \alpha)}\left[1 + \cos(\beta' + \beta + \alpha)\right]$$

Angles come from grating equation (d – grating spacing)

$$\sin\beta - \sin\beta' = \frac{2\pi c}{\omega_0 d} \qquad \sin\beta - \sin(\beta' + \alpha) = \frac{2\pi c}{\Omega d}$$

Yielding

$$\Psi(\Omega) = \frac{\Omega}{c}\overline{ACP}(\Omega) + 2\pi\frac{b}{d}\tan(\alpha + \beta') \quad \Rightarrow \quad \frac{d^2\Psi}{d\Omega^2} = -\frac{\lambda_0}{2\pi c^2}\left(\frac{\lambda_l}{d}\right)^2\frac{b}{\sqrt{r}}\frac{1}{r}$$

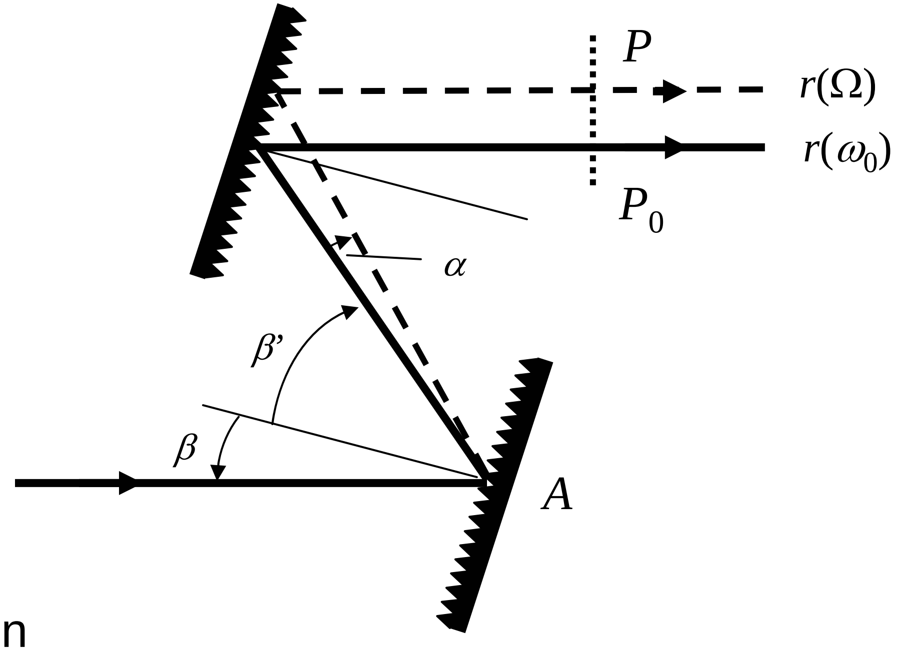
where

$$r = 1 - \left[\frac{2\pi c}{\omega_0 d} - \sin\beta\right]^2 = \cos^2\beta'$$

Anomalous dispersion

$\frac{b}{\sqrt{r}}$ is the distance between the gratings at ω_0

GVD Compensation with Gratings II



Third order dispersion

$$\frac{d^3\Psi}{d\Omega^3} = -\frac{3\lambda_0}{2\pi c r}\left[r + \frac{\lambda_0}{d}\left(\frac{\lambda_0}{d} - \sin\beta\right)\right]\frac{d^2\Psi}{d\Omega^2}\bigg|_{\omega_0}$$

Note: generally positive

Tuning the ratio of 2nd to 3rd order dispersion can be done by changing grating constant and distance between them
→ more difficult than prisms

Comparison: Material, Prisms, Gratings

@800 nm

Object	GVD [fs^2]	TOD [fs^3]
1 cm fused silica	362	280
Prism pair l = 50 cm	-523	-612
Grating pair b = 20 cm, $\beta = 0^o$, d = 1.2 μm	-3 x 10^6	6.8 x 10^6

Prisms:

Small amounts of GVD and TOD, comparable to material
Low loss (Brewsters angle)
Easily adjustable (by insertion, no alignment change)

Combination of prisms & gratings can compensate both GVD and TOD

Gratings:

Large amounts of GVD and TOD
Lossy (~80%)
Not easily adjustable (changes alignment), but more parameters (b, β, d)