ASTR 3730: Problem Set 2
(due Tuesday September 20th)

(1) The collapse of the core of a massive star in a Type 2 supernova results in the formation of a neutron star and the release of large numbers of neutrinos. Neutrinos have a very small cross-section for interaction with matter, but the high densities encountered during stellar collapse mean that there can be circumstances where the neutrinos become temporarily trapped before they scatter and escape.

(i) Consider a static star of mass M, radius r, made up of particles with mass m_n. If the cross-section for interaction between neutrinos and matter is σ, derive an expression for the radius of a star that is just optically thick to neutrino emission from the core.
(ii) Evaluate this radius for a star of mass $M = 1.4\ M_{Sun}$, assuming a neutrino-matter cross-section $\sigma = 10^{-44}\ cm^2$.

(2) Suppose some impulsive process releases a burst of radiation (photons or neutrinos) at the center of a star of radius r. The optical depth between the surface and the center is $\tau >> 1$. Estimate the characteristic time scale of the burst of radiation that would be seen by an observer, after the radiation has diffusively propagated to the surface.

(3) Consider a small, optically thin cloud of gas that lies close to a nearby, luminous point source of radiation. The gas has opacity κ, which we will take to be independent of frequency, while the luminous source has mass M and luminosity L. Suppose that the luminosity *exceeds* the Eddington limit, so that the action of radiation will eject the cloud. If the cloud starts at radius r, and is initially at rest, find an expression for the terminal velocity v that the cloud will have when it is very far from the luminous source.

(4) We will be interested later in radiation transport within stars. Show that in a spherical co-ordinate system, with the center of the star at the origin, the transfer equation can be written in the form,

$$\frac{\cos\theta}{\kappa_\nu \rho} \frac{dI_\nu}{dr} = S_\nu - I_\nu$$

where θ is the angle made between the direction of a ray and the outward radial direction.